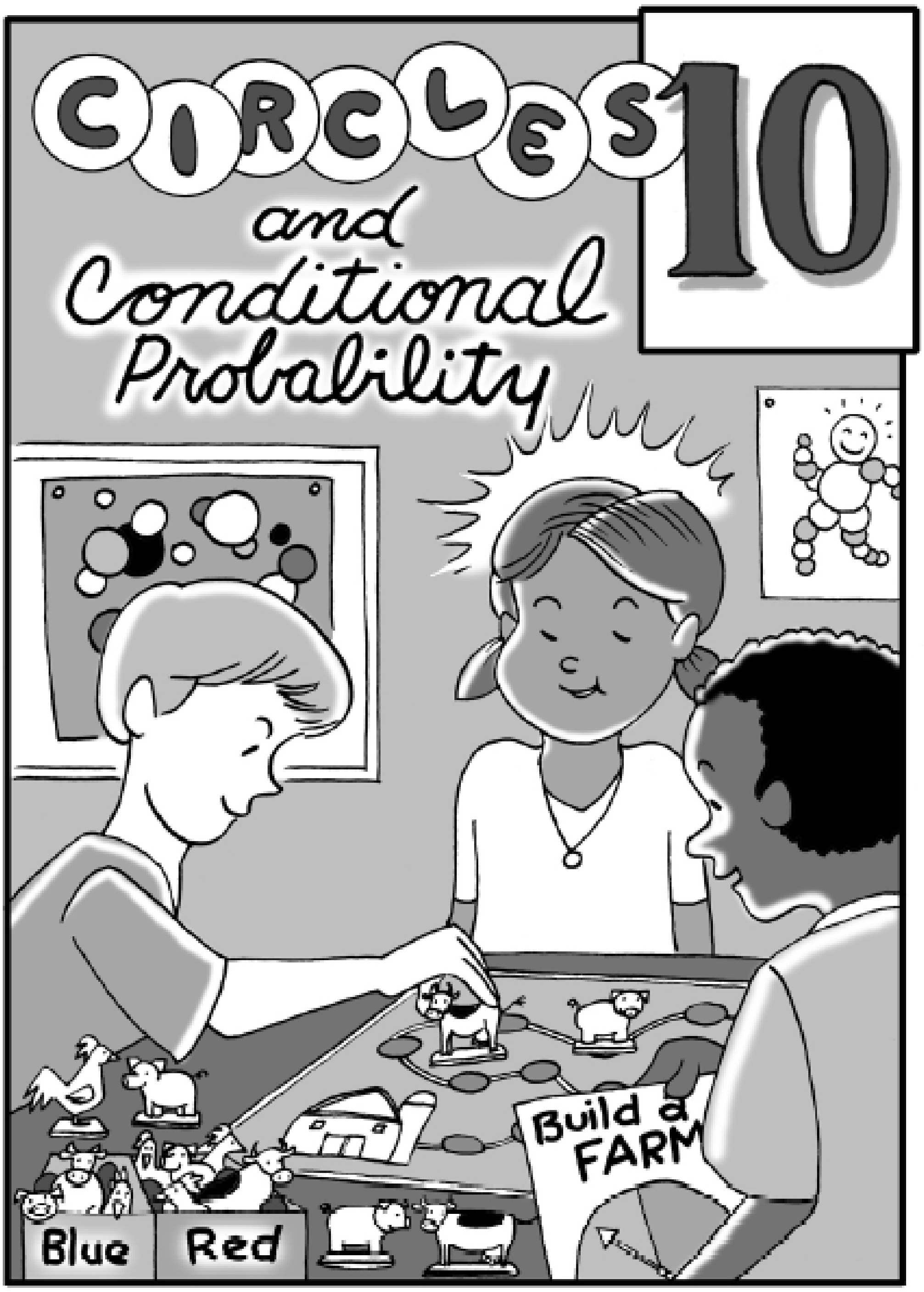

CIRCLES
10
and
Conditional
Probability
Build a
FARM
Blue
Red

CHAPTER 10 Circles and Conditional Probability

In Chapter 8, you developed a method for finding the area and circumference of a circle, and in Chapter 9 you constructed many shapes using circles as a starting point. In Section 10.1, you will explore the relationships between angles, arcs, and chords in a circle.

The focus of your work turns to probability in Section 10.2. As you analyze probabilities, you will develop an understanding of conditional probability and more formal mathematical definitions of independence. With that you can determine if two categorical variables are associated with each other. To calculate and display probabilities, you will add the additional tool of two-way tables to your existing tools of area models and tree diagrams.

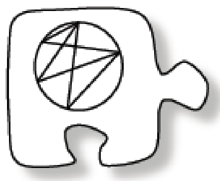
Guiding Question

Mathematically proficient students use appropriate tools strategically.

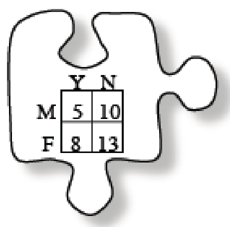
As you work through this chapter, ask yourself:

What tools do I have available to help me solve this problem?

Chapter Outline



Section 10.1 The relationships between angles, arcs, and line segments in a circle will be investigated to develop "circle tools" that can help solve problems involving circles.



Section 10.2 Area models and two-way tables provide the basis for calculating conditional probabilities and determining whether events are independent.

Section 10.3 Some sample spaces are so large that models cannot easily represent them. Based on the Fundamental Principal of Counting, other formulas for permutations and combinations are developed that can be used to solve more complex problems.

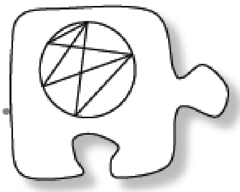
Chapter 10 Teacher Guide

You may wish to omit some of the lessons in this chapter depending on the specific needs of your students. Before skipping any lesson, please refer to the Course Timeline in the Preparing to Teach This Course tab in the front matter of this Teacher Edition.

Section	Lesson	Days	Lesson Objectives	Materials	Homework
10.1	10.1.1	1	Introduction to Chords	• Lesson 10.1.1 Res. Pg. • Tracing paper • Centimeter rulers • Compasses	10-6 to 10-12
	10.1.2	1	Angles and Arcs	• Tracing paper • Compasses and straightedges (opt.)	10-18 to 10-24
	10.1.3	1	Chords and Angles	• 3" x 5" index cards • Compasses and straightedges (opt.)	10-31 to 10-37
	10.1.4	1	Tangents and Secants	• Tracing paper	10-43 to 10-49
	10.1.5	1	Problem Solving with Circles	• Tracing paper • Compasses and straightedges	10-54 to 10-60
10.2	10.2.1	1	Conditional Probability and Independence	None	10-66 to 10-72
	10.2.2	2	Two-Way Tables	None	10-78 to 10-84 and 10-85 to 10-91
	10.2.3	1-2	Applications of Probability	None	10-101 to 10-107
10.3	10.3.1	1	The Fundamental Principle of Counting	None	10-114 to 10-120
	10.3.2	1	Permutations	None	10-127 to 10-133
	10.3.3	1	Combinations	None	10-139 to 10-145
	10.3.4	1	Categorizing Counting Problems	• Lesson 10.3.4 Res. Pg.	10-153 to 10-159
	10.3.5	1-2	Some Challenging Probability Problems	• Lesson 10.3.5 Res. Pg. • Wrapped candies • Dice • Playing cards	10-173 to 10-178 and 10-179 to 10-184
Chapter Closure		Various Options			

Total: 14-16 days plus optional time for Chapter Closure and Assessment

10.1.1 What is the length of the diameter?



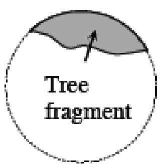
Introduction to Chords

In Chapter 8, you learned that the diameter of a circle is the distance across the center of the circle. This length can be easily determined if the entire circle is in front of you and the center is marked, or if you know the length of the radius of the circle. However, what if you only have part of a circle, called an **arc**? Or what if the circle is so large that it is not practical to measure its diameter using standard measurement tools, such as finding the diameter of the Earth's equator?

Today you will consider a situation that demonstrates the need to learn more about the parts of a circle and the relationships between them.

10-1. THE WORLD'S WIDEST TREE

The baobab tree is a species of tree found in Africa and Australia. It is often referred to as the world's widest tree because it has been known to be up to 45 feet in diameter!



While digging at an archeological site, Rafi found a fragment of a fossilized baobab tree that appears to be wider than any tree on record! However, since he does not have the remains of the entire tree, he cannot simply measure across the tree to find its diameter. He needs your help to determine the length of the radius of this ancient tree. Assume that the shape of the tree's cross-section is a circle.

a. Obtain the Lesson 10.1.1 Resource Page from your teacher. On it, locate $\overset{\frown}{AB}$, which represents the curvature of the tree fragment. Trace this arc as neatly as possible on tracing paper. Then decide with your team how to fold the tracing paper to find the center of the tree. (Hint: This will take more than one fold.) Be ready to share with the class how you found the center.

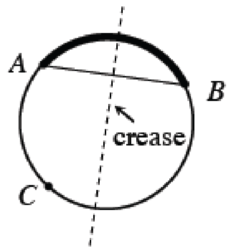
b. In part (a), you located the center of a circle. Use a ruler to measure the radius of that circle. If 1 cm represents 10 feet of tree, find the approximate length of the radius and diameter of the tree. Does the tree appear to be larger than 45 feet in diameter?

10-2. PARTS OF A CIRCLE, Part One

A line segment that connects the endpoints of an arc is called a **chord**. Thus, $\overline{AB}$ in the diagram below is an example of a chord.

a. One way to find the center of a circle when given an arc is to fold it so that the two parts of the arc coincide (lie on top of each other).

If you fold $\overset{\frown}{AB}$ so that A lies on B, what is the relationship between the resulting crease and the chord $\overline{AB}$? Explain how you know.

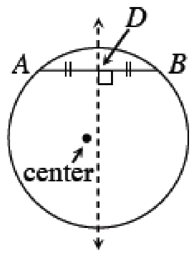


b. The tree fragment in problem 10-1 was an arc between points A and B. However, the missing part of the tree formed another larger arc of the tree. With your team, find the larger arc formed by the circle and points A and B above. Then propose a way to use the points to name the larger arc to distinguish it from $\overset{\frown}{AB}$.

c. In problem 10-1, the tree fragment formed the shorter arc between two endpoints. The shorter arc between points A and B is called the **minor arc** and is written $\overset{\frown}{AB}$. The larger arc is called a **major arc** and is usually written using three points, such as $\overset{\frown}{ACB}$. What do you know about $\overline{AB}$ if the minor and major arcs are the same length? Explain how you know.

10-3. In problem 10-1, folding the arc several times resulted in a point that seemed to be the center of the circle. But how can you prove that the line bisecting an arc (or chord) will pass through the center? To consider this, first assume that the perpendicular bisector does *not* pass through the center. This is an example of a proof by contradiction.

a. According to our assumption, if the perpendicular bisector does not pass through the center, then the center, C, will be off the line in the circle, as shown at right. Copy this diagram onto your paper.



b. Now consider ΔACD and ΔBCD. Are these two triangles congruent? Why or why not?

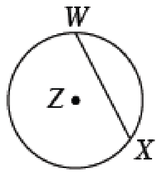
c. Explain why your result from part (b) contradicts the original assumption. That is, explain why the center must lie on the perpendicular bisector of $\overline{AB}$.

10-4. What if you are given two non-parallel chords in a circle and nothing else? How can you use the chords to find the center of the circle?

a. On the Lesson 10.1.1 Resource Page, locate the chords provided for $\odot P$ and $\odot Q$. Work with your team to determine how to find the center of each circle. Then use a compass to draw the circles that contain the given chords. Tracing paper may be helpful.

b. Describe how to find the center of a circle without tracing paper. That is, how would you find the center of $\odot P$ with only a compass and a straightedge? Be prepared to share your description with the rest of the class.

10-5. Examine the chord $\overline{WX}$ in $\odot Z$ at right. If $WX = 8$ units and the length of the radius of $\odot Z$ is 5 units, how far from the center is the chord? Draw the diagram on your paper and show all work.



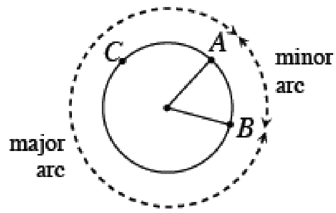
METHODS AND MEANINGS

MATH NOTES

Circle Vocabulary

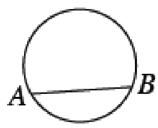
An **arc** is a part of a circle. Remember that a circle does not contain its interior. A bicycle tire is an example of a circle. The spokes and the space in between them are not part of the circle. The piece of tire between any two spokes of the bicycle wheel is an example of an arc.

Any two points on a circle create two arcs. When these arcs are not the same length, the larger arc is referred to as the **major arc**, while the smaller arc is referred to as the **minor arc**.



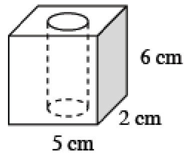
To name an arc, an arc symbol is drawn over the endpoints, such as $\overset{\frown}{AB}$. To refer to a major arc, a third point on the arc should be used to identify the arc clearly, such as $\overset{\frown}{ACB}$.

A **chord** is a line segment that has both endpoints on a circle. $\overline{AB}$ in the diagram at right is an example of a chord. When a chord passes through the center of the circle, it is called a **diameter**.



Review & Preview

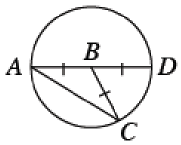
10-6. A rectangular prism has a cylindrical hole removed, as shown at right.



a. If the length of the radius of the cylindrical hole is 0.5 cm, find the volume of the solid.

b. What could this geometric figure represent? That is, if it were a model for something that exists in the world, what might it be? Also, how might you change it to make it a better model?

10-7. In the diagram below, $\overline{AD}$ is a diameter of $\odot B$.

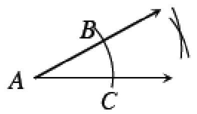


a. If $m\angle A = 35°$, what is $m\angle CBD$?

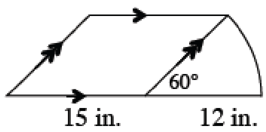
b. If $m\angle CBD = 100°$, what is $m\angle A$?

c. If $m\angle A = x$, what is $m\angle CBD$?

10-8. Lavinia started a construction at right. Explain what she is constructing. Then copy her diagram and finish her construction.



10-9. A sector is attached to the side of a parallelogram, as shown in the diagram at right. Find the area and perimeter of the figure.



10-10. For each recursively defined sequence, list the first five terms and identify the sequence as arithmetic, geometric, or neither.

a. $a_1 = 17,\ a_{n+1} = -a_n$

b. $a_1 = 32,\ a_{n+1} = -5 + \frac{1}{2}a_n$

c. $a_1 = 81,\ a_{n+1} = a_n$

10-11. On the same set of axes, graph both equations listed below. Then name all points of intersection in the form (x, y). How many times do the graphs intersect?

$$y = 4x - 7$$
$$y = x^2 - 2x + 2$$

10-12. **Multiple Choice:** A penny, nickel, and dime are all flipped once. What is the probability that at least one coin comes up heads?

a. $\frac{1}{3}$ b. $\frac{3}{8}$ c. 1 d. $\frac{7}{8}$

Lesson 10.1.1 Resource Page

Finding the Diameter of a Circle

10-1. The arc $\widehat{AB}$ below is a scaled representation of Rafi's tree fragment.

