6.1 Discrete + Random Variables

Determine the probability model for flipping 3 coins. Let X = the number of heads obtained.

Therefore:

$X=0$: TTT

$X=1$: THT, HTT, TTH

$X=2$: HTH, HHT, THH

$X=3$: HHH

The probability distribution is as follows:

value	0	1	2	3
probability	1/8	3/8	3/8	1/8

Ex: Find the probability of at most 2 heads.

$P(X \le 2) = \frac{1}{8} + \frac{3}{8} + \frac{3}{8} = .875$

$1 - P(X > 2) = 1 - \frac{1}{8} = .875$

A random variable takes numerical values that describes the outcomes of a chance process.
The probability distribution of a random variable gives all possible values & corresponding probabilities

Discrete Random Variable, X, takes a fixed set of possible values with gaps between. The probability distribution of a discrete random variable lists values x_i + probabilities p_i

Remember:

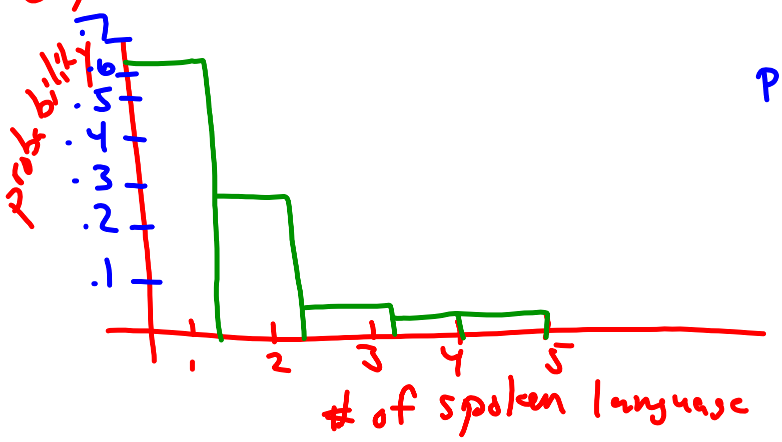
1. $0 \leq p_i \leq 1$
2. $\sum p_i = 1$
3. To find the probability of any event, add the p_i's of the particular x_i's that make up the event.

Ex: Imagine selecting a US high school at random. Define X = number of languages spoken by a randomly selected student. The following is the probability distribution of X:

languages	1	2	3	4	5
probabilities	.630	.295	.065	.008	.002

a) show the probability distribution is legitimate
- All probabilities are between 0 and 1.
- $.630 + .295 + .065 + \ldots + .002 = 1$

b) make a histogram of the probability distribution



probability distribution for # of lang. spoken.

c) What is the probability a randomly selected student speaks at least 3 languages?

$$P(X \geq 3) = .065 + .008 + .002 = .075$$

d) Find $P(X=2) = .295$

Mean of a Discrete Random Variable

For a random variable, X, the mean is μ_x.
This is the average of all possible values of X taking into account not all outcomes are equally likely.

Ex: A roulette wheel has 38 numbers. One possible bet is the "corner bet", where a player places a chip at the intersection of 4 numbers. If you win a $1 bet, you get your $1 back, plus $8. If you lose, the casino keeps your $1. If X = net gain from a single $1 corner bet:

a) make a probability distribution.

net gain	-$1	$8
probability	$34/38$	$4/38$

b) Find the players average gain.

$$-1 \cdot \frac{34}{38} + 8 \cdot \frac{4}{38} = -\$0.05$$

$$\frac{-1 + -1 + -1 + \ldots\ldots + 8 + 8 + 8 + 8}{38} =$$

The mean of a discrete random variable is also called the expected value since it how much we would expect to gain (lose) if we played many times.

$$\mu_X = E(X) = \sum x_i \cdot p_i$$

Ex:

# of lang	1	2	3	4	5
prob	.630	.295	.065	.008	.002

Find E(x) + interpret it's meaning.

$E(X) = 1.457$

$1(.630) + 2(.295) + 3(.065) + 4(.008) + 5(.002)$

The average # of languages spoken is 1.457

The variance, σ_X^2, of a discrete random variable is $\sigma_X^2 = \sum (x_i - \mu_x)^2 \cdot p_i$. Therefore the standard deviation of a discrete random variable X is:

$$\sigma_X = \sqrt{\sum (x_i - \mu_x)^2 \cdot p_i}$$

This is how far the values of a variable typically vary from the mean, μ_x. Since we are using a weighted average, we are giving smaller probabilities to values further from the mean.

Ex: Find σ_X from the language example & interpret its meaning.

$$1\text{ var stats }(\overset{x_i}{L_1}, \overset{p_i}{L_2})$$

$+ \ldots +$

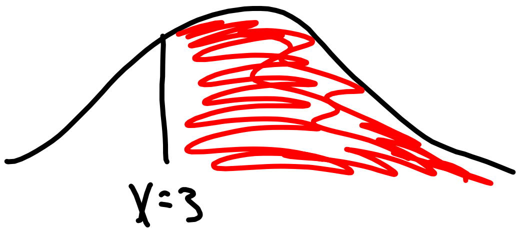
p. 355 check your understanding

1. $(0 \cdot .3) + (1 \cdot .4) + (2 \cdot .2) + (3 \cdot .1) = 1.1$

$\sqrt{(0-1.1)^2 \cdot 0.3 + (1-1.1)^2 (0.4) + (2-1.1)^2 (0.2) + (3-1.1)^2 (0.1)} = 0.94$

A continuous random variable, X, takes all values in an interval of numbers. The probability distribution of X is described by a density curve. The probability of any event is the area under the density curve and above the x axis for that values that make the given event

Ex:



$P(X \geq 3)$