9.2 Tests for a population proportion

Conditions for significance tests for p

① Randomness: SRS or randomized experiment

② 10% condition: if sampling without replacement, $n \le \frac{1}{10} N$

③ Large counts: $np_0 \ge 10$ $\quad n(1-p_0) \ge 10$

p_0 is the parameter value of H_0.

Principles of Significance Tests

① tests compare a statistic calculated from sample data with the value of the parameter stated by the null hypothesis.

② values far from a parameter in the direction of H_a offer strong evidence against the H_0

$H_0: p = .5$

$H_a: p < .5$ $\quad \hat{p} = .9$

③ To assess how far the statistic is from the parameter we standardize the statistic.

$$\text{test statistic} = \frac{\text{statistic} - \text{parameter}}{\text{standard deviation of the statistic}}$$

Ex: In a random sample of 124 couples 83 were observed tilting to the right when kissing. If :

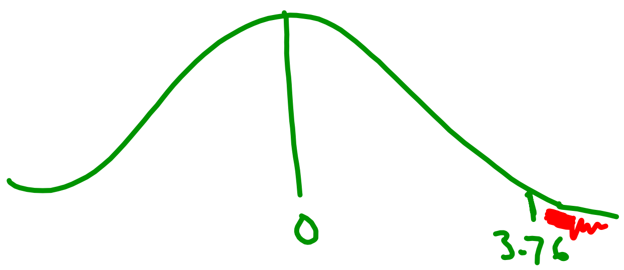
$H_0: p = .5$

$H_a: p > .5$

1. calculate the test statistic

$$z = \frac{.669 - .5}{\sqrt{\frac{.5(1-.5)}{124}}} = 3.76 \qquad \hat{p} = \frac{83}{124} = .669$$

2. Find the P value, and show its area under the Normal curve



$P(z \geq 3.76)$

$1 - .998 \approx .002$

Four step process for significance tests

1. State: hypotheses, significance level, define parameters
2. Plan: Method + conditions
3. Do: Find test statistic + P value
4. Conclude: make a desision (using Pvalue) about the hypothesis in context.

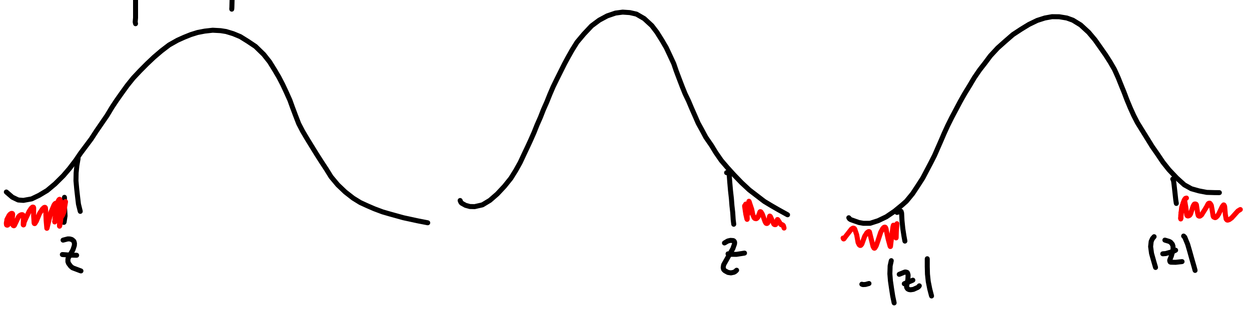
One sample z test for p

Assume conditions are met. to test the hypothesis $H_0: p = p_0$, compute

$$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$$

Find the Pvalue by calculating the probability of getting the z statistic this large or larger in the direction of the specified H_a

$H_a: p < p_0$ $H_a: p > p_0$ $H_a: p \neq p_0$



Ex: On TV shows like American Idol, contestants often wonder if there is an advantage to performing last. To investigate, researchers randomly selected 600 students and show each student the audition video of 12 singers. For each student, the video was shown in random order, so we would expect approximately 1/12 (.083) of the students to prefer the last singer. In this study, 59 of 600 preferred the last singer. Do these data provide convincing evidence at the 5% significance level that there is an advantage to going last?

.0983

p = true proportion of students who prefer the last singer

$H_0: p = .083$

$H_a: p > .083$

We are going to test the claim of p=.083 of the proportion of students who prefer the last singer at the 5% significance level.

We will use the one sample z test for p

- It is a random sample
- 600 students is less than 10% of all students
- large counts: $600(.083) \geq 10$ $\quad 600(1-.083) \geq 10$
  $49.8 \geq 10$ ✓ $\quad 550.2 \geq 10$ ✓

$$z = \frac{.098 - .083}{\sqrt{\frac{.083(1-.083)}{600}}} = 1.36$$

$$P(z \geq 1.36) = 1 - .913 = .0869$$

There is not sufficient evidence to reject the p=.083 as the true proportion of students who prefer the last

Check your understanding p. 563

p = true proportion of employees whose work stress affects their personal life.