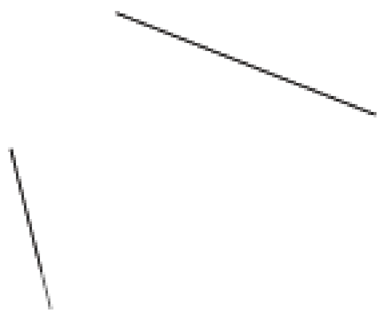
a. Trace this arc as neatly as possible on tracing paper. Then decide with your team how to fold the tracing paper to find the center of the tree. (Note: This will take more than one fold.) Be ready to share with the class how you found the center.

b. If 1 cm represents 10 feet of tree, find the approximate lengths of the radius and diameter of the tree. Does the tree appear to be larger than 45 feet in diameter?

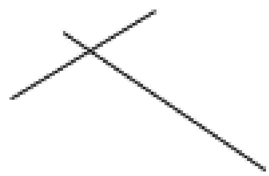
10-4. How can you find the center if all you have is two chords?

- For each pair of chords below, find the center of the circle.
- Then use a compass to draw the circle.

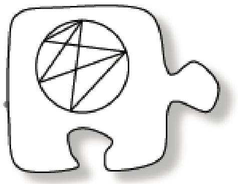
Circle P



Circle Q

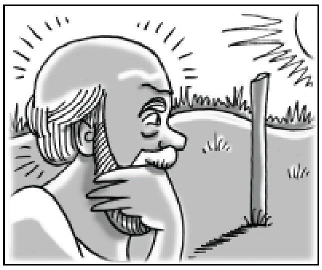


10.1.2 What is the relationship?



Angles and Arcs

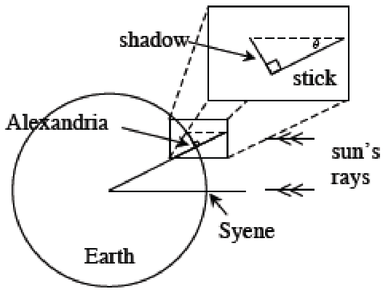
In order to learn more about circles, you need to investigate different types of angles and chords that are found in circles. In Lesson 10.1.1, you studied an application with a tree to learn about the chords of a circle. Today you will study a different application that will demonstrate the importance of knowing how to measure the angles and arcs within a circle.

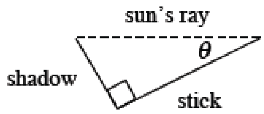
10-13. 

ERATOSTHENES' REMARKABLE DISCOVERY

Eratosthenes, who lived in the 3rd century B.C., was able to determine the circumference of the Earth at a time when most people thought the world was flat! Since he was convinced that the Earth was round, he realized that he could use a shadow to help calculate the length of the Earth's radius.

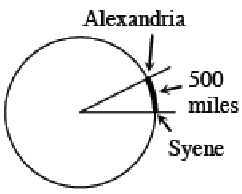
Eratosthenes knew that Alexandria was located about 500 miles north of a town that was closer to the equator, called Syene. When the sun was directly overhead at Syene, a meter stick had no shadow. However, at the same time in Alexandria, a meter stick had a shadow due to the curvature of the Earth. Since the sun is so far away from the Earth, Eratosthenes assumed that the sun's rays were essentially parallel once they entered the Earth's atmosphere and realized that he could therefore use the stick's shadow to help calculate the length of the Earth's radius.





a. Unfortunately, the precise data used by Eratosthenes was lost long ago. However, if Eratosthenes used a meter stick for his experiment today, then the stick's shadow in Alexandria would be 127 mm long. Determine the angle θ that the sunrays made with the meter stick. Remember that a meter stick is 1000 millimeters long.

b. Assuming that the sun's rays are essentially parallel, determine the central angle of the circle if the angle passes through Alexandria and Syene. How did you find your answer?

c. 

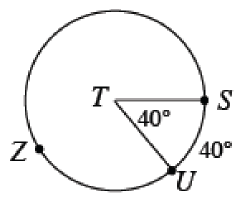
Since the distance along the Earth's surface from Alexandria to Syene is about 500 miles, that is the length of the arc between Alexandria and Syene. Use this information to approximate the circumference of the Earth.

d. Use your result from part (c) to approximate the length of the radius of the Earth.

10-14. PARTS OF A CIRCLE, Part Two

In order to find the circumference of the Earth, Eratosthenes used an angle that had its vertex at the center of the circle. Like the angles in polygons that you studied in Chapter 8, this angle is called a **central angle**.

a. An **arc** is a part of a circle. Every central angle has a corresponding arc. For example, in $\odot T$ at right, $\angle STU$ is a central angle and corresponds to $\overset{\frown}{SU}$. Since the measure of an angle helps us know its part of the whole 360° of a circle, an arc can also be measured in degrees, representing its fraction of an entire circle. Thus, the **measure of an arc** is defined to be equal to the measure of its corresponding central angle.

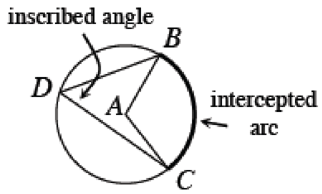


Examine the circle above. What is the measure of $\overset{\frown}{SU}$ (written $m\overset{\frown}{SU}$)? What is $m\overset{\frown}{SZU}$? Show how you got your answer.

b. When Eratosthenes measured the distance from Syene to Alexandria, he measured the length of an arc. This distance is called **arc length** and is measured with units like centimeters or feet. One way to find arc length is to wrap a string about a part of a circle and then to straighten it out and measure its length. Calculate the arc length of $\overset{\frown}{SU}$ above if the length of the radius of $\odot T$ is 12 inches.

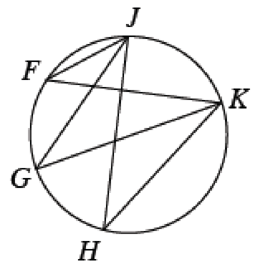
10-15. INSCRIBED ANGLES

In the diagram at right, $\angle BDC$ is an example of an **inscribed angle**, because it lies within $\odot A$ and its vertex lies on the circle. It corresponds to central angle $\angle BAC$ because they both intercept the same arc, $\overset{\frown}{BC}$. (An **intercepted arc** is an arc with endpoints on each side of the angle.)

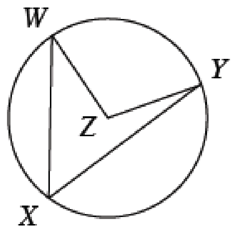


Investigate the measure of inscribed angles as you answer the questions below.

a. In the circle at right, $\angle F, \angle G,$ and $\angle H$ are examples of inscribed angles. Notice that all three angles intercept the same arc ($\overset{\frown}{JK}$). Use tracing paper to compare their measures. What do you notice?

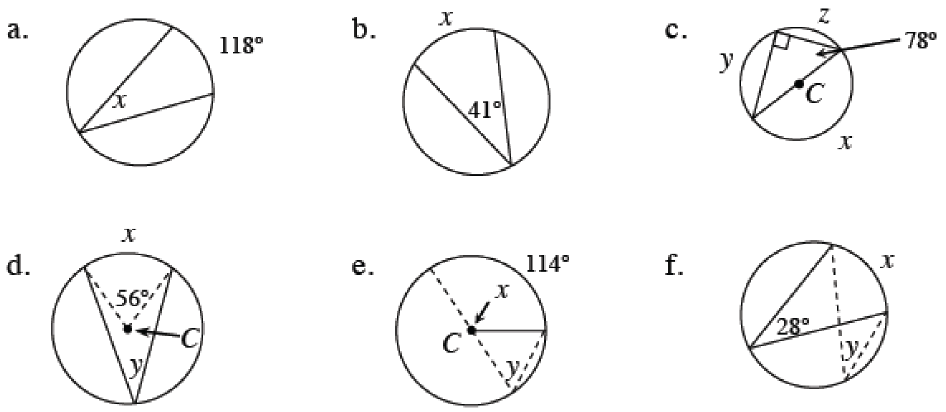


b. Now compare the measurements of the central angle (such as $\angle WZY$ in $\odot Z$ at right) and an inscribed angle (such as $\angle WXY$). What is the relationship of an inscribed angle and its corresponding central angle? Use tracing paper to test your idea.



10-16. In problem 10-15, you found that the measure of an inscribed angle was half of the measure of its corresponding central angle in the cases you tested. Later you will prove that this is always true and since the measure of the central angle always equals the measure of its intercepted arc, then the measure of the inscribed angle must be half of the measure of its intercepted arc.

Examine the diagrams below. Find the measures of the indicated angles. If a point is labeled C, assume it is the center of the circle.



10-17. LEARNING LOG

Reflect on what you have learned during this lesson. Write a Learning Log entry describing the relationships between inscribed angles and their intercepted arcs. Be sure to include an example. Title this entry "Inscribed Angles" and include today's date.

MATH NOTES

METHODS AND MEANINGS

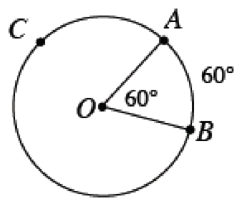
More Circle Vocabulary

The vertex of a **central angle** is at the center of a circle. An **inscribed angle** has its vertex on the circle with each side intersecting the circle at a different point.

One way to discuss an arc is to consider it as a fraction of 360°, that is, as a part of a full circle. When speaking about an arc using degrees, this is called the **arc measure**. The arc between the endpoints of the sides of a central angle has the same measure (in degrees) as its corresponding central angle.

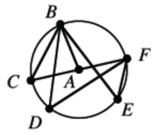
When you want to know how *far* it is from one point to another as you travel along an arc, you call this the **arc length** and measure it in feet, miles, etc.

For example, point O is the center of $\odot O$ at right, and $\angle AOB$ is a central angle. The sides of the angle intersect the circle at points A and B, so $\angle AOB$ intercepts $\overset{\frown}{AB}$. In this case, the measure of $\overset{\frown}{AB}$ is 60°, while the measure of the major arc, $m\overset{\frown}{ACB}$, is 300° because the sum of the major and minor arcs is 360°. The length of $\overset{\frown}{AB}$ is $\frac{60}{360} = \frac{1}{6}$ of the circumference.



Review & Preview

10-18. In $\odot A$ at right, $\overline{CF}$ is a diameter and $m\angle C = 64°$. Find:



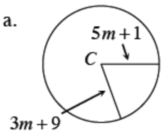
a. $m\angle D$ b. $m\overset{\frown}{BF}$

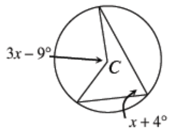
c. $m\angle E$ d. $m\overset{\frown}{CBF}$

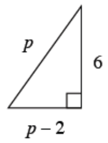
e. $m\angle BAF$ f. $m\angle BAC$

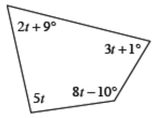
10-19. Find the area of a regular polygon with 100 sides and with a perimeter of 100 units.

10-20. For each of the geometric relationships represented below, write and solve an equation for the given variable. For parts (a) and (b), assume that C is the center of the circle. Show all work.

a. 

b. 

c. 

d. 

10-21. On graph paper, plot ΔABC if $A(-1,-1)$, $B(1, 9)$, and $C(7, 5)$.

a. Find the midpoint of $\overline{AB}$ and label it D. Also find the midpoint of $\overline{BC}$ and label it E.

b. Find the length of the midsegment, $\overline{DE}$. Use it to predict the length of $\overline{AC}$.

c. Now find the length of $\overline{AC}$ and compare it to your prediction from part (b).

10-22. $ABCDE$ is a regular pentagon inscribed in $\odot O$, meaning that each of its five vertices just touches the circle.

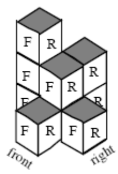
a. Draw a diagram of $ABCDE$ and $\odot O$ on your paper.

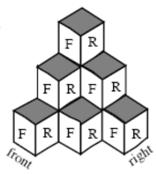
b. Find $m\angle EDC$. How did you find your answer?

c. Find $m\angle BOC$. What relationship did you use?

d. Find $m\overset{\frown}{EBC}$. Is there more than one way to do this?

10-23. Create mat plans from the following isometric views and find the volume of each.

a. 

b. 

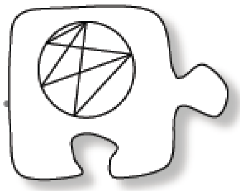
c. Did you have to make any assumptions about hidden cubes when you drew the mat plans? If so, what assumptions did you make in each case and why?

d. What other view would you need to see to be sure how many cubes there are? Explain.

10-24. **Multiple Choice:** Jill's car tires are spinning at a rate of 120 revolutions per minute. If her car tires' radii are each 14 inches, how far does she travel in 5 minutes?

a. 140π in. b. 8400π in. c. 3360π in. d. 16800π in.

10.1.3 What more can I learn about circles?

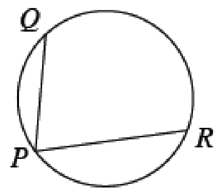


Chords and Angles

As you investigate more about the parts of a circle, look for connections that you can make to other shapes and relationships you have studied so far in this course.

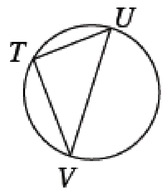
10-25. WHAT IF IT'S A SEMICIRCLE?

What is the measure of an angle when it is inscribed in a **semicircle** (an arc with measure 180°)? Consider this as you answer the questions below.



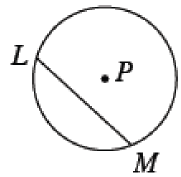
a. Assume that the diagram at right is not drawn to scale. If $m\overset{\frown}{QR} = 180°$, then what is $m\angle P$? Why?

b. Since you have several tools to use with right triangles, the special relationship you found in part (a) can be useful. For example, $\overline{UV}$ is a diameter of the circle at right. If $TU = 6$ units and $TV = 8$ units, what is the length of the radius of the circle? What is its area?



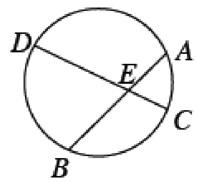
10-26. In Lesson 10.1.1, you learned that a chord is a line segment that has its endpoints on a circle. What geometric tools do you have that can help find the length of a chord?

a. Examine the diagram of chord $\overline{LM}$ in $\odot P$ at right. If the length of the radius of $\odot P$ is 6 units and if $m\overset{\frown}{LM} = 150°$, find LM. Be ready to share your method with the class.



b. What if you know the length of a chord? How can you use it to reverse the process? Draw a diagram of a circle with radius with length 5 units and chord $\overline{AB}$ with length 6 units. Find $m\overset{\frown}{AB}$.

10-27. Timothy asks, "*What if two chords intersect inside a circle? Can triangles help me learn something about these chords?*" Copy his diagram at right in which chords $\overline{AB}$ and $\overline{CD}$ intersect at point E.



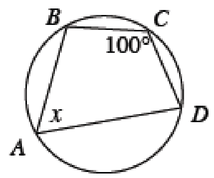
a. Timothy decided to create two triangles (ΔBED and ΔACE). Add line segments $\overline{BD}$ and $\overline{AC}$ to your diagram.

b. Compare $\angle B$ and $\angle C$. Which is bigger? How can you tell? Likewise, compare $\angle D$ and $\angle A$. Write down your observations.

c. How are ΔBED and ΔACE related? Justify your answer.

d. If $DE = 8$, $AE = 4$, and $EB = 6$, then what is EC? Show your work.

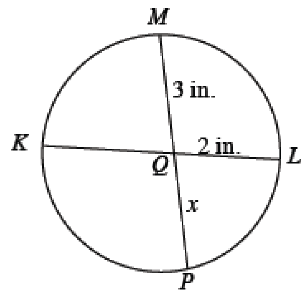
10-28. A polygon is said to be **inscribed** in a circle when each of is vertices touch the circle. How are the angles of a quadrilateral inscribed in a circle related? Consider this as you answer the questions below.



a. For *ABCD* inscribed in the circle at right, solve for x. Explain how you found your answer.

