6.2 Transforming + combining random variables

Ex: At El Dorado Tech, the tuition for full time students is \$50 per credit. That is, if T = tuition charges for a randomly selected full time student, T = 50X. The probability distribution is:

# of credits	12	13	14	15	16	17	18
Tuition charges	\$600	\$650	\$700	\$750	\$800	\$850	\$900
Probability	.25	.10	.05	.30	.10	.05	.15

Find: μ_X + μ_T

$\mu_X = 14.65 \cdot 50$

$\mu_T = \$732.50$

Ex: In addition to tuition charges, El Dorado Tech assesses a student fee of \$100. If C = overall cost of a randomly selected full time student, $C = 100 + T$. So:

Overall cost	\$700	\$750	\$800	\$850	\$900	\$950	\$1000
probability	.25	.10	.05	.30	.10	.05	.15

Find μ_T + μ_C $\mu_T = \$732.50$ $\mu_C = \$832.50$

Find σ_T + σ_C $\sigma_T = \$102.80$ $\sigma_C = \$102.80$

If $Y = a + bX$ is a linear transformation of the random variable X, then:

1. The probability distribution of Y has the same shape as the probability distribution of X.
2. $\mu_Y = a + b\mu_X$
3. $\sigma_Y = |b|\sigma_X$
4. There is no change to the standard deviation of random variable when adding a constant.

Combining Random Variables

Sum of a random variable

The mean of the sum of random variables, $X+Y=T$

is $E(T) = \mu_T = \mu_X + \mu_Y$

Ex: El Dorado Tech also has a downtown campus. Full time students take only 3 credit courses. Let Y = # of credits taken by a randomly selected student at the downtown campus. Then:

# of credits	12	15	18
probability	.30	.40	.30

a) Find $\mu_Y = 12(.3) + 15(.4) + 18(.3) = 15$

b) Find $\mu_T = \mu_{X+Y} = 15 + 14.65 = 29.65$

c) What is the range of X? $18 - 12 = 6$

Y? $18 - 12 = 6$

X+Y? $36 - 24 = 12$

Independent Random Variable : If knowing whether any event involving X alone has occurred, tells us nothing about the occurance of any event involving Y alone, and vice versa, then X + Y are independent random variables.

* we can't find σ_T^2 if we don't know the variables are independent.

Ex: Lets find the prob. distribution X+Y. at El Dorado Tech.

P(X=12) P(Y=12)

# of credits	24	25	26	27	28	29	30	31	32	33	34	35	36
prob.	.075	.03	.015	.19	.07	.055	.240	.07	.035	.15	.03	.015	.045

.25·.3

P(Y=12)
P(X=13)
·3·.1

P(Y=12)·
P(X=14)
·3·.05

verify: μ_{X+Y} Find σ_{X+Y}^2 , σ_{X+Y}

$\sigma_{X+Y}^2 = 9.63$ $\sigma_X^2 = 4.23$ $\sigma_Y^2 = 5.4$

$\sigma_{X+Y} = 3.10$

For any 2 random variables X + Y, the variance of T (T = X+Y) is:

$$\sigma_T^2 = \sigma_X^2 + \sigma_Y^2$$

Ex: Find σ_T , σ_X , σ_Y

$\sigma_T = 3.10$

$\sigma_X = 2.06$

$\sigma_Y = 2.32$

* You can never add standard deviation of random variables.

Mean + Variance of a Difference of Random Variables

For any 2 random variables X + Y, if D = X - Y

then:

$$\mu_D = E(D) = \mu_X - \mu_Y$$

$$\sigma_D^2 = \sigma_X^2 + \sigma_Y^2$$ (X + Y must be independent)

p. 378 #1 + 2

1. $\mu_D = \mu_X - \mu_Y$

$= 1.1 - .7 = .4$

2. σ_D

$\sigma_D^2 = \sigma_X^2 + \sigma_Y^2$

$= (.943)^2 + (.64)^2$

$\sqrt{\sigma_D^2} = \sqrt{1.29}$

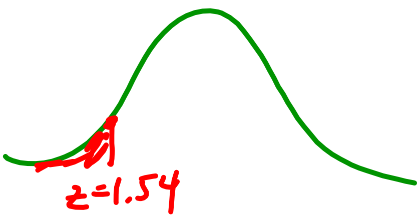
$\boxed{\sigma_D = 1.13}$

Like the women's height example showed us, if a random variable is Normally distributed we can use its μ + σ to compute probabilities. Furthermore, the sum or difference of independent random variables is also then Normally distributed.

Ex: Suppose a certain variety of apple has weights that follow $N(9oz, 1.5oz)$. If a bag of apples is filled by randomly selecting 12 apples, what is the probability that the sum of the weights is less than 100 oz?

$P(T<100)$

$T = X_1 + X_2 + X_3 + \ldots + X_{12}$



$\mu_T = 9+9+9+\ldots+9 = 108$

$\dfrac{T-\mu_T}{\sigma_T}$

$\sigma_T^2 = (1.5)^2 + (1.5)^2 + \ldots + (1.5)^2 =$

$\sqrt{\sigma_T^2} = \sqrt{27}$

$\sigma_T = 5.2$

$z < \dfrac{100-108}{5.2}$

$z < -1.54$

$P(T<100) = .0618$

6% chance of the bag of apples being less than 100 oz.