


2.2 Normal Calculations: Do you have a VOI, graph, and context for your answer????

#1) Serving Speed

In the 2008 Wimbledon tennis tournament, Rafael Nadal averaged 115 miles per hour (mph) on his first serves. Assume that the distribution of his first serve speeds is Normal with a mean of 115 mph and a standard deviation of 6 mph.

(a) About what proportion of his first serves would you expect to be slower than 103 mph?


$X < 103 \text{ mph}$
 $Z < \frac{103 - 115}{6}$
 $Z < -2$



The proportion of serves slower than 103 mph is .0228.
 .0228

(b) About what proportion of his first serves would you expect to exceed 120 mph?


$X > 120 \text{ mph}$
 $Z > \frac{120 - 115}{6}$
 $Z > .83$



The proportion of serves exceeding 120 mph is .2033.
 1 - .7967
 .2033

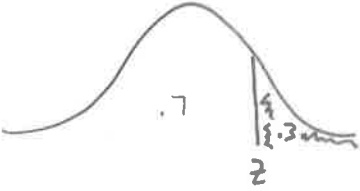
(c) What percent of Rafael Nadal's first serves are between 100 and 110 mph?

$100 \text{ mph} < X < 110 \text{ mph}$
 $\frac{100 - 115}{6} < Z < \frac{110 - 115}{6}$
 $-2.50 < Z < -.83$



19.71% of Nadal's serves are between 100 + 110 mph.
 $.2033 - .0062$
 $.1971$


(d) The fastest 30% of Nadal's first serves go at least what speed?



$\text{invNorm}(\text{area} = .7, \mu = 0, \sigma = 1)$
 $Z > .52$
 $\frac{X - 115}{6} > .52$
 $X > 118.12 \text{ mph}$

The fastest 30% of Nadal's serves are at least 118.12 mph.

(e) A different player has a standard deviation of 8 mph on his first serves and 20% of his serves go less than 100 mph. If the distribution of his serve speeds is approximately Normal, what is his average first serve speed?



$\text{invNorm}(\text{area} = .2, \mu = 0, \sigma = 1)$
 $Z = -.84$
 $\frac{100 - \mu}{8} = -.84$
 $\mu = 106.72 \text{ mph}$

His average serve speed is 106.72 mph

#2) **Baseball**

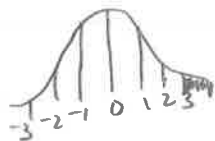
Suppose that Clayton Kershaw of the Los Angeles Dodgers throws his fastball with a mean velocity of 94 miles per hour (mph) and a standard deviation of 2 mph and that the distribution of his fastball speeds can be modeled by a Normal distribution.

(a) About what proportion of his fastballs will travel at least 100 mph?

$$X > 100 \text{ mph}$$

$$z > \frac{100 - 94}{2}$$

$$z > 3$$



$$1 - .9987 = .0013$$

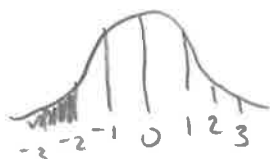
The proportion of fastballs that travel at least 100 mph is .0013.

(b) About what proportion of his fastballs will travel less than 90 mph?

$$X < 90 \text{ mph}$$

$$z < \frac{90 - 94}{2}$$

$$z < -2$$



$$.0228$$

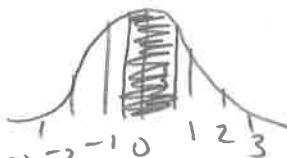
The proportion of fastballs that travel less than 90 mph is .0228.

(c) About what proportion of his fastballs will travel between 93 and 95 mph?

$$93 \text{ mph} < X < 95 \text{ mph}$$

$$\frac{93 - 94}{2} < z < \frac{95 - 94}{2}$$

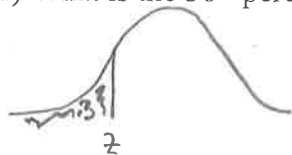
$$-.50 < z < .50$$



$$.6915 - .3085 = .383$$

The proportion of fastballs traveling between 93 and 95 mph is .383.

(d) What is the 30th percentile of Kershaw's distribution of fastball velocities?



$$\text{invNorm}(\text{area} = .3, \mu = 0, \sigma = 1)$$

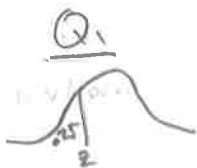
$$z = -.52$$

$$-.52 = \frac{X - 94}{2}$$

$$X = 92.93 \text{ mph}$$

A fastball at 92.93 mph is at the 30th percentile.

(e) What fastball velocities would be considered low outliers for Kershaw?



$$\text{invNorm}(\text{area} = .25, \mu = 0, \sigma = 1)$$

$$z = -.67$$

$$-.67 = \frac{X - 94}{2}$$

$$X = 92.66 \text{ mph for } Q_1$$

$$Q_3$$

$$.67 = \frac{X - 94}{2}$$

$$X = 95.34 \text{ mph for } Q_3$$



$$\text{invNorm}(\text{area} = .75, \mu = 0, \sigma = 1)$$

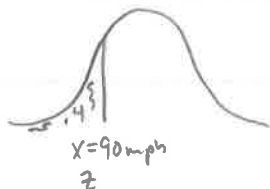
$$z = .67$$

$$IQR = 95.34 - 92.66 = 2.68$$

$$92.66 - 1.5(2.68) = 88.64 \text{ mph}$$

Any fastball below 88.64 mph would be a low outlier.

(f) Suppose that a different pitcher's fastballs have a mean velocity of 92 mph and 40% of his fastballs go less than 90 mph. What is his standard deviation of his fastball velocities, assuming his distribution of velocities can be modeled by a Normal distribution?



$$\text{invNorm}(\text{area} = .4, \mu = 0, \sigma = 1)$$

$$z = -.25$$

$$-.25 = \frac{90 - 92}{\sigma}$$

$$\sigma = 8 \text{ mph}$$

The standard deviation of his fastball is 8 mph.