# 5.3 Conditional Probability + Independence

| | Not Female | Female | Total |
|---|---|---|---|
| Blue | 7 | 5 | 12 |
| Not Blue | 10 | 11 | 21 |
| Total | 17 | 16 | 33 |

Ex:

P(female given blue eyes)

$\frac{5}{12} = .42$

P(blue eyes | female)

P(blue eyes given female)

$\frac{5}{16} = .31$

these probabilities are known as <u>conditional probabilities</u>: The probability that one event happens give that another event has already happened.

Notation: Suppose we know event A has already happened. Then the probability that event B happens given A has already occurred is: $P(B|A)$ ← given

## Calculating Conditional Probabilities

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$ ← AP Test

$$P(B|A) = \frac{P(B \cap A)}{P(A)} = \frac{P(\text{Both events occurs})}{P(\text{given event occur})}$$

Ex:

| | cell | no cell | total |
|---|---|---|---|
| land line | .51 | .09 | .6 |
| no land line | .38 | .02 | .4 |
| total | .89 | .11 | 1 |

$P(\text{cell} \mid \text{landline})$

$$\frac{.51}{.6} = .85$$

$P(\text{landline} \mid \text{no cell})$

$$\frac{.09}{.11} = .82$$

1) $P(L) = P(\text{lower than a B}) = \frac{3656}{10000} = .3656$

2) $P(E|L) = P(\text{EPS classes given } \underline{\text{lower than a B}})$

$= \frac{800}{3656} = .22$

$P(L|E) = P(\text{lower than B give EPS classes}) = \frac{800}{1600} = .5$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \cap B)$$

## General multiplication rule

Tells us how to find the probability of both events happening.

$$P(A \text{ and } B) : P(A)\,\frac{P(B\cap A)}{P(A)} = P(B|A)\cdot P(A)$$

$$\boxed{P(A\cap B) = P(B|A)\cdot P(A)}$$

This rule does imply that one event happens first and then another event must happen.

Ex: About 55% of high school students participate in athletics, and 5% of those students play at college. What is the probability a student plays athletics in high school and college?

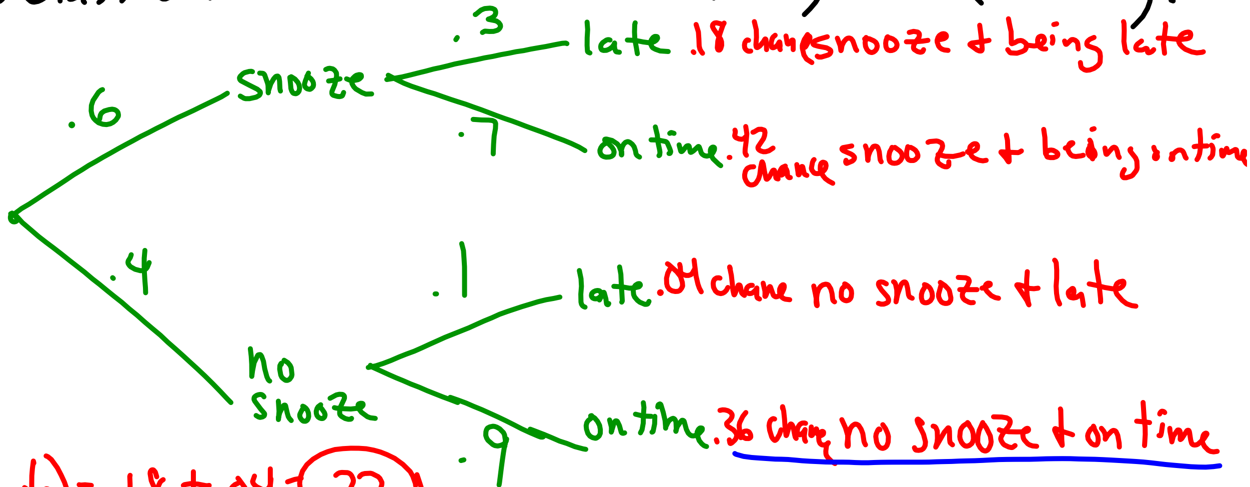
$P(\text{athletics in high school}) = .55$

$P(\text{college} \mid \text{high school}) = .05$

$P(\text{Both}) = (.55)(.05) = .0275$

The general multiplication rule is useful when a chance process involves a sequence of outcomes. We can use a <u>tree diagram</u> to display the sample space + probabilities.

Ex: Beth hits the snooze bar on her alarm clock on 60% of school days. If she doesn't hit the snooze bar, there is a .9 probability that she is on time to class. However, if she hits the snooze bar there is a 7/10 chance she makes it to class on time. What is P(late) + P(on time).