$H_0: p = .75$

$H_a: p \neq .75$

We are testing the claim that p = .75 of the proportion of employees whose work stress effects their personal life at 5% significance level

method is a one sample z test for p

- random sample
- 100 is less than 10% of all employees
- large counts $100(.75) = 75 \geq 10$ $100(.25) = 25 \geq 10$ ✓

on the calc. use 1propztest,

$p_0 = .75$, $x = 68$, $n = 100$, $\neq p_0$

$z = -1.62$

$\boxed{2 \cdot} P(z \leq -1.62)$

$= .106$

There is not sufficient evidence to reject p = .75 as the true prop of employees who say work stress affects them.

Assumm 50 answered.

This would give us a z statistic of 5.7 which gives a P-value of almost 0.

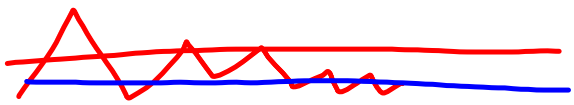
There is sufficient evidence to reject $p = .75$ as the true prop. of dissatisfied emp.

The 95% CI is (.402, .598).

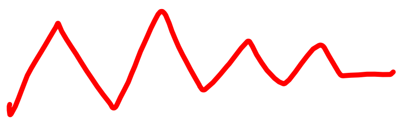
The big idea is the 2 sided significance test at level α give essentially the same information as a $100(1-\alpha)\%$ CI.

Type II errors and Power

Recall a Type II error fails to reject H_0 when it is false. There are many P values that could work, the further the value of P is from H_0, the less likely we are to make this error.

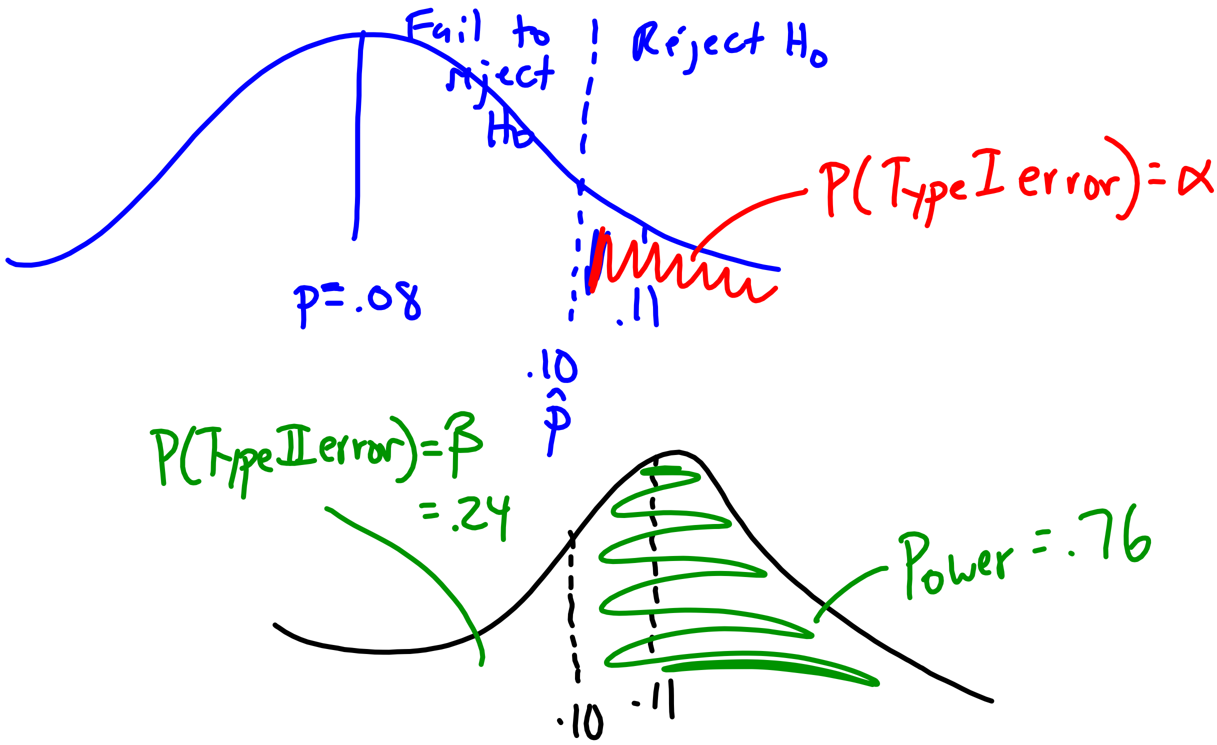


$H_0: p = .8$



It is usually reported differently. We report the probability of a significance test does reject H_0 when the H_a is true. This is the <u>Power</u> of a significance test.

$$Power = 1 - \beta$$, β is the probability of a type II error.


Fail to reject Ho
Reject Ho
P(Type I error) = α
p = .08
.10
.11
p̂
P(Type II error) = β
= .24
Power = .76
.10
.11

Define Power

(1) A Power close to 0 means a test has almost no chance of detecting a wrong H_0

(2) A Power close to 1 means a test is very likely to reject H_0 when it is wrong.

(3) Finds the probability of reaching the right conclusion when the alternative hypothesis is true.

To increase Power (Decrease P(Type II error))

1. increase sample size
2. increase significance level α
3. increase the difference between the null + alternative parameter.

Ex: check your understanding p. 569

1. Type I error: Reject a good a shipment
2. Type II error: Fail to reject a bad shipment