8.3 Estimating a Population mean

If we try to estimate a pop. mean, μ,
using our method for the population proportion p
we would end up with:

$$\bar{x} \pm z^* \cdot \frac{\sigma}{\sqrt{n}}$$

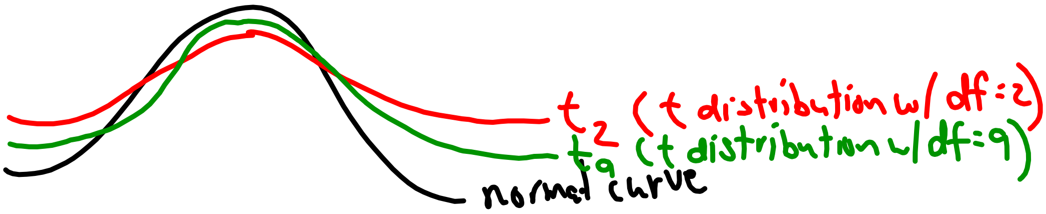
What if we try S_x? We would not capture
μ C% of the time. We would have more variability
using S_x in place of σ.

t-distributions

Draw an SRS of size n from a large population that has $N(\mu, \sigma)$ distribution. The statistic $t = \frac{\bar{x} - \mu}{s_x / \sqrt{n}}$ has the t distribution with degrees of freedom, $df = n-1$. When the population distribution is not Normal, the statistic will have an approximate t_{n-1} distribution if the sample is large enough.

t distribution characteristics

- shape is similar to the normal curve (unimodal, symmetric about 0, bell shaped) but has more area in the tails (less in the center)
- as degrees of freedom increase, t density curve approaches the normal curve since as sample size increases, s_x approaches σ.
- spread is greater for the t distribution since s_x has more variability than σ.



Find a t critical value, t^*, for a 99% CI from an SRS of size 10.

$$df = n - 1$$
$$= 10 - 1$$
$$= 9$$

$$t^* = 3.25$$

Ex: Find t^* for a 96% CI with $n = 22$

$$t^* = 2.189$$

Condition for constructing a CI about μ

Random: SRS and 10% condition (assuming no replacement)

Large counts: Population distribution is Normal or $n \geq 30$. If $n < 30$, graph the data to assess the normality, i.e. no outliers and not strongly skewed.

Standard error of sample mean $\bar{x}$,

$$SE_{\bar{x}} = \frac{s_x}{\sqrt{n}}$$ where s_x

is the sample standard deviation.
It describes how far $\bar{x}$ will typically be from μ in repeated samples of size n.

The one sample t interval for population mean

when the conditions are met, a C % CI for unknown population mean μ is:

$$\bar{x} \pm t^* \frac{s_x}{\sqrt{n}}$$

where t^* is the critical value for the t_{n-1} distribution with C% of the area between $-t^*$ & t^*

Ex: Ann wanted to estimate the average weight of an Oreo to determine if the average weight was less than advertised. She randomly selected 36 cookies + found the weight of each cookie in grams. The mean weight was $\bar{x} = 11.392$ grams with $S_x = .0817$ grams.

a) Construct + interpret a 95% CI for the mean weight of the cookie.

μ = actual average weight of all oreo cookies in grams

with 95% confidence.

random: randomly selected 36 oreos
10% condition is met since 36 oreos is less than 10% all the oreos that are made.

large counts: sample size is greater than 30 cookies.

Conditions are met, so I will use 1 sample t interval for a population mean.

$$\bar{x} \pm \boxed{t^* \frac{S_x}{\sqrt{n}}}$$

$$11.392 \pm 2.03 \cdot \frac{.0817}{\sqrt{36}}$$

$$(11.36, 11.42)$$

We are 95% confident that the interval from 11.36g to 11.42 g will capture the true population mean of the weight of oreo's in grams.

b) On the packaging, the standard serving size is 3 oreos (34 g). Does the interval in part a provide convincing evidence the average weight is less than advertised? Explain.

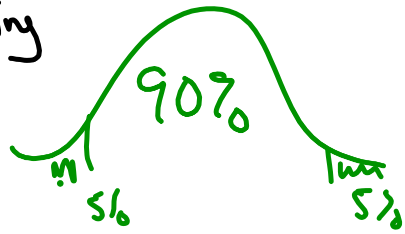
(11.36 g to 11.42 g) $\frac{34\text{g}}{3 \text{ cookies}} = 11.33$ grams/cookie

Choosing a sample size for a desired margin of error when estimating the μ

1. Get a reasonable σ from a previous study.
2. Find the critical value z^* from the normal curve for confidence level C.
3. Solve $z^* \frac{\sigma}{\sqrt{n}} \le ME$

p. 524 check your understanding

$z^* = 1.645$



$\sqrt{n}\left(1.645 \cdot \frac{154}{\cancel{\sqrt{n}}}\right) \le 30\sqrt{n}$

$\frac{(1.645)(154)}{30} \le \frac{30\sqrt{n}}{30}$

$(8.44)^2 \le (\sqrt{n})^2$

$71.2 \le n$

$\boxed{\text{72 students in the sample}}$