28. $f(x) = \frac{4}{x^2-4}$ $g(x) = \sqrt{x}$

a) $(f \circ g)(x)$

$f(g(x))$

$\frac{4}{(\sqrt{x})^2-4} = \frac{4}{x-4}$

b) $(g \circ f)(x)$

$g(f(x))$

$\sqrt{\frac{4}{x^2-4}}$

c) $(f \circ g)(-2)$

$\frac{4}{-2-4} = \frac{4}{-6} = -\frac{2}{3}$

a) $(g \circ f)(1)$

$\sqrt{\frac{4}{(1)^2-4}} = \sqrt{\frac{4}{-3}}$

N.S.

40. $f(x) = x^2 - 3x \quad g(x) = 5x + 3$

$$\frac{f(x+h) - f(x)}{h}$$

$$\frac{(x+h)^2 - 3(x+h) + (-x^2 + 3x)}{h}$$

$$\frac{x^2 + 2hx + h^2 - 3x - 3h - x^2 + 3x}{h}$$

$$\frac{2hx + h^2 - 3h}{h}$$

$$2x + h - 3$$

6.2 Inverse Fnns

a fnn, $f^{-1}(x)$, that "undoes" $f(x)$

Ex: $f(x) = 4x$ $\quad g(x) = x+3$

$f^{-1}(x) = \frac{x}{4}$ $\quad g^{-1}(x) = x-3$

Inverse fnns:

if f and g are fnns such that $f(g(x)) = x$ and $g(f(x)) = x$ then g is the inverse of f.

Ex: verify if $f(x) = 2x+4$ and $g(x) = \frac{1}{2}x - 2$ are inverses.

$f(g(x))$

$2\left(\frac{1}{2}x - 2\right) + 4$

$x - 4 + 4$

x ✓

$g(f(x))$

$\frac{1}{2}(2x+4) - 2$

$x + 2 - 2$

x ✓

Yes!

Ex verify $f(x) = 3x - 2$ and $g(x) = \frac{1}{3}x + 2$

$f(g(x))$

$3\left(\frac{1}{3}x + 2\right) - 2$

$x + 6 - 2$

$x + 4$

Not inverses

Finding an inverse fnn

1. replace f(x) with y
2. switch the x and y
3. Solve for y if possible
4. replace y with $f^{-1}(x)$
5. verify results

$f(x) = \sqrt{2x+9}$

$y = \sqrt{2x+9}$

$x^2 = \sqrt{2y+9}^{\,2}$

$x^2 = 2y+9$

$-9 \quad -9$

$\frac{x^2-9}{2} = \frac{2y}{2}$

$\frac{x^2-9}{2} = y$

$\frac{x^2-9}{2} = f^{-1}(x)$

$f(f^{-1}(x))$

$\sqrt{2\left(\frac{x^2-9}{2}\right)+9}$

$\sqrt{x^2-9+9}$

$\sqrt{x^2}$

x ✓

$f^{-1}(f(x))$

$\frac{\left(\sqrt{2x+9}\right)^2-9}{2}$

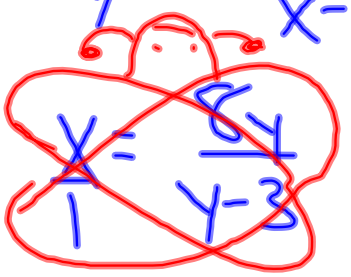
$\frac{2x+9-9}{2}$

$\frac{2x}{2}$

x ✓

$$h(x) = \frac{8x}{x-3}$$

$$y = \frac{8x}{x-3}$$



$$x = \frac{8y}{y-3}$$

$$x(y-3) = 8y$$

$$xy - 3x = 8y$$

$$-xy \qquad -xy$$

$$-3x = 8y - xy$$

$$\frac{-3x}{8-x} = \frac{y(8-x)}{8-x}$$

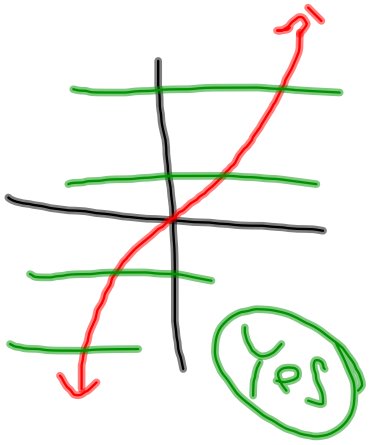
$$\frac{-3x}{8-x} = y$$

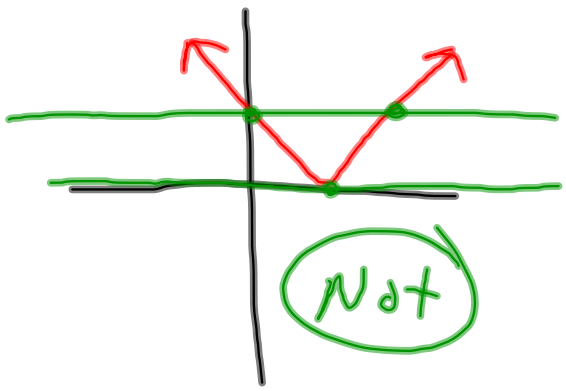
$$\frac{-3x}{8-x} = h^{-1}(x)$$

Horizontal line test

A fnn has an inverse if no horizontal line intersects the fnn at more than 1 point. Such a fnn is called a one to one fnn

Ex: Use the HLT to determine if the following have inverses:





Hw: p. 517

12-70 even

skip 56-62