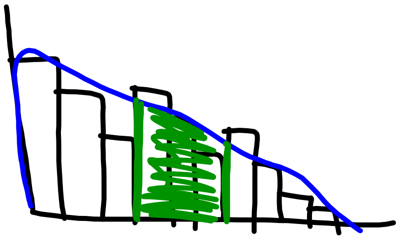
2.2 Normal Distribution

A density curve is always on or above the horizontal axis and always has an area equal to 1. It approximates the overall pattern of a distribution.

Ex:

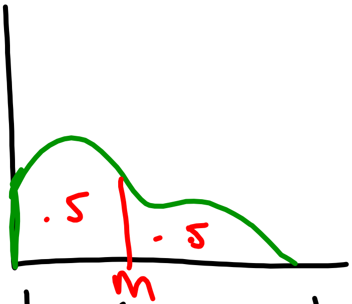


The area under the curve is the proportion of the area of the observations that fall in the given interval.

Describing Density Curves

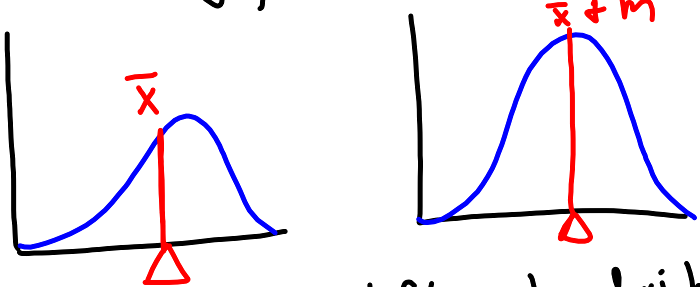
Median: divides the area under the curve in half.

Ex:



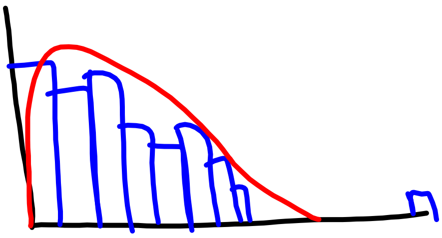
Mean: Balancing point of the curve.

Ex:

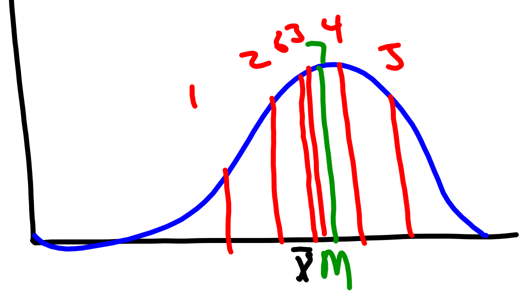


Shape: symmetric, skewed left or skewed right, but we don't plot outliers.

Ex:

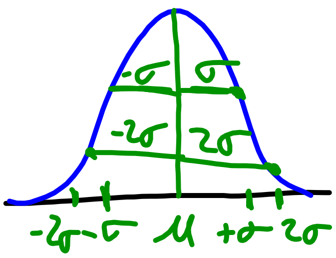


Ex: Determine which line represents the median and the mean.

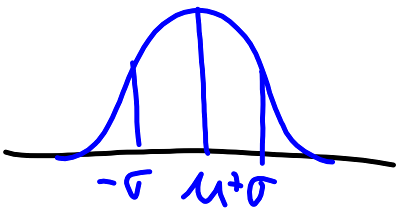


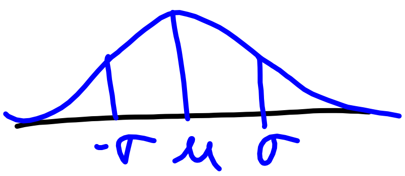
Normal Distribution: Is described by the Normal Density curve. It is completely described by its mean, μ, and by its standard deviation, σ. The mean is centered in a symmetric Normal curve, while the standard deviation is the distance from μ to the change of curvature on either side.

Ex.



- This is the Normal Distribution with mean, μ, and standard deviation, σ.
- $N(\mu, \sigma)$



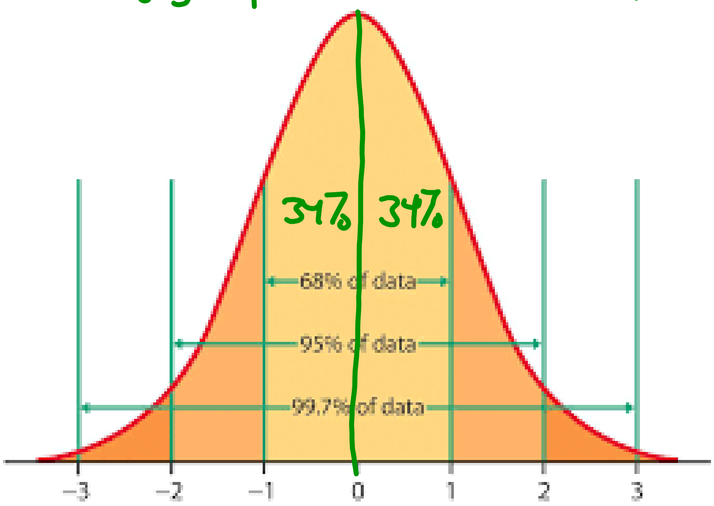


<u>68-95-99.7 rule</u> : In a $N(\mu, \sigma)$:

$\approx$ 68% of the observations fall within 1σ of the mean.

