6.3 Binomial + Geometric Distributions

A binomial setting arises when we perform several independent trials of the same chance process and record the number of times a particular outcome occurs.

Conditions for a binomial setting: BINS

Binary: possible outcomes can be classified as a success or a failure.

Independent: Knowing the result of 1 trial doesn't tell us anything about future trials.

Number: the number of trials, n, must be fixed.

Success: there is the same probability, p, for each successful trial.

The count X of successes in a binomial setting is a binomial random variable. The probability distribution of X is the binomial distribution with parameters n and p. The possible values of X are whole numbers from 0 to n.

Ex: Do the following follow a binomial distribution. Justify.

Roll a dice 10 times. Let X = # of 6's.

Binary because it is a 6 or not a 6.

Independent all dice rolls on a fair dice are ind.

Number: there are exactly 10 trials

Success: 1/6 of getting a 6 on each roll.

Yes.

observe the next 100 cars that drive by. Let C = color.

Binary: no because there are more than 2 options for color.

Shoot the ball 20 times from various places on the court. Let Y = # of shots made.

Binary: Yes, make or miss

Independent: yes, one shot would not have an impact on the next ones.

Number: yes, shooting 20 times

Sucess: No, typically the further you the less chance of making it.

Binomial Probability formula: If X has the binomial distribution with n trials and probability p of success on each trial, the possible values of X are $0, 1, \ldots, n$. If k is any one of these values:

$$P(X=k) = \binom{n}{k} p^k (1-p)^{n-k}$$

$_nC_k$

binomial coefficient

Ex: When rolling 2 dice the probability of rolling doubles is 1/6. Suppose you roll the dice 4 times hoping for doubles.

a) Find the probability of getting double twice (k) in 4 attempts

$$P(X=2) = {}_4C_2 \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^2 = .116$$

12% chance of rolling doubles twice

b) Should you be surprised if you get more than 2 doubles in 4 rolls?

$$P(X>2) = P(X=3) + P(X=4)$$

binomcdf(n, p, k) if $P(X \le k)$
binompdf(n, p, k) if $P(X=k)$

$1 - P(X \le 2) = 1 -$ binomcdf($n=4$ trials, 1/6 = prob. of success, $k=2$ 2 doubles)
$= 1 - .98$
$= .02$

2% chance of rolling more than 2 doubles so I would be surprised

How to find binomial probabilities

1. state the distribution + values of interest.
   (BINS) (n, p, k)
2. Perform the calculation by hand or on calculator
   w/ labeled entries.
3. Answer the question

p. 397 check your understanding.

1. Binary because it is right or wrong
   Indepent how you answer 1 ? has no bearing on the next.
   number: 10 ?'s per test
   success: probability is $\frac{1}{5}$ for a right answer

2. $P(X=3)$ = binompdf(n=10 trials, p=.2 for a right answer, k=3 right answer)
   = .20 = 20% chance of getting exactly 3 correct.

3. $P(X \geq 6) = 1 - P(X \leq 5) = 1 -$ binomcdf(n=10 trials, p=$\frac{1}{5}$ for a right answer, k=5 right answer)
   = .01

   Yes I would be surprised since there is only a 1% chance of getting 6 or more correct

If a count X of successes has the $B(n,k)$ and probability p of success, then

$$\mu_X = np \qquad \sigma_x = \sqrt{np(1-p)}$$

Binomial Distribution in statistical sampling

10% condition: when taking a SRS of size n from of a population of size N, we can use the binomial distribution to model the count of successes as long as $n \le \frac{1}{10}N$.

Ex: Suppose you have 8 AAA batteries and only 6 of them work. You need to choose 4 for your calculator. Explain why the answer is not

$$P(X=4) = \binom{4}{4}(.75)^4(.25)^0 = .3164$$

$6/8 = .75$

$5/7 = .714$

$6/800 = .0075$

$5/799 = .0063$

Geometric Random Variables

A Geometric setting occurs when independent trials are performed and we record the number of trials until we get a success. On each trial the probability p of success is the same.

The number of trials, Y, it takes to get a success in a geometric setting the geometric random variable. The probability distribution of Y is the geometric distribution with parameter, p. Possible values are 1, 2, ...

Geometric Probability Formula

If Y has the geometric distribution with probability, p, of success, the possible values for Y are 1, 2..... If k is one of these values, then:

$$P(Y=k) = (1-p)^{k-1}p$$

Ex: Find the probability Matt rolls a 6 on a die.

$$\left(\frac{5}{6}\right)\left(\frac{5}{6}\right)\left(\frac{5}{6}\right)\left(\frac{1}{6}\right) = 10\%$$

10% chance Matt gets a 6 on the 4th roll.

geometpdf (p, k) = $P(Y=k)$

geometcdf (p, k) = $P(Y \leq k)$

Ex: Find the probability of rolling a 6 on the 4th roll.

$$P(Y=4) = \left(\frac{5}{6}\right)\left(\frac{5}{6}\right)\left(\frac{5}{6}\right)\left(\frac{1}{6}\right)$$

Mean of a Geometric Random Variable

If Y is a geometric random variable with probability p of succes on each trial, the the mean (expected value) is: $\mu_Y = E(Y) = \frac{1}{p}$