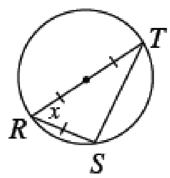
b. Alejandra noticed something. *"I think that the opposite angles of a quadrilateral inscribed in a circle are always supplementary*." Is she correct? Prove your conclusion.

10-29. Use the relationships in the diagrams to solve for x. Justify your solutions.

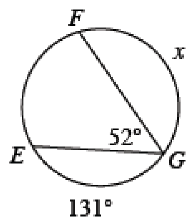


a. $\overline{KL}$ and $\overline{MP}$ intersect at Q and $KL = 8$ inches

b. $\overline{RT}$ is a diameter



c.



10-30. LEARNING LOG

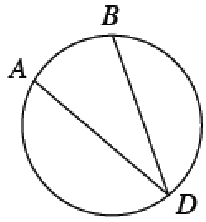
Look over your work from today. Consider all the geometric tools you applied to learn more about angles and chords of circles. In a Learning Log entry, describe which connections you made today. Title this entry "Connections with Circles" and include today's date.

METHODS AND MEANINGS

MATH NOTES

Inscribed Angle Theorem

The measure of any inscribed angle is half of the measure of its intercepted arc. Likewise, any intercepted arc is twice the measure of any inscribed angles whose sides pass through the endpoints of the arc.

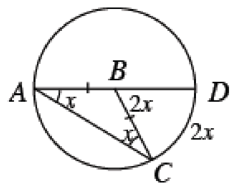


For example, in the diagram at right:

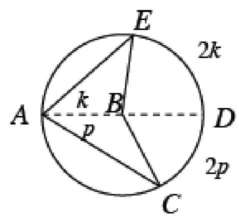
$$m\angle ADB = \tfrac{1}{2} m\widehat{AB} \text{ and } m\widehat{AB} = 2m\angle ADB$$

Proof:

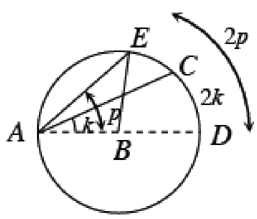
To prove this relationship, consider the relationship between an inscribed angle and its corresponding central angle. In problem 10-7, you used the isosceles triangle ΔABC to demonstrate that if one of the sides of the inscribed angle is a diameter of the circle, then the inscribed angle must be half of the measure of the corresponding central angle. Therefore, in the diagram at right, $m\angle DAC = \frac{1}{2} m\widehat{DC}$.



But what if the center of the circle instead lies in the interior of an inscribed angle, such as $\angle EAC$ shown at right? By extending $\overline{AB}$ to construct the diameter $\overline{AD}$, the work above shows that if $m\angle EAD = k$ then $m\widehat{ED} = 2k$ and if $m\angle DAC = p$, then $m\widehat{DC} = 2p$. Since $m\angle EAC = k + p$, then $m\widehat{EC} = 2k + 2p = 2(k + p) = 2m\angle EAC$.



The last possible case to consider is when the center lies outside of the inscribed angle, as shown at right. Again, constructing a diameter $\overline{AD}$ helps show that if $m\angle CAD = k$ then $m\widehat{CD} = 2k$ and if $m\angle EAD = p$, then $m\widehat{ED} = 2p$. Since $m\angle EAC = p - k$, then $m\widehat{EC} = 2k - 2p = 2(k - p) = 2m\angle EAC$.

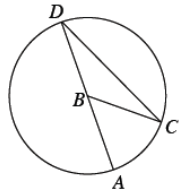


Therefore, an arc is always twice the measure of any inscribed angle that intercepts it.

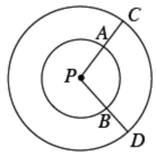
Review & Preview

10-31. Assume point B is the center of the circle below. Match each item in the left column with the best description for it in the right column.

a.	$\overline{AB}$	1.	inscribed angle
b.	$\overline{CD}$	2.	semicircle
c.	$\overparen{AD}$	3.	radius
d.	$\angle CDA$	4.	minor arc
e.	$\overparen{AC}$	5.	central angle
f.	$\angle ABC$	6.	chord



10-32. The figure at right shows two concentric circles.

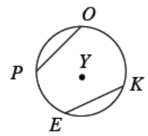


a. What is the relationship between $\overparen{AB}$ or $\overparen{CD}$? How do you know?

b. Of $\overparen{AB}$ or $\overparen{CD}$, which has greater measure? Which has greater length? Explain.

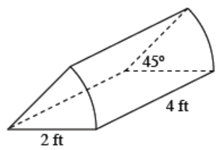
c. If $m\angle P = 60°$ and $PD = 14$, find the length of $\overparen{CD}$. Show all work.

10-33. In $\odot Y$ at right, assume that $m\overparen{PO} = m\overparen{EK}$. Prove that $\overline{PO} \cong \overline{EK}$. Use the format of your choice.



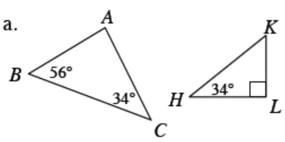
10-34. While working on the quadrilateral hotline, Jo Beth got this call: "*I need help identifying the shape of the quadrilateral flowerbed in front of my apartment. Because a shrub covers one side, I can only see three sides of the flowerbed. However, of the three sides I can see, two are parallel and all three are congruent. What are the possible shapes of my flowerbed?*" Help Jo Beth answer the caller's question.

10-35. Compute the volume of the solid shown at right.

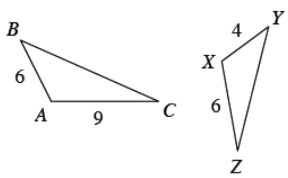


10-36. For each pair of triangles below, decide if the triangles are similar or not and explain how you know. If the triangles are similar, complete the similarity statement $\Delta ABC \sim \Delta$ ______.

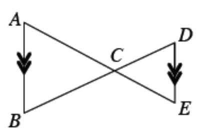
a.



b.



c.

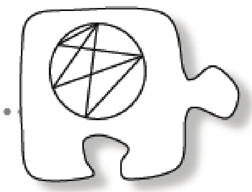


10-37. **Multiple Choice:** Which equation below is perpendicular to $y = \frac{-2}{5}x - 7$ and passes through the point $(4, -1)$?

a. $2x - 5y = 13$ b. $2x + 5y = 3$ c. $5x - 2y = 22$

d. $5x + 2y = 18$ e. None of these

10.1.4 What is the relationship?



Tangents and Secants

So far, you have studied the relationships that exist between angles and chords (line segments) in a circle. Today you will extend these ideas to include the study of lines and circles.

10-38. Consider all the ways a circle and a line can intersect. Can you visualize a line and a circle that intersect at exactly one point? What about a line that intersects a circle twice? On your paper, draw a diagram for each of the situations below, if possible. If it is not possible, explain why.

a. Draw a line and a circle that do not intersect.

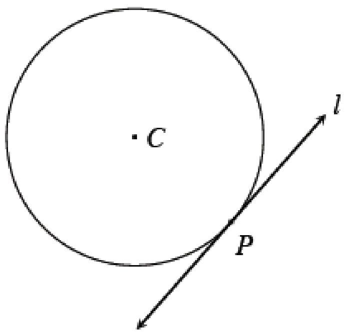
b. Draw a line and a circle that intersect at exactly one point. When this happens, the line is called a **tangent**.

c. Draw a line and a circle that intersect at exactly two points. A line that intersects a circle twice is called a **secant**.

d. Draw a line and a circle that intersect three times.

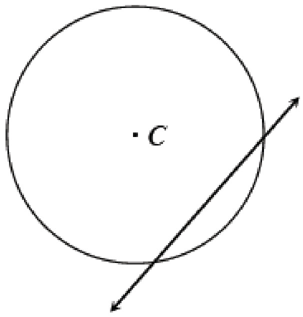
10-39. A line that intersects a circle exactly once is called a **tangent**. What is the relationship of a tangent to a circle?

To investigate this question, carefully copy the diagram showing line l tangent to $\odot C$ at right onto tracing paper. Fold the tracing paper so that the crease is perpendicular to line l through point P. Your crease should pass through point C. What does this tell you about the tangent line?



10-40. Ventura began to think about perpendicularity in a circle. He wondered, "*If a radius is perpendicular to a line at a point on the circle, how do we know if that the line is a secant or a tangent?*" His team decided to tackle his question.

a. Ho says, "*Let's assume the line perpendicular to the radius is a secant*." On your paper, draw a diagram, like the one at right, with $\odot C$ and a secant. Label the points where the secant intersects the circle *A* and *B*. Since Ventura's question assumes that the line is perpendicular to a radius, assume that $\overleftrightarrow{AB}$ is perpendicular to $\overline{CA}$ at *A*.



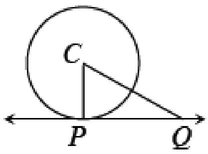
b. Ventura adds "*I think* ΔCAB *is isosceles*." Do you agree? Explain how you know.

c. Sandra chimes in with, "*Then* $\angle CBA$ *must be a right angle too*." Ventura quickly adds, "*But that's impossible!*" What do you think? Discuss this with your team and give reasons to support your conclusions.

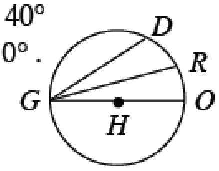
d. Explain to Ventura what this contradiction reveals about the line perpendicular to the radius of a circle at a point on the circle.

10-41. Use the relationships in the diagrams below to answer the following questions. Be sure to name what relationship(s) you used.

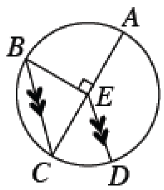
a. $\overleftrightarrow{PQ}$ is tangent to $\odot C$ at P. If $PQ = 5$ and $CQ = 6$, find CP and $m\angle C$.



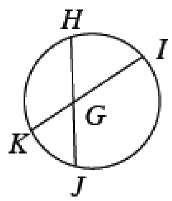
b. In $\odot H$, $m\widehat{DR} = 40°$ and $m\widehat{GOR} = 210°$. Find $m\widehat{GD}$, $m\widehat{OR}$, and $m\angle RGO$.



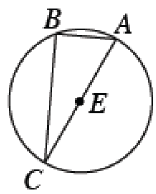
c. $\overline{AC}$ is a diameter of $\odot E$ and $\overline{BC} \parallel \overline{ED}$. Find the measure of $\widehat{CD}$.



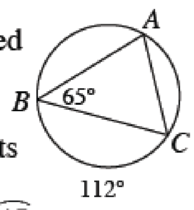
d. $\overline{HJ}$ and $\overline{IK}$ intersect at G. If $HG = 9$, $GJ = 8$, and $GK = 6$, find IG.



e. $\overline{AC}$ is a diameter of $\odot E$, the area of the circle is 289π units^2, and $AB = 16$ units. Find BC and $m\widehat{BC}$.



f. ΔABC is inscribed in the circle at right. Using the measurements provided in the diagram, find $m\widehat{AB}$.



10-42. In Chapter 9, it was stated that the intersection of the angle bisectors of a triangle is the center of a circle inscribed in the triangle. You now have enough information to prove this relationship.

a. Assume that in ΔABC, $\overleftrightarrow{AG}$ and $\overleftrightarrow{BH}$ are angle bisectors of $\angle CAB$ and $\angle CBA$, respectively. Draw this diagram on your paper. Label the intersection of the angle bisectors *P*.

b. You need to show that *P* is the same distance from sides $\overline{AB}$ and $\overline{AC}$. Draw a perpendicular from *P* to $\overline{AB}$ and label its intersection *D*. Similarly, draw a perpendicular from *P* to $\overline{AC}$ and label its intersection *E*. How can you prove that $\Delta ADP \cong \Delta AEP$?

c. Explain why $PE = PD$.

d. Use similar reasoning to show that *P* must also be the same distance from $\overline{BC}$. For example, if the perpendicular from *P* to $\overline{BC}$ intersects $\overline{BC}$ at *F*, why is $PF = PD$?

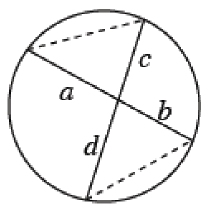
e. Explain why there must be a circle through points *D*, *E*, and *F* with center *P* and that each side of ΔABC must be tangent to this circle, making the circle inscribed in the triangle.

MATH NOTES

METHODS AND MEANINGS

Intersecting Chords

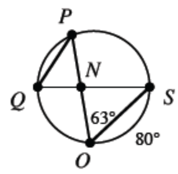
When two chords in a circle intersect, an interesting relationship between the lengths of the resulting segments occurs. If the ends of the chords are connected as shown in the diagram, similar triangles are formed (see problem 10-27). Then, since corresponding sides of similar triangles have a common ratio, $\frac{a}{d}=\frac{c}{b}$, and so



$$ab = cd.$$

Review & Preview

10-43. If $\overline{QS}$ is a diameter and $\overline{PO}$ is a chord of the circle at right, find the measure of the geometric parts listed below.

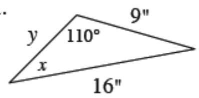


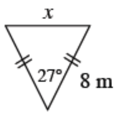
a. $m\angle QSO$ b. $m\angle QPO$

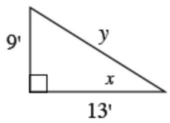
c. $m\angle ONS$ d. $m\widehat{PS}$

e. $m\widehat{PQ}$ f. $m\angle PQN$

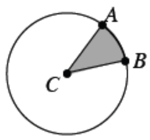
10-44. For each triangle below, solve for the given variables.

a.


b.


c.


10-45. The spinner at right is designed so that if you randomly spin the spinner and land in the shaded sector, you win \$1,000,000. Unfortunately, if you land in the unshaded sector, you win nothing. Assume point C is the center of the spinner.

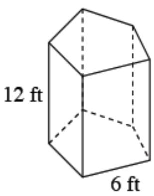


a. If $m\angle ACB = 90°$, how many times would you have to spin to reasonably expect to land in the shaded sector at least once? How did you get your answer?

b. What if $m\angle ACB = 1°$? How many times would you have to spin to reasonably expect to land in the shaded sector at least once?

c. Suppose $P(\text{winning } \$1{,}000{,}000) = \frac{1}{5}$ for each spin. What must $m\angle ACB$ equal? Show how you got your answer.

10-46. Calculate the total surface area and volume of the prism at right. Assume that the base is a regular pentagon.



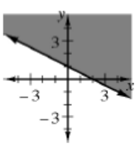
10-47. Quadrilateral $ABCD$ is graphed so that $A(3, 2)$, $B(1, 6)$, $C(5, 8)$, and $D(7, 4)$.

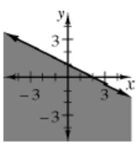
a. Graph $ABCD$ on graph paper. What shape is $ABCD$? Justify your answer.

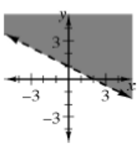
b. $ABCD$ is rotated 180° about the origin to create $A'B'C'D'$. Then $A'B'C'D'$ is reflected across the x-axis to form $A''B''C''D''$. Name the coordinates of C' and D''.

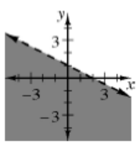
10-48. Polly has a pentagon with angle measures $3x-26°$, $2x+70°$, $5x-10°$, $3x$, and $2x+56°$. Find the probability that if one vertex is selected at random, then the measure of its angle is more than or equal to 90°.

10-49. **Multiple Choice:** Which graph below represents $y > -\frac{1}{2}x + 1$?

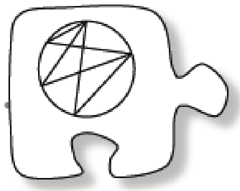
a.


b.


c.


d.