$\approx$ 95% of the observations fall within 2σ of the mean

$\approx$ 99.7% of the observation fall within 3σ of the mean



p.112 check your understanding

Standard Normal Distribution

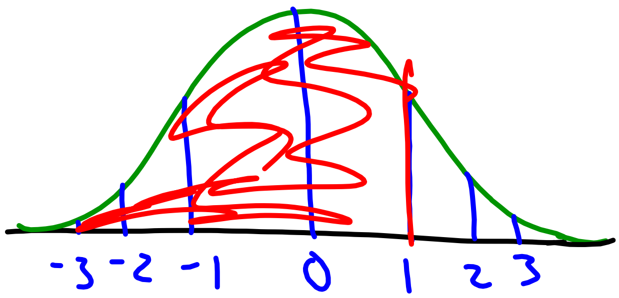
All Normal distributions are the same if we measure units of σ from the μ.

We can standardize the observations using

$$z = \frac{x - \mu}{\sigma}$$

If the observations followed a Normal Distribution so do the standardized scores of the observations.

Furthermore, it will have a $\mu = 0$ and $\sigma = 1$ so it will have $N(0,1)$.

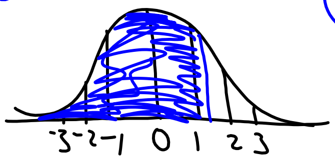


By standardizing values, this allows us to find the area under the curve for any interval using the standard Normal table (Table A)

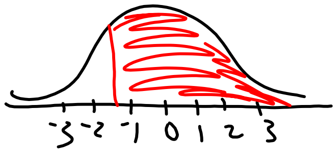
Normal Distribution Calculations

1. Draw a picture of a labeled $N(\mu, \sigma)$, shaded area, and identified variables of interest.
2. Perform any calculations if necessary using table A or technology.
3. Answer the question.

Ex: Find $z < 1.39 = .9177$

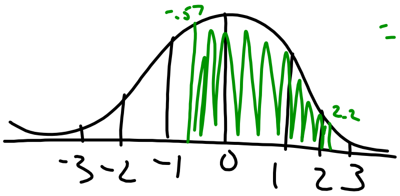


Ex: Find $z > -1.32 = 1 - .0934 = \boxed{.9066}$

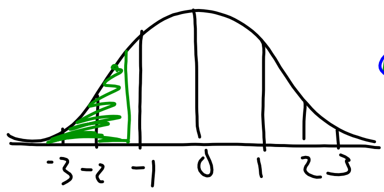


Ex: Find $-.57 < z < 2.2 = .9861 - .2843$

$= \boxed{.7018}$



Ex: Find the z-Score corresponding to the 15th percentile

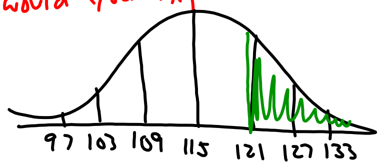


z

$\boxed{-1.04 \rightarrow .1492}$

$-1.03 \rightarrow .1515$

Ex: In a recent tennis tournament, Rafael Nadal, averaged 115 mph on his serve. Assume serve speed follows N(115, 6). What percent of his serves would you expect to exceed 120 mph?



$X > 120$

$z = \frac{X - \mu}{\sigma}$

$z = \frac{120 - 115}{6} = .83$

$z > .83 = 1 - .7967$

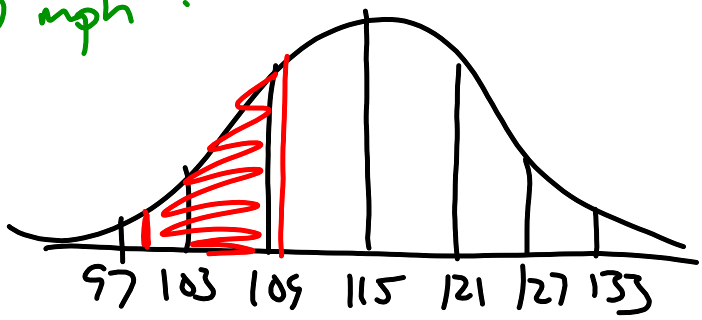
$= .2033$

Rafael Nadal hits 20.33% of serves over 120 mph.

On my TI calc, I used normalcdf (.83 lower bound, 1000 upper bound, 0 = mean, 1 = standard deviation)

= .2033

Ex: What proportion of serves are between 100 and 110 mph?



$100 \leq X \leq 110$

on my TI calc, ~~[illegible]~~

normalcdf(100 lower bound, 110 upper bound, $\mu = 115$, $\sigma = 6$) = .1961

Rafael hits 19.61% of his serves between 100 & 110 mph.

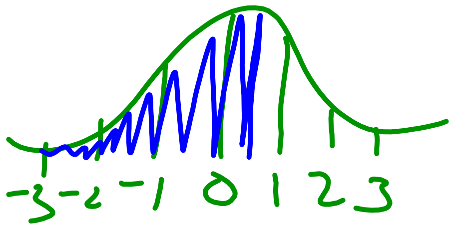
what is the Q_3 of Nadal's serving speed?

75th percentile

on my TI calc, using invNorm(area=.75, μ=0, σ=1)

=.6745

$$z = \frac{x-\mu}{\sigma}$$



$$.6745 = \frac{x-115}{6}$$

$x = 119.047$ mph

Rafael's 3rd quartile serve speed is 119.047 mph

Normal Probability Plot

p.124 Guinea Pig Survival