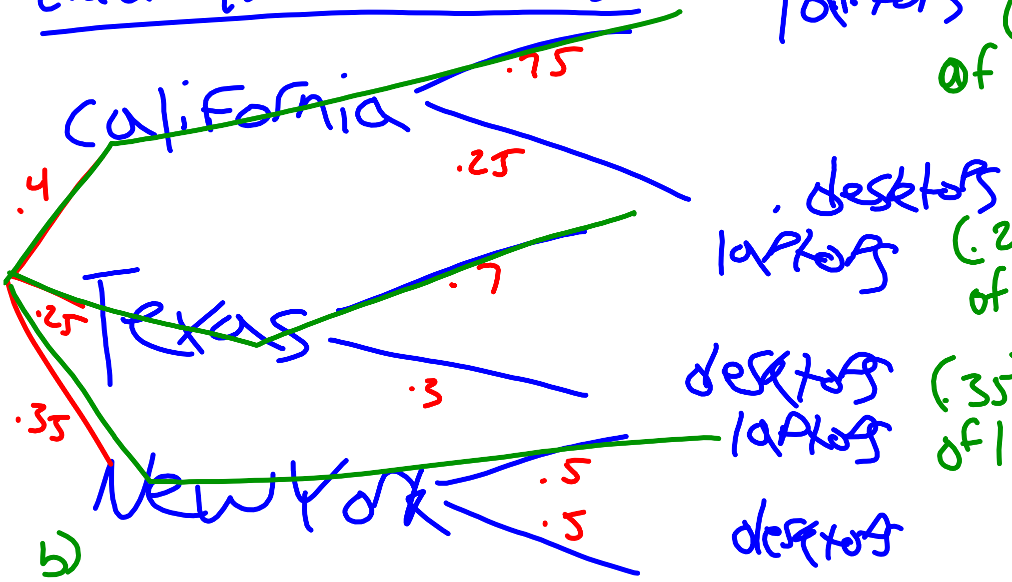

$P(\text{late}) = .18 + .04 = .22$

$P(\text{on time}) = 1 - P(\text{late}) = 1 - .22 = .78$

Suppose Beth is late for school. What is the probability she hit the snooze bar?

$$P(\text{snooze} \mid \text{late}) = \frac{P(\text{Both})}{P(\text{late})} = \frac{.18}{.22} = .82$$

check your understanding




$(.4)(.75) = .3$ prob. of laptop in cal.

$(.25)(.7) = .175$ prob of laptop in Texas

$(.35)(.5) = .175$ prob. of laptop in NY

b)

$P(\text{laptop}) = .175 + .175 + .3 = .65$

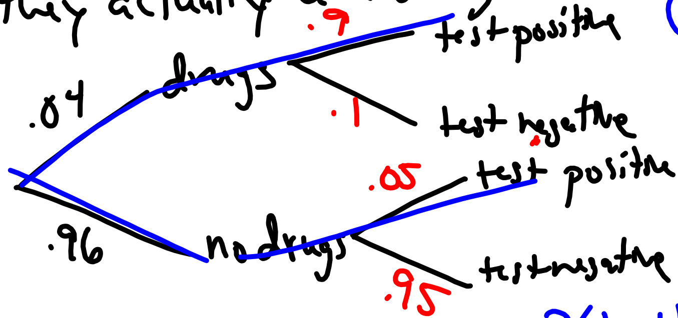
c) $P(\text{cal.} \mid \text{laptop}) = \dfrac{P(\text{cal. and laptop})}{P(\text{laptop})} = \dfrac{.3}{.65} = .46$

## False Positives & False Negatives

False Positive: suggest that one has a quality when they actually don't.

False Negative: suggests that one does not have a quality when they actually do.

Ex: Many employers require prospective employees to take a drug test. A positive result indicates the prospective employee uses illegal drugs. However, not all people who test positive actually use drugs. Suppose 4% of prospective use drugs. The false positive rate is 5% and the false negative rate is 10%. If a randomly selected prospective employee tests positive, what is the probability they actually use drugs?



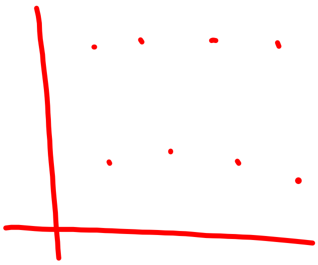

$(.04)(.9) = .036$ prob of taking drugs + testing positive

$(.96)(.05) = .048$ prob of not taking drugs + testing positive.

$$P(\text{drugs} \mid \text{test positive}) = \frac{P(\text{both})}{P(\text{positive test})} = \frac{.036}{.036 + .048} = .43$$

Independent Events : Two events A + B, are independent if the occurance of one does not change the probability that the 2nd will happen.

$$P(A|B) = P(A) \quad \text{and} \quad P(B|A) = P(B)$$



We can compute probabilities to determine if events are independent or not.

| | Female | Male | Total |
|---|---|---|---|
| index finger | 78 | 45 | 123 |
| ring finger | 82 | 152 | 234 |
| same length | 52 | 43 | 95 |
| Total | 212 | 240 | 452 |

Are the events "female" and "longer ring finger" independent?

$$P(\text{ring finger}) \stackrel{?}{=} P(\text{ringer} \mid \text{female})$$

$$\frac{234}{452} \qquad \frac{82}{212}$$

$$.52 \neq .39$$

Not independent, if your a female you are less likely to have a longer ring finger than index.

p. 328 check your understanding

Multiplication Rule for independent events:

If A + B are independent events, then the probability of A+B is:

$$P(A \cap B) = P(A) \cdot P(B)$$

since $P(B|A) = P(B)$

Ex: Find the probability of picking an ace out of a deck cards, replacing it, and then choosing an even numbered card.

$P(\text{ace}) = \frac{4}{52}$

$P(\text{even}) = \frac{20}{52}$

$P(\text{ace} + \text{even}) = \frac{4}{52} \cdot \frac{20}{52} = .0295$

$\boxed{\text{www.whfreeman.com/tps5e}}$

Is there a connection between association and independence.

$$P(\text{ring finger} \mid \text{female}) = P(\text{ring finger})$$

If there is no association, there is independence

Is there a connection between mutually exclusive + independent?

Ex: A: red card B: club
mutually exclusive? Yes, no red clubs
Independent? $P(A) = .5$ $P(A|B) = 0$
no

Ex: A: red card B: 7
mutually exclusive?: no, 7♢ + 7♡
independent?: $P(A) = .5$ $P(A|B) = .5$
Yes

Ex: A: red B: heart
mutually exclusive: no, all the hearts
independent: $P(A) = .5$ $P(A|B) = 1$
no

Two mutually exclusive events cannot be independent because if one happens, the other is gaurenteed not to happen.