10.1.5 How can I solve it?



Problem Solving with Circles

Your work today is focused on consolidating your understanding of the relationships between angles, arcs, chords, and tangents in circles. As you work today, ask yourself the following focus questions:

Is there another way?

What is the relationship?

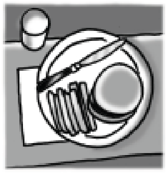
10-50. On a map, the coordinates of towns *A*, *B*, and *C* are $A(-3, 3)$, $B\ (5, 7)$, and $C\ (6, 0)$. City planners have decided to connect the towns with a circular freeway.

a. Graph a map of the towns on graph paper. Once the freeway is built, $\overline{AB}$, $\overline{BC}$, and $\overline{AC}$ will be chords of the circle. Use this information to find the center of the circle.

b. Draw triangle *ABC* and use a compass to draw the circle connecting all three towns on your graph paper. Then, find the length of the radius of the circular freeway.

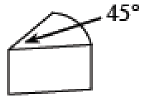
The circle that you drew **circumscribes** triangle ΔABC, because ΔABC is inscribed in the circle. The center of the circle is called the **circumcenter** of the triangle, because it is the center of the circle that circumscribes the triangle.

c. The city planners also intend to locate a new restaurant at the point that is an equal distance from all three towns. Where on the map should that restaurant be located? Justify your conclusion.

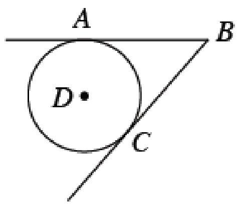
10-51. 

An 8-inch dinner knife is sitting on a circular plate so that its ends are on the edge of the plate. If the minor arc that is intercepted by the knife measures 120°, find the length of the diameter of the plate. Show all work.

10-52. A cylindrical block of cheese has a 6-inch diameter and is 2 inches thick. After a party, only a sector remains that has a central angle of 45°. Find the volume of the cheese that remains. Show all work.



10-53. Dennis plans to place a circular hot tub in the corner of his backyard so that it is tangent to a fence on two sides, as shown in the diagram at right.



a. Prove that $\overline{AB} \cong \overline{CB}$.

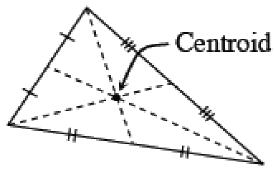
b. The switch to turn on the air jets is located at point B. If the length of the diameter of the hot tub is 6 feet and $AB = 4$ feet, how long does his arm need to be for him to reach the switch from the edge of the tub? (Assume that Dennis will be in the tub when he turns the air jets on and that the switch is level with the top edge of the hot tub.)

MATH NOTES

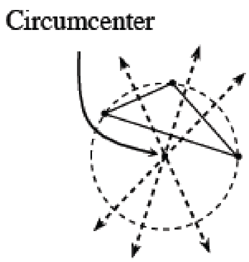
METHODS AND MEANINGS

Points of Concurrency

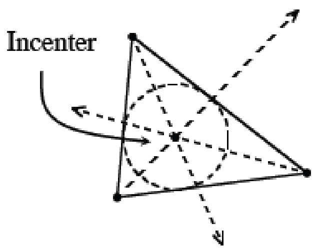
You learned that the **centroid** of a triangle is the point at which the three medians of a triangle intersect, as shown at right. When three lines intersect at a single point, that point is called a **point of concurrency**. Refer to the Math Notes box in Lesson 9.2.4.



A circle that **circumscribes** a triangle touches all three vertices of the triangle. The center of this circle is called the **circumcenter**. The circumcenter is another point of concurrency because it is located where the perpendicular bisectors of each side of a triangle meet. See the example at right.

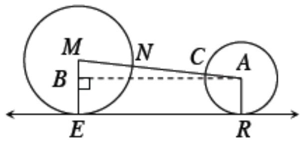


A circle that **inscribes** a triangle touches all three sides of the triangle just once. The center of this circle is called the **incenter**. The incenter is yet another point of concurrency because it is located where the three angle bisectors of a triangle meet.

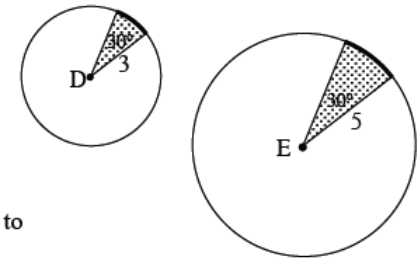


Review & Preview

10-54. In the diagram at right, $\odot M$ has radius length of 14 feet and $\odot A$ has radius length of 8 feet. $\overleftrightarrow{ER}$ is tangent to both $\odot M$ and $\odot A$. If $NC = 17$ ft feet, find ER.



10-55. Use the sectors in circles D and E to answer the following questions.

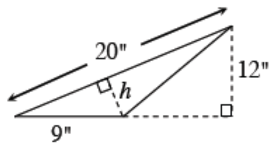


a. What is the ratio of the arc length to the radius of circle D? Leave your answer in terms of π.

b. What is the ratio of the arc length to the radius of circle E?

c. The ratio of the arc length to the radius of a circle is called a **radian**. Why will all sectors with a central angle of 30° have the same radian measure?

10-56. In the figure at right, find the interior height (h) of the obtuse triangle. Show all work.



10-57. A cylinder with volume 500π cm^3 is similar to a smaller cylinder. If the scale factor is $\frac{1}{5}$, what is the volume of the smaller cylinder? Explain your reasoning.

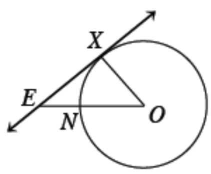
10-58. A six-year old house, now worth \$175,000, has had an annual appreciation of 5%.

a. What is the multiplier?

b. What did it cost when new?

c. Write a function of the form $f(t) = ab^t$, where t is the time in years, that represents the value of the house since it was new.

10-59. In the figure at right, $\overleftrightarrow{EX}$ is tangent to $\odot O$ at point X. $OE = 20$ cm and $XE = 15$ cm.

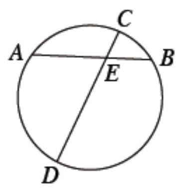


a. What is the area of the circle?

b. What is the area of the sector bounded by $\overline{OX}$ and $\overline{ON}$?

c. Find the area of the region bounded by $\overline{XE}$, $\overline{NE}$, and $\widehat{NX}$.

10-60. **Multiple Choice:** In the circle at right, $\overline{CD}$ is a diameter. If $AE = 10$, $CE = 4$, and $AB = 16$, what is the length of the radius of the circle?

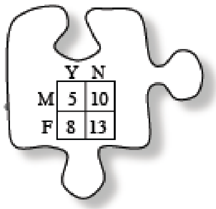


a. 15 b. 16

c. 18 d. 19

e. None of these

10.2.1 What does independence tell me?



Conditional Probability and Independence

When two events can occur, either simultaneously or one after the other, how can you calculate the probability of one of them when you know the other has already happened? You will investigate this question today. Then you will see how finding **conditional probabilities** will help you determine if two events are independent of each other.

10-61. EIGHT THE HARD WAY

Maribelle is playing the board game Eight The Hard Way with her friends. Each player rolls two dice on their turn, and moves according to the sum on the dice. However, if a player rolls two fours (called "eight the hard way"), they instantly win the round of play and a new round is started.

Shayna stepped into the kitchen to get snacks when she heard Maribelle shout "*Woo Hoo! I got an eight!*"

Shayna knows Maribelle got an eight. With your team, help Shayna investigate the probability that Maribelle rolled two fours and won the round of play. In other words calculate the **conditional probability** that Maribelle rolled two fours, given that you know she already rolled a sum of eight.

a. Use an area model to represent all of the possible sums of numbers when rolling two dice.

b. However, since you know that Maribelle rolled a sum of eight, the sample space is changed. Now the sample space is only all the ways a sum of eight can be rolled. On your area model, shade all of the ways a sum of eight can be rolled. How many different ways can a sum of eight be rolled, that is, how many outcomes are in the new sample space?

c. You are interested in the event {eight the hard way}. How many different ways can two fours be rolled?

d. What is the probability of the event {eight the hard way} given that you know Maribelle already rolled a sum of eight?

e. Becca rolls "high" (meaning that she rolled a sum of nine or more). What is the conditional probability that she rolled an odd number, given that you know she rolled high?

10-62. At Einstein Technical University (ETU), data on engineering majors was collected:

	Engineering majors	Other majors
Live Off Campus	30	170
Live On Campus	6	34

a. What is the probability of a student living on campus at ETU?

b. Copy the table and shade the cells with engineering majors. What is the conditional probability of a student living on campus, given that you know a student is an engineering major?

c. Two events, A and B, are **independent** if knowing that B occurred does not change the probability of event A occurring. That is, two events, A and B, are independent if P(A given B) = P(A). Are the events {live on campus} and {engineering} independent?

d. Two events are **mutually exclusive** (or **disjoint**) if they cannot both occur at the same time. That is, two events are mutually exclusive if P(A and B) = 0. Are the events {on campus} and {engineering} mutually exclusive?

10-63. The following data was collected about students in Mr. Rexinger's high school statistics class.

	Wearing jeans	Not wearing jeans
Male	7	7
Female	5	13

a. Mr. Rexinger is playing a game with his students. He randomly chooses a student from his class roster. If a player guesses the gender of the student correctly, the player gets an early-lunch pass. Madeline is the next player. Which gender should she guess to have the greatest chances of winning the lunch pass? Explain.

b. Mr. Rexinger tells Madeline that the student is wearing jeans. Should Madeline change her guess? Explain.

c. In a previous course, you may have studied the **association** of two *numerical* variables by analyzing scatterplots and least squares regression lines. Associations between *categorical* variables are determined by independence – if two variables are independent then they are not associated.

Are the events {female} and {wearing jeans} associated for the students in Mr. Rexinger's class today? Explain using the independence relationship from part (c) of problem 10-62.

d. Are the events {female} and {wearing jeans} mutually exclusive? Explain.

10-64. At Digital Technical Institute, the following data was collected:

	Engineering majors	Other majors
Live Off Campus	30	170
Live On Campus	0	40

a. Are the events {live on campus} and {engineering majors} associated at this institute?

b. Are the events {live on campus} and {engineering} mutually exclusive at this institute? What outcomes are in the intersection of {live on campus} and {engineering}?

10-65. LEARNING LOG

Explain in your Learning Log what it means for two *numerical* variables to be associated, and give an example. What does it mean for two *categorical* variables to be associated? Give an example. Think of a new situation in which two events are associated mathematically, and explain the difference between mutually exclusive and independent events in your own words. Title this entry, "Independent or Mutually Exclusive?" and include today's date.

METHODS AND MEANINGS

MATH NOTES

Mutually Exclusive

Mutually exclusive events, also called **disjoint** events, can never both happen at the same time. When one of the events occurs, it means the other cannot possibly occur. If event B occurs, then you know that event A cannot occur: P(A and B) = 0 and the intersection of {A} and {B} contains no outcomes.

If events A and B are mutually exclusive, the occurrence of B tells you precisely about the probability of A occurring (A cannot occur). The probabilities of mutually exclusive events depend on each other. Mutually exclusive events are never independent (and thus always associated).

For example, suppose natural blondes occur in about 10% of the students at your school. Being naturally blonde and having naturally black hair are mutually exclusive – if one occurs, the other cannot possibly occur. If your friend tells you that a randomly selected person has naturally black hair, the probability they have naturally blonde hair is 0%. The probability of blonde has changed, knowing that the person has black hair. The events {blonde hair} and {black hair} are not independent.

Review & Preview

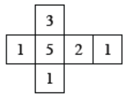
10-66. Natalie has a bag that contains eight marbles. She draws out a marble, records its color, and puts it back.

a. If Natalie repeats this eight times and does not record any red marbles, can she conclude that there are not any red marbles in the bag? Explain.

b. If she repeats this 100 times and does not record any red marbles, can she conclude that there are not any red marbles in the bag? Explain.

c. How many times would she have to draw marbles (putting them back each time) to be absolutely certain that there are no red marbles in the bag?

10-67. When the net at right is folded, it creates a die with values as shown.

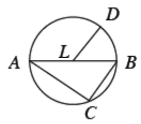


a. If the die is rolled randomly, what is P(even)? P(1)?

b. If the die is rolled randomly 60 times, how many times would you expect an odd number to land side-up? Explain how you know.

c. Now create your own net so that the resulting die has P(even) = $\frac{1}{3}$, P(3) = 0, and P(a number less than 5) = 1.

10-68. In the diagram at right, $\overline{AB}$ is a diameter of $\odot L$. If $BC = 5$ and $AC = 12$, use the relationships shown in the diagram to solve for the quantities listed below.



a. AB b. length of the radius of $\odot L$

c. $m\angle ABC$ d. $m\widehat{AC}$

10-69. When Erica and Ken explored a cave, they each found a gold nugget. Erica's nugget is similar to Ken's nugget. They measured the length of two matching parts of the nuggets and found that Erica's nugget is five times as long as Ken's. When they took their nuggets to the metallurgist to be analyzed, they learned that it would cost \$30 to have the surface area and weight of the smaller nugget calculated, and \$150 to have the same analysis done on the larger nugget.

"*I won't have that kind of money until I sell my nugget, and then I won't need it analyzed!*" Erica says.

"*Wait, Erica. Don't worry. I'm pretty sure we can get all the information we need for only \$30.*"

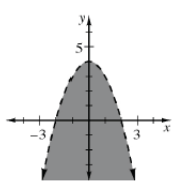
a. Explain how they can get all the information they need for \$30.

b. If Ken's nugget has a surface area of 20 cm^2, what is the surface area of Erica's nugget?

c. If Ken's nugget weighs 5.6 g (about 0.2 oz), what is the weight of Erica's nugget?

10-70. Find x if the angles in a quadrilateral are $2x$, $3x$, $4x$, and $5x$.

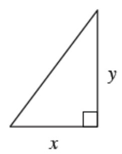
10-71. A graph of an inequality is shown at right. Decide if each of the points (x, y) listed below would make the inequality true or not. For each point, explain how you know.



a. (1, 1) b. (–3, 2)

c. (–2, 0) d. (0, –2)

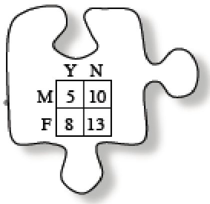
10-72. **Multiple Choice:** Which expression below represents the length of the hypotenuse of the triangle at right?



a. $\frac{y}{x}$ b. $\sqrt{x^2+y^2}$ c. $x+y$

d. $\sqrt{y^2-x^2}$ e. None of these

10.2.2 Is there another way to organize data?

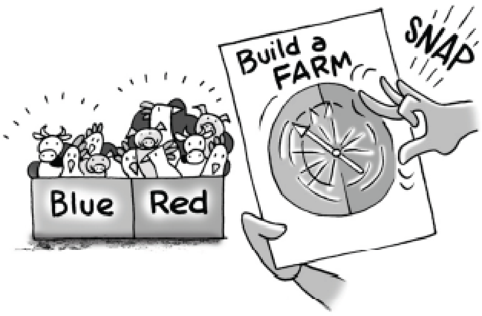


Two Way Tables

In the previous lesson, you were given the counts (or the frequency) of the number of people or objects in a given situation, and then, from this, you computed conditional probabilities. In this lesson, you will extend your understanding of conditional probabilities by starting from probabilities rather than counts. You will see how data or probabilities are often organized into **two-way tables**, and you will continue to investigate the association of two categorical variables.

10-73. BUILD-A-FARM

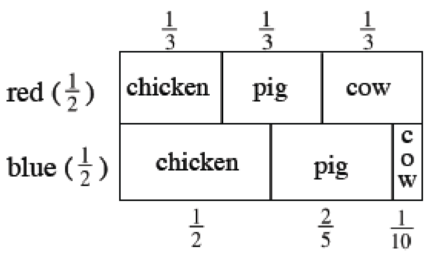
In the children's game, Build-a-Farm, each player first spins a spinner. Half of the time the spinner comes up red and half of the time the spinner comes up blue. If the spinner is red, the player reaches into the red box. If the spinner is blue, the player reaches into the blue box. The red box has 10 chicken counters, 10 pig counters, and 10 cow counters, while the blue box has 5 chicken counters, 4 pig counters, and 1 cow counter.



a. Draw a tree diagram, including probabilities, to represent the sample space for this game.

b. What is the probability of getting a cow counter in one turn?

c. Even though the events {spin} and {animal counter} are not independent, a modified area model, as shown at right, is possible for this situation.



Copy the area model and shade the parts of the diagram that correspond to getting a cow counter. Using the diagram, verify that P(cow) is the same as the one that you found in part (b).

d. Let's investigate the conditional probability that a child's spin was red, given that you know the child got a cow counter.

Since you know that the child got a cow counter, the new sample space is limited to only the outcomes that contain cow – which is the area that you shaded in part (c). Considering only the outcomes that contain cow, what is the conditional probability that a child spun red, knowing that the child got a cow counter?

e. Using the method in part (d), find the conditional probability that if you got a pig counter, your spin was blue.

10-74. FLIP TO SPIN OR ROLL

On the midway at the county fair, there are many popular games to play. One of them is Flip to Spin or Roll. First, the player flips a coin. If a head comes up, the player gets to spin the big wheel, which has ten equal sections: three red, three blue, and four yellow. If the coin shows a tail, the player gets to roll a cube with three red sides, two yellow sides, and one blue side. If the wheel spin lands on blue, or if the blue side of the cube comes up, the player wins a stuffed animal.

a. Draw a modified area model to represent the sample space for Flip to Spin or Roll. Note that the rectangles for heads will have different areas than the rectangles for tails.

b. Suppose that you know that Tyler won a stuffed animal. Discus this with your team and then shade the appropriate parts of the modified area model to help you figure out the probability that he started off by getting a head. Be prepared to share your ideas with the class.

10-75. Raul is conducting a survey for the school news blog. He surveyed 200 senior-class students and found that 78 students had access to a car on weekends, 54 students had regular chores assigned at home, and 80 students neither had access to a car, nor had regular chores to do. Raul said he couldn't figure out how to put the data into a table like the one at right.

	car	no car
chores		
no chores		

a. Copy and complete Raul's table to figure out the number of students in each cell.

b. This type of table is called a **two-way table** and is often used to organize information and calculate probabilities. Two-way tables often include row and column totals also. If you have not already done so, add row and column totals to your two-way table. Is there an association between car privileges and having regular chores for this group? Explain your answer in the context of the problem.

10-76. There are 30 students in Mr. Cooper's class; 18 boys and 12 girls. Mr. Cooper chooses a student at random to take the attendance folder to the office. Four of the boys have previously taken the folder to the office, and 3 of the girls have previously taken it.

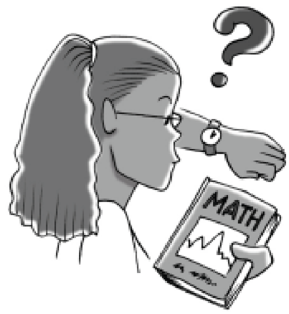
a. Create a two-way table to display this data.

b. If Mr. Cooper randomly selects a student, what is the probability he selects a boy who previously took the folder? Make a new two-table table, and fill in that probability. Then fill in the remaining cells with their respective probabilities. Include row and column totals.

c. If a student is chosen at random, what is the probability that the student is a girl or is a student that has taken the folder previously? Use the probabilities from the table that you made in part (b).

d. Shade the cells in your table from part (c) where a student has previously taken the folder. If a student previously took the folder, what is the probability that the student is a girl?

10-77. If Letitia studies for her math test tonight, she has an 80% chance of getting an A. If she does not study, she only has a 10% chance. Whether she can study or not depends on whether she has to work at her parents' store. Earlier in the day, her father said there is a 50% chance that Letitia would be able to study.



a. Draw a modified area model for the situation.

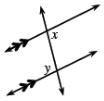
b. Find the probability that Letitia gets an A on the math test.

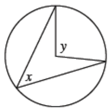
c. What are the chances that Letitia studied, given that she got an A? Show how you shaded the diagram.

d. Create a two-way table that shows the probabilities for this situation. Include row and column totals. Verify using your table that if she studies, Letitia has an 80% chance of getting an A as described in the beginning of this problem.

Review & Preview

10-78. For each diagram below, write an equation to represent the relationship between x and y.

a.


b.


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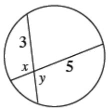

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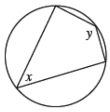
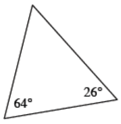


10-79. For each triangle below, use the information in the diagram to decide if it is a right triangle. Justify each conclusion. Assume the diagrams are not drawn to scale.

a.


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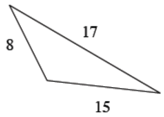


10-80. A cement block is the shape of a prism with length 1.5 ft, width 1 ft and height 1 ft. Centered on the top of the block and passing all the way through the block are two 0.25 ft by 0.2 ft rectangular holes.

a. Draw a diagram of the block.

b. What is the volume of the block?

10-81. A spinner is divided into two regions. One region, red, has a central angle of 60°. The other region is blue.

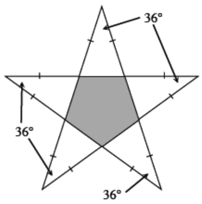
a. On your paper, sketch a picture of this spinner.

b. If the spinner is spun twice, what is the probability that both spins land on blue?

c. If the radius of the spinner is 7 cm, what is the area of the blue region?

d. A different spinner has three regions: purple, mauve, and green. If the probability of landing on purple is $\frac{1}{4}$ and the probability of landing on mauve is $\frac{2}{3}$, what is the central angle of the green region?

10-82. After doing well on a test, Althea's teacher placed a gold star on her paper. When Althea examined the star closely, she realized that it was really a regular pentagon surrounded by 5 isosceles triangles, as shown in the diagram at right. If the star has the angle measurements shown in the diagram, find the sum of the angles inside the shaded pentagon. Show all work.



10-83. Remember that the radian measure of a central angle of a circle is the ratio of the arc length to the radius (see problem 10-55).

a. What is the radian measure for a 45-degree central angle on a circle with radius 5 cm? What is the radian measure for a 45-degree central angle on a circle with radius 1 cm? Answer in terms of π.

b. The central angle of a circle has a radian measure of $\frac{\pi}{3}$. What is the measure of the central angle of the sector in degrees?

10-84. **Multiple Choice:** ΔABC is a right triangle and is graphed on coordinate axes. If $m\angle B = 90°$ and the slope of $\overline{AB}$ is $-\frac{4}{5}$, what is the slope of BC?

a. $\frac{4}{5}$ b. $\frac{5}{4}$ c. $-\frac{5}{4}$ d. $-\frac{4}{5}$

e. Cannot be determined

10-85. At the University of the Great Plains the following data about engineering majors was collected:

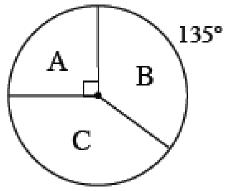
	Engineering major	Other major	
Live Off Campus	800	7200	8000
Live On Campus	120	11,880	12,000
	920	19,080	20,000

a. What is the conditional probability of living on campus, given that you know a student is an engineering major?

b. Compare your answer to part (a) to the probability of living on campus.

c. Are the two events, {living on campus} and {engineering major} associated? Use the probabilities to explain why or why not.

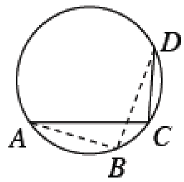
10-86. The spinner at right has three regions: A, B, and C. If it is spun 80 times, how many times would you expect each region to result? Show your work.



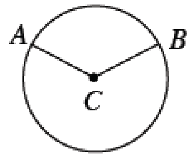
10-87. Review what you know about the angles and arcs of circles below.

a. A circle is divided into nine congruent sectors. What is the measure of each central angle?

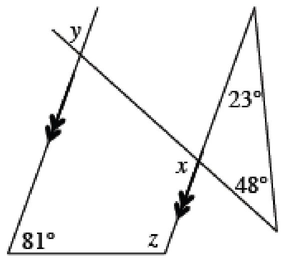
b. In the diagram at right, find $m\widehat{AD}$ and $m\angle C$ if $m\angle B = 97°$.



c. In $\odot C$ at right, $m\angle ACB = 125°$ and $r = 8$ inches. Find $m\widehat{AB}$ and the length of $\widehat{AB}$. Then find the area of the smaller sector.



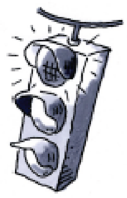
10-88. Examine the diagram at right. Use the given geometric relationships to solve for x, y, and z. Be sure to justify your work by stating the geometric relationship and applicable theorem.



10-89. Solve each equation below for x. Check your work.

a. $\frac{x}{2} = 17$ b. $\frac{x}{4} = \frac{1}{3}$ c. $\frac{x+6}{2} + 2 = \frac{5}{2}$ d. $\frac{4}{x} = \frac{5}{8}$

10-90. Mrs. Cassidy solved the problem $(w-3)(w+5) = 9$ and got $w = 3$ and $w = -5$. Is she correct? If so, show how you know. If not, show how you know and find the correct solution.

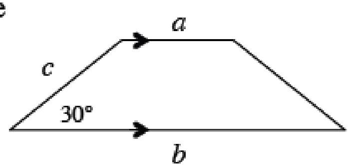


10-91. **Multiple Choice:** Which expression represents the area of the trapezoid at right?

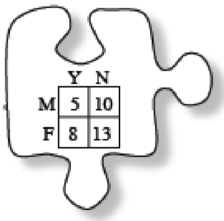
a. $\frac{c(a+b)}{4}$ b. $\frac{c(a+b)}{2}$

c. $\frac{bc}{2}$ d. $\frac{a+b+c}{2}$

e. None of these



10.2.3 How can I pull it all together?



Applications of Probability

Probability has uses far beyond its origins in games of chance. Often, probabilities are based on survey data or data taken from a sample population. Today you will learn two new probability rules, and at the end of the lesson you will be given an opportunity to summarize the probability rules that you know.

10-92. In a recent survey of college freshman, 35% of students checked the box next to "Exercise regularly," 33% checked the box next to "Eat five servings of fruits and vegetables a day," and 57% checked the box next to "Neither."

a. Create a two-way table to represent this situation. Include row and column totals.

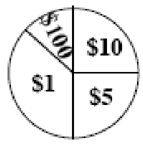
b. What is the probability that a freshman in this study exercises regularly *and* eats 5 servings of fruits and vegetables each day?

c. What is the probability that a freshman in this study exercises regularly *or* eats 5 servings of fruits and vegetables each day?

d. Do you think that freshmen who eat 5 servings of fruit and vegetables per day are more likely to exercise? In other words, are exercising and eating associated?

e. Compare and contrast a two-way table with an area model.

10-93. DOUBLE SPIN



Remember Double Spin, the game at the fair from Chapter 4? The player gets to spin a spinner twice, but only wins if the same amount comes up both times. The \$100 sector is $\frac{1}{8}$ of the circle. Nick is currently playing the game.

a. Make an area model to show the sample space of every possible outcome for two spins. What is the probability that Nick wins?

b. When Nick came home from the fair, he told Zack that he had won some money in the Double Spin game. Knowing that Nick won some money, what are the chances that he won \$100?

c. Make a two-way table that shows the probabilities for the Double Spin game. How does your table compare to the area model from part (a)? Explain.

d. A mathematical way to express the conditional probability relationship is:

$$P(\text{A given B}) = \frac{P(\text{A and B})}{P(\text{B})}.$$

This relationship is called the **Multiplication Rule**. (You will learn why it is called that in the next problem.) Verify your answer to part (b) using the Multiplication Rule. Be sure to define events A and B.

10-94. ANOTHER DEFINITION FOR INDEPENDENCE

a. To learn how the Multiplication Rule got its name, rewrite the Multiplication Rule starting with "P(A and B) = ".

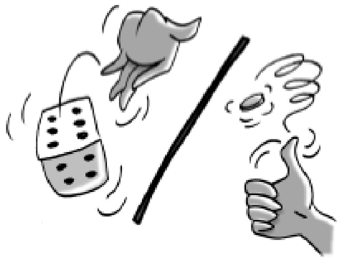
b. Write the relationship between event A and event B for when they are independent using symbols.

c. Substitute the independence relationship from part (b) into the Multiplication Rule that you wrote for part (a) to get another definition for independence.

10-95. The Laundry Shop sells washers and dryers. The owner of the store, Mr. McGee, thinks that a customer who purchases a washer is more likely to purchase a dryer than a customer that did not purchase a washer. He analyzes the sales from the last month and finds that a total of 240 customers made purchases. He counts 180 washers that were purchased and 96 dryers that were purchased. Mr. McGee then counts the number of sales that included both a washer and a dryer and finds 72 customers purchased both.

Is there an association between the purchase of washers and dryers? Explain and show your reasoning using the relationships that you have learned in this lesson.

10-96. SHIFTY SHAUNA



Shauna has a bad relationship with the truth – she doesn't usually tell it! In fact, whenever Shauna is asked a question, she rolls a die. If it comes up 6, she tells the truth. Otherwise, she lies.

a. If Shauna flips a fair coin and you ask her how it came out, what is the probability that she says "heads" *and* is telling the truth? Choose a method to solve this problem and carefully record your work. Be ready to share your solution method with the class.

b. Suppose Shauna flips a fair coin and you ask her whether it came up heads or tails. What is the probability that she says "heads"? (Hint: The answer is not $\frac{1}{12}$!)

c. Suppose Shauna tells you that the coin says heads. What is the probability that she really did flip heads?

d. Is whether Shauna lies or tells the truth independent of whether the coin lands on heads or tails?

10-97. It is generally assumed that there is no relationship between height and IQ (a measure of intelligence). Thus, the heights for 175 people randomly selected people are independent of their IQ's.

Using this assumption, complete the two-way table below.

	Below average height	Above average height	
Below average IQ			70
Above average IQ			105
	50	125	175

10-98. A spinner has just two colors, red and blue. The probability the spinner will land on blue is x.

a. What is the probability it will land on red?

b. Sketch an area model for spinning this spinner twice.

c. When the spinner is spun twice, what is the probability that it will land on the same color both times?

d. Given that the spinner lands on the same color twice, what is the probability that it landed on blue both times?

10-99. On another spinner, blue occurs a fraction x of the time, while the red and green portions have equal area. There are no other colors on the spinner.

a. Find the probability that the spinner will land on green.

b. Sketch an area diagram for spinning the spinner twice.

c. Shade the region on your area diagram corresponding to getting the same color on the spinner twice.

d. What is the probability that both spins give the same color?

e. If you know that you got the same color twice, what is the probability that the color was blue?

10-100. LEARNING LOG

Title this Learning Log entry, "Probability Rules" and include today's date. First, consider the Build A Farm game described in problem 10-73. What are three different models you can use to find probabilities?

Then summarize the probability rules you know as follows:

- State the Addition Rules. Also state the rule as a union or intersection, whichever is appropriate.
- State both versions of the Multiplication Rule (the multiplication and the division versions).
- State two different rules that define independence.
- For each rule, create examples from the Build A Farm game.

METHODS AND MEANINGS

MATH NOTES

Conditional Probability and Independence

When you are calculating a probability, but have been given additional information about an event that has already occurred, you are calculating a **conditional probability**. For the conditional probability P(A given B), you know that event B has occurred, so event B becomes the sample space of all possible outcomes. P(A given B) is the fraction of event B's outcomes that also include event A, which is formally stated as the **Multiplication Rule**:

$$\text{P(A given B)} = \frac{\text{P(A and B)}}{\text{P(B)}}$$

Two events are **independent** when the outcome of one does not influence the outcome of the other. Two independent events could both occur, but knowing event B has occurred does not change the probability of event A occurring, thus P(A given B) = P(A). When events are not independent, you say that they are associated. That is, one event influences the other.

If you substitute the definition for independence, P(A given B) = P(A), into the Multiplication Rule and rearrange the result, you get an alternate definition for independence: If events A and B are independent, then P(A and B) = P(A) · P(B). The converse of this statement is also true: If P(A and B) = P(A) · P(B), then A and B are independent.

Review & Preview

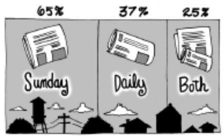
10-101. A technology group wants to determine if bringing a laptop on a trip that involves flying is related to people being on business trips. Data for 1000 random passengers at an airport was collected and summarized in the table below.

	Laptop	No laptop
Traveling for business	236	274
Not traveling for business	93	397

a. What is the probability of traveling with a laptop if someone is traveling for business?

b. Does it appear that there is an association between bringing a laptop on a trip that involves flying and traveling for business?

10-102. In a certain small town, 65% of the households subscribe to the daily paper, 37% subscribe to the weekly local paper, and 25% subscribe to both papers.



a. Make a two-way table to represent this data.

b. If a household is selected at random, what is the probability that it subscribes to at least one of the two papers? Shade these areas in your table.

c. Charlie's neighbor subscribes to a paper. What is the probability that he receives the daily paper?

10-103. The Sunshine Orange Juice Company wants its product in a one-quart container (1 quart equals 107.75 cubic inches). The manufacturer for their containers makes cylindrical cans that have a base that is 5 inches in diameter. What will be the height of the one-quart container?

10-104. The radian measure of a central angle of 90° is $\frac{\pi}{2}$. What must the radian measure of a central angle of 180° be? How can you tell without actually computing the radian measure?

10-105. The mat plan for a three-dimensional solid is shown at right.

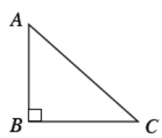
0	0	0	RIGHT
0	1	0	
2	3	1	
FRONT			

a. On graph paper, draw *all* of views of this solid. (There are six views.) Compare the views. Are any the same?

b. Find the volume and surface area of the solid. Explain your method.

c. Do the views you drew in part (a) help calculate volume or surface area? Explain.

10-106. For the triangle at right, find each trigonometric ratio below. The first one is done for you.



a. $\tan C = \frac{AB}{BC}$ b. $\sin C$

c. $\tan A$ d. $\cos C$

e. $\cos A$ f. $\sin A$

10-107. Review circle relationships as you answer the questions below.

a. On your paper, draw a diagram of $\odot B$ with $\overset{\frown}{AC}$. If $m\overset{\frown}{AC} = 80°$ and the length of the radius of $\odot B$ is 10, find the length of chord $\overline{AC}$.

b. Now draw a diagram of a circle with two chords, $\overline{EF}$ and $\overline{GH}$, that intersect at point K. If $EF = 15$, $EK = 6$, and $HK = 3$, what is GK?

10.3.1 What if the sample space is very large?

The Fundamental Principle of Counting

Phone numbers in the U.S. are composed of a three-digit area code followed by seven digits. License plates in some states are made up of three letters followed by a three-digit number. Postal ZIP codes are made up of five digits, and another four digits are often added. To win the lottery in one state you need to select the correct five numbers from all the possible choices of five numbers out of 56. Consider these questions:

- How likely is it that you could win the lottery?
- Are there enough phone numbers for the dramatic increase in cell phones, tablets, and e-readers for books, many of which use unseen phone numbers to download information?
- Jay wants to know the probability of randomly getting JAY on his license plates so he can avoid paying the extra amount for a personalized license plate.

The sample spaces for these questions are very large. Imagine trying to draw a tree diagram! Tree diagrams with three or four branches, branching two or three times, are messy enough. You need a way to count possibilities without having to draw a complete diagram. In this and the next two lessons, you and your team will develop some strategies that will allow you to account for all possibilities without having to make a complete list or draw a complete tree diagram. As you work on the problems in this lesson, discuss the following questions with your team:

What decisions am I making when I make a systematic list?

How many decisions do I need to make?

How many ways are there to make each decision?

How can I use the patterns in a tree diagram to find the total number of branches?

10-108. Nick came across the following problem: If a 4-digit number is randomly selected from all of the 4-digit numbers that use the digits 1, 2, 3, 4, 5, 6, and 7, with repeated digits allowed, what is the probability that the selected number is 2763? Nick knew that he had to figure out how many numbers were possible, in order to know the size of his sample space.

a. Nick started to make a systematic list of the possibilities, but after the first few he gave up. What is the difficulty in trying to create a list?

b. Next he started a tree diagram. What problem did he encounter with the tree?

c. Nick decided he needed a shortcut strategy for organizing this problem; otherwise he was going to be up all night. He started by asking himself, *"How many decisions (about the digits) do I need to make?"* With your team discuss his first question and then consider his next question, *"How many choices do I have for each decision (each digit)?"*

d. Audrey was at her house working on the same problem. She was thinking of a tree diagram, when she asked herself, *"How many branch points will this tree have?"* and *"How many branches at each point?"* What are the answers to her questions? How are these questions related to the one that Nick was pondering?

e. At the same moment, they text messaged each other that they were stuck. When they talked, they realized they were on the same track. The problem asks for four-digit numbers, so there are four decisions. Simultaneously they said, *"We need a **decision chart**."* They wrote the following on their papers:

______	______	______	______
1st digit	***2nd digit***	***3rd digit***	***4th digit***

How many choices are there for each decision? How many four digit numbers are there?

f. What is the probability that the randomly selected number will be 2763?

10-109. How many four-digit numbers could you make with the digits 1, 2, 3, 4, 5, 6, and 7 if you could not use any digit more than once in the four-digit number? Make a decision chart and explain the similarities and differences between this situation and the one described in problem 10-108.

10-110. The basis for a decision chart is the **Fundamental Principle of Counting**. Read the Math Notes box at the end of this lesson to help you understand the Fundamental Principle of Counting. Then use a decision chart to answer each question below.

a. A game contains nine discs, each with one of the numbers 1, 2, 3, 4, 5, 6, 7, 8, or 9 on it. How many different three-digit numbers can be formed by choosing any three discs, without replacing the discs?

b. A new lotto game called Quick Spin has three wheels, each with the numbers 1, 2, 3, 4, 5, 6, 7, 8, and 9 equally spaced around the rim. Each wheel is spun once, and the numbers the arrows point to are recorded in order. How many three-digit numbers are possible?

c. Explain the similarities and differences between part (a) and part (b).

10-111. Marcos is selecting classes for next year. He plans to take English, physics, government, pre-calculus, Spanish, and journalism. His school has a six-period day, so he will have one of these classes each period.

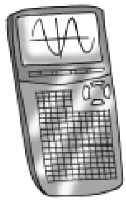
a. How many different schedules are possible?

b. How many schedules are possible with first-period pre-calculus?

c. What is the probability that Marcos will get first-period pre-calculus?

d. What is the probability that Marcos will get both first-period pre-calculus *and* second-period physics?

10-112. CAN MY CALCULATOR FIND IT FASTER?



a. How many possible ways can the letters in the word MATH be arranged?

b. On your calculator, find the **factorial** function, $n!$ or !. On many scientific calculators, it can be found by pressing the PRB key. On many graphing calculators, it is a function in the math menu and probability submenu.

Find the value of 7 factorial (written 7!), then 6!, then 5!, 4!, … , 1!

c. How do you think your calculator computes 5!

d. Explain why 4! gives the correct solution to the possible number of ways to arrange the letters M A T H.

e. What happens when you try to find 70! with your calculator? Why?

10-113. Remembering what $n!$ means can help you do some messy calculations quickly, as well as help you do problems that might be too large for your calculator's memory.

For instance, if you wanted to calculate $\frac{9!}{6!}$, you could use the $n!$ button on your calculator and find that 9! = 362,880 and 6! = 720, so $\frac{9!}{6!} = \frac{362880}{720} = 504$.

You could also use a simplification technique. Since $9! = 9 \bullet 8 \bullet 7 \bullet 6 \bullet 5 \bullet 4 \bullet 3 \bullet 2 \bullet 1$ and $6! = 6 \bullet 5 \bullet 4 \bullet 3 \bullet 2 \bullet 1$, you can rewrite $\frac{9!}{6!} = \frac{9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 9 \cdot 8 \cdot 7 = 504$.

Use this simplification technique to simplify each of the following problems before computing the result.

a. $\frac{10!}{8!}$ b. $\frac{70!}{68!}$ c. $\frac{7!}{4!3!}$ d. $\frac{20!}{18!2!}$

METHODS AND MEANINGS

MATH NOTES

Fundamental Principle of Counting

The **Fundamental Principle of Counting** is a method for counting the number of outcomes (the size of the sample space) of a probabilistic situation, often where the order of the outcomes matters. If event {A} has m outcomes, and event {B} has n outcomes after event {A} has occurred, then the event {A} followed by event {B} has $m \bullet n$ outcomes.

For a sequence of events, a tree diagram could be used to count the number of outcomes, but if the number of outcomes is large a **decision chart** is more useful.

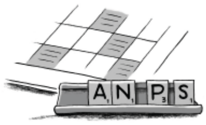
For example, how many three-letter arrangements could be made by lining up any three blocks, chosen from a set of 26 alphabet blocks, if the first letter must be a vowel? There are three decisions (three blocks to be chosen), with 5 choices for the first letter (a vowel), 25 for the second, and 24 for the third. According to the Fundamental Principle of Counting, the total number of possibilities is:

$$\underset{\text{1st decision}}{\underline{\quad 5 \quad}} \cdot \underset{\text{2nd decision}}{\underline{\quad 25 \quad}} \cdot \underset{\text{3rd decision}}{\underline{\quad 24 \quad}} = 3000 .$$

This decision chart is a way to represent a tree with 5 branches for the first alphabet block, followed by 25 branches for each of those branches; each of those 125 branches would then have 24 branches representing the possibilities for the third alphabet block.

Review & Preview

10-114. A Scrabble® player has four tiles with the letters A, N, P, and S.



a. How many arrangements of these letters are possible?

b. Draw a tree diagram that shows how to get the arrangements and explain how a decision chart represents the tree.

c. What is the probability of a two-year-old randomly making a word using the four letters?

10-115. Five students are running for Junior class president. They must give speeches before the election committee. They draw straws to see who will go first, second, etc. In how many different orders could they give their speeches?

10-116. Parents keep telling their teens to "turn down the music" or "turn off the computer" when studying. But teens insist that these "distractions" actually help them study better! In order to put this argument to rest, a psychologist studied whether subjects were able to memorize 20 index cards while listening to loud music or studying in silence. The sixty subjects had these results:

	Able to memorize	Not able to memorize
Loud music	9	36
Silence	3	12

a. What is the probability that a randomly chosen subject is able to memorize the index cards?

b. What is the probability that a music listener memorizes the index cards?

c. According to the data from this study, is the ability to memorize independent of listening to loud music?

10-117. Marty and Gerri played Pick a Tile, in which the player reaches into two bags. One bag contains square tiles and the other circular tiles. The bag with squares contains three yellow, one blue, and two red squares. The bag with circles has one yellow and two red circles. In order to win the game (and a large stuffed animal), a player must choose one blue square and one red circle.

a. Complete the two-way table below.

CIRCLES

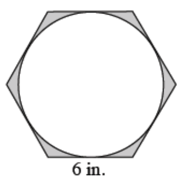
SQUARES		red	yellow	
	yellow			$\frac{3}{6}$
	red			
	blue			
		$\frac{2}{3}$		1

b. What is the probability of a player choosing the wining blue-red combination?

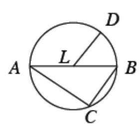
c. When Marty pulled her hand out of the bag, Gerri squealed with delight because she thought she saw something blue. If it was something blue, what is the probability that Marty won a stuffed animal?

10-118. The *sum* of the lengths of the edges of a cube is 1200 cm. Find the surface area and the volume of the cube.

10-119. The circle at right is inscribed in a regular hexagon. Find the area of the shaded region.



10-120. **Multiple Choice:** Examine $\odot L$ at right. Which of the mathematical statements below is not necessarily true?



a. $LD = AL$

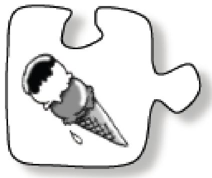
b. $m\angle DLB = m\widehat{DB}$

c. $\overline{LD} \parallel \overline{CB}$

d. $m\widehat{BC} = 2m\angle BAC$

e. $2AL = AB$

10.3.2 How can I count arrangements?



Permutations

There are many kinds of counting problems. In this lesson you will learn to recognize problems that involve arrangements. In some cases outcomes will be repeated, but in others they will not. A list of **permutations** includes different arrangements of distinct objects chosen from a set of objects. In other words, permutations are arrangements of elements without using any element more than once, and without repetition. As you work on the problems in this lesson discuss the following questions with your team:

When I make a decision chart, how many choices do I have after I make the first choice? The second? The third? …

Can I use the same choice again?

Can this situation be represented as a permutation?

What patterns can I find in these problems?

10-121. Jasper finally managed to save enough money to open a savings account at the credit union. When he went in to open the account, the accounts manager told him that he needed to select a four-digit PIN (personal identification number). She also said that he could not repeat a digit, but that he could use any of the digits 0, 1, 2, …, 9 for any place in his four-digit PIN.

a. How many different PIN's are possible?

b. Notice that the decision chart for this problem looks like the beginning of 10!, but it does not go all the way down to 1. Factorials can be used to represent this problem, but you must compensate for the factors that you do not use, so you can write $\frac{10!}{6!}$. Discuss with your team how this method gives the same result as your decision chart.

10-122. With your team, discuss how you could use factorials to represent each of the following situations. Then find the solutions. Four of the five problems involve permutations, and one does not. As you work, discuss with your team which problems fit the definition for permutations and why or why not. Write your answers both as factorials and as whole numbers.

a. Fifty-two contestants entered a contest for a new school logo design. In how many different ways can the judges pick the best logo and the runners-up one, two, and three?

b. The volleyball team is sponsoring a mixed-doubles sand court volleyball tournament and sixteen pairs have signed up for the chance to win one of the seven trophies and cash prizes. In how many different ways can the teams finish in the top seven slots?

c. Carmen is getting a new locker at school, and the first thing she must do is decide on a new locker combination. The three-number locker combination can be picked from the numbers 0 through 35. How many different locker combinations could she create if none of the numbers can be repeated?

d. How many three-digit locker combinations could Carmen make up if zero could only be the second or third number and none of the numbers can be repeated?

e. How many locker combinations can Carmen have if she can use any of the numbers 0 through 35 and she can repeat numbers? Is this still a permutation? Explain why you think that it is or is not.

10-123. Problems about the order of teams or winners, and questions about how many numbers you could make without repeating any digits, are called **permutations**.

a. Below is a list of all of the license plate letter triples that can be made with the letters A, B, and C.

AAA	BBB	CCC	AAB	ABA
BAA	AAC	ACA	CAA	ABB
BAB	BBA	ACC	CAC	CCA
ABC	ACB	CAB	BAC	CBA
BCA	BCC	CBC	CCB	CBB
BCB	BBC			

How is this list different from all the arrangements a child can make on a line on the refrigerator door with three magnetic letters A, B, and C. Make the list of arrangements the child can make with the refrigerator magnets. Why are the lists different? Which one is a permutation?

b. Imagine a group of 8 candidates: one will become president, one vice president, and one secretary of the school senate. Now imagine a different group of 8 applicants, three of whom will be selected to be on the spirit committee. How will the lists of three possible people selected from the 8 people differ? Which list would be longer? Which is a permutation?

c. Consider these two situations. Decide if they are permutations. Why or why not?

- The possible 4-digit numbers you could write if you could choose any digit from the numbers 2, 3, 4, 5, 6, 7, 8, and you could use digits several times.
- All the 4-digit numbers you could make using seven square tiles numbered 2, 3, 4, 5, 6, 7, and 8.

d. What are the important characteristics that a counting problem has to have in order to classify it as a permutation problem? Discuss this with your team and then write a *general* method for counting the number of arrangements in any problem that could be identified as a permutations problem.

10-124. WHAT IS THE FORMULA?

a. In part (a) of problem 10-122 you calculated how many ways judges could pick the logo contest winner and three runners up from 52 contestants. The answer can be written using factorials as $\frac{52!}{48!}$. Explain where these numbers came from.

b. The logo contest situation can be thought of as finding the number of possible arrangements of 52 elements arranged 4 at a time. Reexamine your answers to parts (b) and (c) of problem 10-122 and use your answers to write a general formula to calculate the number of possible arrangements of n objects arranged r at a time. Begin your formula with ${}_nP_r =$.

c. Use your formula from part (b) above to calculate:

i. ${}_7P_4$ *ii.* ${}_{52}P_4$ *iii.* ${}_{16}P_7$

10-125. ANAGRAMS

a. How many distinct ways can the letters in the word MASH be arranged?

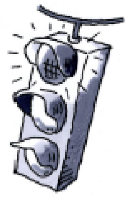
b. How many distinct ways can the letters in the word SASH be arranged? Use a tree diagram if it helps.

c. How many distinct ways can the letters in the word SASS be arranged?

d. Express your answers to parts (b) and (c) using fractions with factorials. The numerators should both be 4!.

e. How can you use fractions with factorials to account for repeated letters when counting the number of arrangements?

10-126. Sasha wonders how many distinct ways she can arrange the letters in her name. She thinks the answer is $\frac{5!}{4!} = 5$. What is her mistake? What is the correct answer, written using factorials?



MATH NOTES

METHODS AND MEANINGS

n! and Permutations

A **factorial** is shorthand for the product of a list of consecutive, descending whole numbers from the largest down to 1:

$$n! = n(n-1)(n-2) \ldots (3)(2)(1)$$

For example, 4 factorial or $4! = 4 \bullet 3 \bullet 2 \bullet 1 = 24$ and $6! = 6 \bullet 5 \bullet 4 \bullet 3 \bullet 2 \bullet 1 = 720$.

A **permutation** is an arrangement of items in which the order of selection matters and items cannot be selected more than once. The number of permutations that can be made by selecting r items from a set of n items can be represented with tree diagrams or decision charts, or calculated ${}_nP_r = \frac{n!}{(n-r)!} = n(n-1)(n-2)\ldots(n-r+1)$.

For example, eight people are running a race. In how many different ways can they come in first, second, and third? The result can be represented ${}_8P_3$, which means the number of ways to choose *and* arrange three different (not repeated) things from a set of eight.

$${}_8P_3 = \frac{8!}{(8-3)!} = \frac{8!}{5!} = \frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 8 \cdot 7 \cdot 6 = 336$$

Review & Preview

10-127. For the homecoming football game the cheerleaders at High Tech High printed each letter of the name of the school's mascot, WIZARDS, on a large card. Each card has one letter on it, and each cheerleader is supposed to hold up one card. At the end of the first quarter, they realize that someone has mixed up the cards.

a. How many ways are there to arrange the cards?

b. If they had not noticed the mix up, what would be the probability that the cards would have correctly spelled out the mascot?

10-128. Twelve horses raced in the CPM Derby.

a. How many ways could the horses finish in the top three places?

b. If you have not already done so, write your answer to part (a) as a fraction with factorials.

10-129. An engineer is designing the operator panel for a water treatment plant. The operator will be able to see four LED lights in a row that indicate the condition of the water treatment system. LEDs can be red, yellow, green, or off. How many different conditions can be signaled with the LEDs?

10-130. An insurance company wants to charge a higher premium to drivers of red cars because they believe that they get more speeding tickets. A research company collected the following data to investigate their claim. Use the data below to decide if the insurance company should be charging a higher premium to drivers of red cars.

Total: 20,000 cars with 507 speeding tickets

Red Cars: 348 red cars with 9 speeding tickets.

10-131. A survey of local car dealers revealed that 64% of all cars sold last month had a Green Fang system, 28% had alarm systems, and 22% had both Green Fang and alarm systems.

a. What is the probability one of these cars selected at random had neither Green Fang nor an alarm system?

b. What is the probability that a car had Green Fang and was not protected by an alarm system?

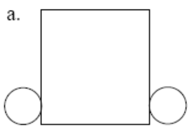
c. Are having Green Fang and an alarm system disjoint (mutually exclusive) events?

d. Use the alternative definition of independence (see the Math Notes box in Lesson 10.2.3) to determine if having Green Fang is associated with having an alarm.

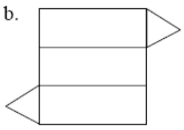
10-132. In the past, many states had license plates composed of three letters followed by three digits (0 to 9). Recently, many states have responded to the increased number of cars by adding one digit (1 to 9) ahead of the three letters. How many more license plates of the second type are possible? What is the probability of being randomly assigned a license plate containing ALG 2?

10-133. Describe the solid represented by each net below.

a.



b.



c.

10.3.3 How many groups are possible?

Combinations

In the previous lesson you learned a method for counting arrangements, including permutations. In this lesson you will consider questions such as how many five-card poker hands are possible, how many spirit committees can be selected from the Junior class, or your chances of winning the lottery.

In a five-card poker hand, the *arrangement* of the cards does *not* matter. Since all the spirit committee members have equal status, the *order* in which they are selected does *not* matter. If you have a winning lottery ticket, you will not care about the order in which the numbers are drawn. In these situations, you need to count the **combinations**. As you work with your team on the problems in this lesson, use the following questions to help focus your discussion:

Does the particular arrangement matter?

What are the relationships among these situations?

10-134. Five members of the Spirit Club have volunteered for the club governing board. These members are Al, Barbara, Carl, Dale, and Ernie. The club members will select three of the five as board members for the next year. One way to do this would be to elect a governing committee of three in which all members would have the same title. A second way would be to select a president, vice-president, and secretary.

a. How many different lineups of officers are possible? This means a president, vice-president, and a secretary are chosen. Thus, Al as president, Barbara as vice-president, and Carl as secretary would be a different possibility from Al as president, Barbara as secretary, and Carl as vice-president.

b. How many different three-member committees are possible? In this case, it is a good idea to make a list of all the possibilities, which are called **combinations**.

c. Felicia decides that she wants to volunteer as well.

i. How many different possibilities for officers are possible now?

ii. How many different governing committees are possible now? Again, make the list of all of the possibilities, or combinations.

d. Since there are more volunteers, the spirit club has decided to appoint another committee member.

i. If they add a treasurer to the list of officers, how many different ways are there to select the four officers are possible?

ii. If they choose a governing committee of four, how many possibilities are there?

10-135. Compare the results you got for each set of numbers in problem 10-134 when the roles were determined (permutations) and when there were no specific roles (combinations).

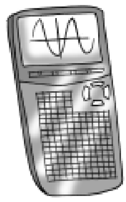
a. How do the number of combinations and permutations compare in each situation?

b. Work with your team to develop a conjecture about the mathematical relationship between permutations and combinations chosen from the same sized groups. Be prepared to share your thinking with the class.

c. Test your conjecture by calculating the number of permutations and combinations of 2 items chosen from 6. Does it work?

d. How can you generalize your conjecture so that it can be applied to permutations and combinations of r items chosen from n? Write a formula relating permutations (written ${}_nP_r$) and combinations (written ${}_nC_r$).

10-136. Now you will use your calculator to test the formula you wrote in problem 10-135.



a. Try 4 items chosen from 20. Does your formula work?

b. With your team, find a way to justify the logic of your formula. How can you convince someone that it has to be correct for all numbers?

10-137. In one state lottery, there are 56 numbers from which a player can choose six.

a. Does the order in which the numbers are chosen matter?

b. Find the number of possible combinations for a set of 6 winning lottery numbers.

c. What is the probability of selecting the six winning numbers?

10-138. In the game of poker called Five-Card Draw, each player is dealt five cards from a standard deck of 52 cards. While players tend to arrange the cards in their hands, the order in which they get them does not matter. How many five-card poker hands are possible? Use the methods you developed in today's investigation to answer this question.

METHODS AND MEANINGS

MATH NOTES

Combinations

When selecting committees, it matters who is selected but not the **order** of selection or any **arrangement** of the groups. Selections of committees, or of lists of groups without regard to the order within the group, are called **combinations**. Note that combinations do not include repeated elements.

For example: Eight people are eligible to receive $500 scholarships, but only three will be selected. How many different ways are there to select a group of three?

This is a problem of counting combinations. ${}_8C_3$ represents the number of ways to choose three from a set of eight. This is sometimes read as "eight *choose* three."

To compute the number of combinations, first calculate the number of permutations and then divide by the number of ways to arrange each permutation.

$$ {}_8C_3 = \frac{{}_8P_3}{3!} = \frac{8!}{5!3!} = 56 $$

In general: Number of ways to choose = $\frac{\text{\# of ways to choose and arrange}}{\text{\# of ways to arrange}}$

$$ {}_nC_r = \frac{{}_nP_r}{r!} = \frac{n!}{(n-r)!r!} $$

Review & Preview

10-139. How many different batting orders can be made from the nine starting players on a baseball team? Write the answer using factorials and as a number.

10-140. What do you think 0! is equal to?

a. Try it on your calculator to see what you get.

b. What does ${}_8P_8$ mean? What *should* ${}_8P_8$ be equal to? Write ${}_8P_8$ using the factorial formula. Why is it necessary for $0!$ to equal 1?

c. Do you remember how to show that $2^0 = 1$? You can use a sequence of powers of two like this: $\frac{2^4}{2} = 2^3$, $\frac{2^3}{2} = 2^2$, $\frac{2^2}{2} = 2^1$, so $\frac{2^1}{2} = 2^0$. Since $\frac{2^1}{2} = 1$ you also know that $2^0 = 1$.

You can construct a similar pattern for $0!$, starting with $\frac{5!}{5} = 4!$ and then $\frac{4!}{4} = 3!$. Continue the pattern and make an argument to justify that $0! = 1$.

10-141. For the each of the following, write down a factorial expression and then compute the value. Use the ${}_nC_r$ and ${}_nP_r$ functions on your calculator to make the computation.

a. ${}_{10}P_8$ b. ${}_{10}C_8$ c. ${}_6C_1$

10-142. Of the 63 drinks at Joe's Java and Juice Hut, 42 contain coffee and 21 contain dairy products. If Alexei randomly chooses a drink, what is the probability of getting a drink with both coffee and a dairy product? What is the probability of getting neither coffee nor a dairy product? Assume that choosing a coffee drink is independent of choosing a dairy-product drink. See if you can answer these questions without making a two-way table first.

10-143. Akio coaches the girls volleyball team. He needs to select players for the six different starting positions from his roster of 16 players. On Akio's teams, each position has its own special responsibility: setter, front left-side and middle hitters, back right- and left-side passers, and libero.

a. If he chooses randomly, how many ways can Akio form his starting lineup?

b. How many of those teams have Sidney playing in the libero position?

c. If Akio chooses starting teams randomly, what is the probability (in percent) that Sidney gets chosen as the starting libero?

10-144. If $f(n) = n!$, evaluate each of the following ratios.

a. $\frac{f(5)}{f(3)}$ b. $\frac{f(6)}{f(4)}$ c. $\frac{f(9)}{f(7)f(2)}$

10-145. Consider the following anagrams.

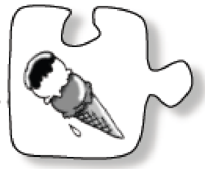
a. How many distinct ways can the letters in the word ITEMS be arranged?

b. How many distinct ways can the letters in the word STEMS be arranged?

c. How many distinct ways can the letters in the word SEEMS be arranged?

d. What makes these counts different?

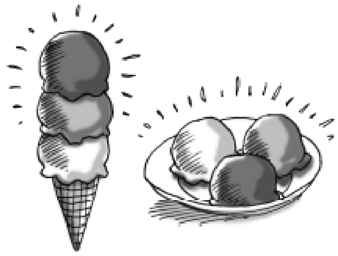
10.3.4 What kind of counting problem is this?



Categorizing Counting Problems

One of the biggest challenges in solving problems that involve counting techniques is deciding which method of counting to use. Selecting a counting method depends on whether different arrangements of elements will be considered to be different outcomes and on whether elements can be repeated in an outcome. As you work with your team on the Ice Cream Shop problem, starting a list of possibilities will be a useful strategy. The list may be too long to complete, but starting it might help you decide which counting technique to use.

10-146. THE ICE-CREAM SHOP



Friday was the seventh day of the heat wave with temperatures over $95°$, and DJ's Gourmet Ice-Cream Shop had only five flavors left: chocolate fudge, French vanilla, maple nut, lemon custard, and blueberry delight. Some customers ordered their ice cream in cones and some in a dish, but everyone ordered three scoops, the maximum DJ was allowing to ensure that the inventory would last.

On Saturday the temperature hit 100°. DJ still had five flavors and both cones and dishes, but he decided to allow no more than one scoop of a particular flavor per customer in order to keep a balanced variety on hand. On Friday, the customers had more choices than on Saturday because they could order a cone (which most people eat from the top down) or a dish (where scoops can be eaten in any order) and they could have three different flavors or more than one scoop of their favorite.

DJ's advertises that it has ***Over 100 Choices!*** When DJ's customers complained that he did not have 100 flavors, he responded, "*But I still offer more than 100 choices!*" Was that true on both Friday and Saturday?

Your Task: There are four counting problems here, two for Friday and two for Saturday. Describe each situation and show how to calculate the number of choices customers have once they decide on a cone or a dish.

Discussion Points

What are some possible outcomes for this situation? Can I start a list?

Does the arrangement or order of the scoops matter?

Can the choices be repeated?

Does the description of the outcomes for this situation fit any of the counting formulas I know?

Could this situation involve several different counting situations?

Further Guidance

10-147. It is useful to organize the information in a large 2×2 chart with columns for Friday and Saturday and rows for dishes and cones. With your team, set up a 2×2 chart or obtain the Lesson 10.3.4 Resource Page from your teacher and use it to organize the different possibilities. Describe each problem in relation to whether it involves arrangements or repeats elements, and make a prediction about which situation has the greatest number of choices and which has the least.

10-148. Use what you have learned about the Fundamental Principal of Counting, permutations, and combinations to solve three of the four problems.

10-149. The fourth problem is more cumbersome because the order of the scoops does not matter and all of the scoops could be different, two could be the same and one different, or all could be the same. This problem has a number of subproblems, and for at least one of them you may need to make a list. Work with your team to identify and solve each subproblem.

Further Guidance section ends here.

10-150. Charlie and his nephew, Jake, who is a bottomless hunger pit, went to the state fair. Charlie had promised he would buy Jake three snacks, one when they arrived, one mid-afternoon, and one when they were about to leave. As they were arriving, Jake was trying to negotiate to get all of the snacks all at once. At the food stand the menu included seven items:

Corn Dogs	Popcorn
Root Beer	Orange Soda
Sno Cones	Cotton Candy
Candied Apples	

Jake has a dilemma. He likes everything on the menu so much that he would not mind having any three items or even any two or three of the same thing. Uncle Charlie thinks variety is good so he wants Jake to choose three different things.

Your Task: With your team, categorize the alternatives for Jake and Charlie in terms of arrangements and repetition. Then describe and justify the solution method you would use to count the number of possibilities for each situation. Finally, figure out how many possible ways there are for Jake to choose his snacks for each situation.

10-151. When Jake and Charlie disagree, Jake has a two-thirds chance of getting his way. Draw an area model or tree diagram and calculate the probability that Charlie prevails and Jake has to order three different items and have his snacks spread out.

10-152. LEARNING LOG

Summarize the differences between combinations and permutations, anagrams, and other counting problems that involve the Fundamental Principle of Counting. Make your explanation clear and thorough enough that a student who is just transferring into your class could understand counting techniques. Include information about whether arrangements are important and whether elements can be repeated and give examples that illustrate the different possibilities. Title this entry, "Counting Problems and Strategies" and label it with today's date.

Review & Preview

10-153. From a batch of 500 light bulbs, how many ways can three be tested to see if they are defective?

10-154. Mr. K wants to bring a variety of language textbooks to the classroom in which he teaches French. From his library at home, he found some that would be appropriate for the classroom: 4 Russian texts, 7 German texts, 1 Japanese text, 2 Italian texts, and 3 Danish texts. If he decides to bring one book of each language to the classroom, and put them in alphabetical order by language on his bookshelf (Danish, German, Italian, and so forth), how many different ways can Mr. K arrange the new language books?

10-155. Joaquin is getting a new locker at school and the first thing he must do is decide on a new combination. The three number locker combination can be selected from the numbers 0 through 21.

a. How many different locker combinations can Joaquin choose if none of the numbers can be repeated?

b. With your understanding of permutations, combinations, and factorials, decide if the name "combination lock" is appropriate.

c. How many mathematical combinations are possible?

d. How many choices would there be if you could repeat a number, but not use the same number twice in a row?

10-156. This problem is a checkpoint for finding angles in and areas of regular polygons. It will be referred to as Checkpoint 10.

a. What is the measure of each interior angle of a regular 20-gon?

b. Each angle of a regular polygon measures 157.5°. How many sides does this polygon have?

c. Find the area of a regular octagon with sides 5 cm.

Check your answers by referring to the Checkpoint 10 materials located at the back of your book.

If you needed help solving these problems correctly, then you need more practice. Review the Checkpoint 10 materials and try the practice problems. Also, consider getting help outside of class time. From this point on, you will be expected to do problems like these quickly and easily.

10-157. The first four factors of 7! are 7, 6, 5, and 4 or 7, $(7 - 1)$, $(7 - 2)$, and $(7 - 3)$.

a. Show the first four factors of 12! in the same way the factors of 7! are shown above.

b. What are the first six factors of $n!$?

c. What is ${}_nP_6$?

10-158. Here is another way to think about the question: *"What is 0! ?"*

a. How many ways are there to choose all five items from a group of five items? What happens when you substitute into the factorial formula to compute ${}_5C_5$? Since you know (logically) what the result has to be, use this to explain what 0! must be equal to.

b. On the other hand, how many ways are there to choose *nothing* from a group of five items? And what happens when you try to use the factorial formula to compute ${}_5C_0$?

10-159. A European high-speed passenger train is made up of two first-class passenger cars, five second-class cars, and a restaurant car. How many ways can the train be made up?

Lesson 10.3.4 Resource Page

	Repetition	**No Repetition**
Order Matters		
Order Does Not Matter		

10.3.5 What are my chances of winning?

Some Challenging Probability Problems

In this lesson, you will have the opportunity to apply what you have learned about probability and counting principles to solve some interesting (and very challenging) problems. As you work with your team on one of the following problems, you may get stuck at some point along the way. Below are discussion questions that can help you to get started again.

What subproblems do I need to solve?

What simpler problem would help us to understand this problem?

How would I start a tree or a list?

Does order matter? Are the outcomes combinations, permutations, or something else?

Are these separate groups of outcomes? Are the probabilities independent?
Should I add or do I need to multiply?

Would it be easier to consider what is *not* an outcome?

10-160. THE CANDY DISH

A bowl contains three candies: two red and one green. Work with a partner and decide who is player A and who is player B. Then take turns choosing a candy from the bowl without looking. Player A takes one and holds on to it, then player B takes one. If the colors match, player A gets a point; if they differ, player B gets a point. Is this a fair game?

a. First try the game experimentally. Then show your analysis of the probabilities.

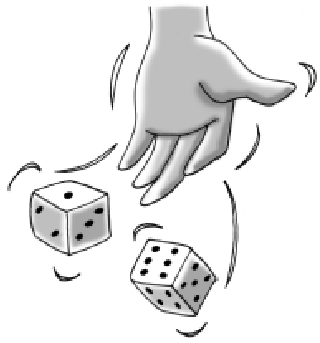
b. Now put four candies in the bowl, three of one color and one of another. Will this game be fair? Again, check experimentally then give your analysis using probabilities.

c. Are there other ways to put different numbers of two colors of candy in the bowl that would lead to a fair game while keeping the rest of the rules the same as in the previous two problems? Try a number of different possibilities (up to at least a total of 20 candies). Analyze each one using probability, make some hypotheses, and report any patterns you see in the results, conclusions, or generalizations that you can justify mathematically.

Your Task: Prepare a report or poster that shows:

- The number and variety of cases you investigated and analyzed.
- Your organization of the data, your analyses, and your general conclusions.
- The extent to which you can mathematically generalize your observations and justify your generalizations.

10-161. CASINO DICE GAME



To play this game, you roll two dice. If your total on the first roll is 7 or 11 points, you win. If your total is 2, 3, or 12 points, you lose. If you get any other number (4, 5, 6, 8, 9, or 10), that number becomes your point. You then continue to roll until your point comes up again or until a 7 comes up. If your point comes up before you roll a 7, you win. If 7 comes up first, you lose. You ignore any outcomes that are not your point or 7.

a. In pairs, play the game ten times. Record how many wins and losses your team has. Combine your information with other teams working on the problem. Are the results fairly even or were there many more wins or losses?

b. The game you have been playing is the basic dice game played in casinos worldwide. What is the probability of winning?

Your Task: To calculate the probability of winning, you will need to identify and solve several subproblems. Prepare a report that shows each of the subproblems clearly, as well as how you solved each one. Your report should also show the exact probability of winning as a fraction as well as a decimal approximation.

10-162. **An extra challenge:** Most casinos allow bettors to bet against the dice roller. In this case, the bettor wins whenever the roller would lose *except when the roller gets a 12 on the first roll*. When 12 comes up, the bettor does not win or lose and he or she just waits for the next roll. What is the probability of winning a bet against the roller? Which is the better bet, for or against? By how much?

Further Guidance

10-163. Start with a list of the ways to get each sum 2, 3, …, 12. For the remaining parts of this work, it will help to keep answers in fraction form.

a. Find the probability of winning on the first roll.

b. Find the probability of losing on the first roll.

c. Find the probability of the game ending on the first roll.

10-164. Now consider the other ways to win by rolling the point before rolling a 7.

a. Find the probability of rolling a 4.

b. Find the probability of rolling a 4 before a 7. (Note that you are only interested in 4's and 7's for this problem.)

c. Find the probability of rolling a 4 and then rolling another 4 before a 7. In other words, what is the probability of getting the outcome in part (a) and then the outcome in part (b)?

d. Find the probability of rolling a 5.

e. Find the probability of rolling a 5 before a 7. (You only care about 5's and 7's here.)

f. Find the probability of rolling a 5 and another 5 before a 7.

g. Find the probabilities for winning when your first roll is 6, 8, 9, or 10. Look for symmetry as you do this.

10-165. Make a list of the all the ways to win.

a. What is the probability of winning?

b. If you won the game, what is the probability that you won by throwing 7 or 11 on the first throw?

c. What is the probability of losing this game?

d. Is it a fair game? Is it close to fair? Explain why casinos can allow betting on this game without expecting to lose money.

Further Guidance section ends here.

10-166. TRIANGLES BY CHANCE

Obtain three dice from your teacher. You may also want some string, linguini, a compass, or some other building material.

a. Roll the three dice and use the numbers on the dice to represent the lengths of sides of a triangle. Build (or draw) the triangle. Record the three numbers in a table according to the type of triangle formed (scalene, isosceles, equilateral, or no triangle). For example, if 3, 3, and 5 came up on the dice, you would record 3, 3, 5 under the heading isosceles since a triangle with sides of length 3, 3, and 5 is isosceles.

b. Repeat this ten times, and then combine your information with the other teams working on this problem. Examine the data and discuss the results.

c. Based on your discussion, make an estimate for the probability of each outcome. Then calculate the theoretical probabilities.

Your Task: Complete a team report or poster that includes:

- Initial estimates of probabilities with your team justification for each one.
- The subproblems you solved, including how you counted the possible outcomes.
- The theoretical probability for each case.

Further Guidance

10-167. First you will need to calculate the size of the sample space for rolling three dice.

a. How many ways come up so that the result is an equilateral triangle?

b. How many ways can the dice come up so that the result is an isosceles triangle?

c. How many ways can the dice come up so that the result is a scalene triangle?

d. How many outcomes lead to no triangle?

e. Use your results from parts (a) through (d) to compute the probabilities for each outcome.

Further Guidance section ends here.

10-168. POKER

In the basic game of five-card-draw poker, five cards are dealt to each player from a standard deck of 52 cards. Players place bets based on their estimate of their chances of winning. They then draw any number of cards (up to five) to see if they can improve their hands, and they make another round of placing bets.

The winning poker hands (assuming no wild cards) are described below, in order from best to worst. Poker is a game that has been played for many centuries. Players had established the order of winning hands centuries before mathematicians developed the counting techniques, which verified that the order was mostly correct based on the probability of getting the hand.

(Note: In the list below, J stands for Jack, Q stands for Queen, K stands for King, A stands for Ace, and X stands for any card.)

1. Royal flush: 10-J-Q-K-A, all the same suit.
2. Straight flush: such as, 7-8-9-10-J, any five in a row, all the same suit (A can be used before 2 or after K).
3. Four of a kind: 2-2-2-2-X, four of a number or face card, and any other card.
4. Full house: 7-7-7-A-A-, three of one kind and two of another.
5. Flush: any five cards of the same suit, not all consecutive.
6. Straight: 3-4-5-6-7, any five in a row, a mixture of 2 or more suits.
7. Three of a kind: 8-8-8-J-A, three of a number or face card, the other two different.
8. Two pair: 9-9-5-5-2, pairs of two different numbers or face cards, with one other number or face card.
9. Two of a kind: A-A-7-8-J, any pair with three random others that do not match.
10. Bust: no matches, no runs of five in a row, different suits.

Your Task: Calculate the number of five-card hands that can be selected from a deck of 52 cards, and then, for the first six of the above hands, calculate the number of ways the hand can be dealt, and the probability that a player will be dealt that hand. Prepare a team report or poster that describes your work on both the counting problems and the probabilities.

Further Guidance

10-169. The most difficult Poker hand to get is a royal flush. To calculate the probability of getting a royal flush, you first need to determine the size of the sample space. How many five-card hands are possible if there are 52 cards to choose from? Then decide how many ways there are to make a royal flush.

a. What is the probability of getting a royal flush?

b. How many straight flushes are there that are all spades? Making a list will help you decide. Then how many straight flushes are there altogether?

c. What is the probability of getting a straight flush that is not a royal flush?

10-170. Flushes are five cards of one suit.

a. How many flushes are possible?

b. What is the probability of getting a flush?

10-171. Straights are five cards in a row, such as 4-5-6-7-8 of any suit.

a. How many straights are possible that include 2 or more suits? In other words, how many straights are possible that are not also straight flushes or royal flushes?

b. What is the probability of getting a straight?

10-172. How many ways are there to draw four cards that are the same number? Making a list will help. And how many ways are there to get the fifth card? What do you need to do to get the total number of five-card hands that contain four of a kind?

a. What is the probability of getting four of a kind?

b. Now consider a full house. First, think of listing the number of ways to get exactly three cards that are the same number. Once you know the three cards, how many ways are there to get the other two cards in your hand the same? What should you do with these two results to get the number of full houses possible?

c. What is the probability of getting a full house?

d. Recall your result for the number of ways to get three of a kind and figure out how many ways there are to get two cards that are different from the rest of the deck. Use this information to calculate the number of five-card hands with three matching numbers.

e. What is the probability of getting three of a kind?

f. Use a similar method for calculating the number of ways to get one pair and the probability of getting another pair.

g. Think about how you calculated the number of full houses and about how you calculated the number of hands with four of a kind. Then calculate the number of hands with two pairs.

h. What is the probability of drawing a hand that is a "bust?" How can you use the probabilities you have already calculated?

Further Guidance section ends here.

METHODS AND MEANINGS

MATH NOTES

Definition of 0!

The use of the combinations formula when $r = n$ (when the number to be chosen is the same as the total number in the group) leads to a dilemma, as illustrated in the following example.

Suppose the Spirit Club has a total of three faithful members. Only one three-member governance committee is possible. If you apply the formula for combinations, you get ${}_3C_3 = \frac{{}_3P_3}{3!} = \frac{3!}{(3-3)!3!} = \frac{3!}{0!3!} = 1$. Does this make sense?

To resolve this question and make the formulas useful for all cases, mathematicians decided on this definition: $0! = 1$.

Review & Preview

10-173. From a new shipment of 100 video games, how many ways can three games be tested to see if they are defective?

10-174. Which is greater: $(5-2)!$ or $(5-3)!$? Justify your answer.

10-175. Write an equivalent expression for each of the following situations that does not include the factorial (!) symbol.

a. The first five factors of $(n-3)!$

b. The first five factors of $(n+2)!$

c. $\frac{n!}{(n-3)!}$

d. $\frac{(n+2)!}{(n-2)!}$

10-176. Of the students who choose to live on campus at Coastal College, 10% are seniors. The most desirable dorm rooms are in the newly constructed OceanView dorm, and 60% of the seniors live there, while 20% of the rest of the students live there.

a. Represent these probabilities in a two-way table.

b. What is the probability that a randomly selected resident of the OceanView dorm is a senior?

c. Use the alternative definition of independence (see the Math Notes box in Lesson 10.2.3) to determine if being a senior is associated with living in the Ocean View dorm.

10-177. At 10:00 a.m. a radioactive material weighed 2 grams but at 6:00 p.m. it only weighed 0.45 grams. What were the approximate hourly multiplier and the hourly percent of decrease?

10-178. A triangular prism has a volume of 600 cubic cm. The base is a right triangle with a hypotenuse of length 17 cm and one leg with length 15 cm.

a. Draw the figure.

b. Find the height of the prism.

c. Find the surface area of the prism.

d. Find all three angles of the triangle in the base of the prism.

10-179. A pizza parlor has 12 toppings other than cheese. How many different pizzas can they create with five *or fewer* toppings? List all subproblems and calculate the solution.

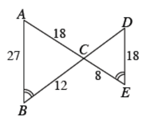
10-180. Mike was asked to make his popular ten-layer dip for the tailgate party at the big football game. The ten layers are: three layers of mashed avocado, two layers of cheddar cheese, and one layer each of refried beans, sour cream, sliced olives, chopped tomatoes, and green onions. How many ways can Mike make the ten layers in a glass serving pan?

10-181. Write an equation and use it to solve this problem.

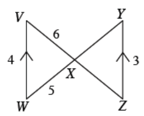
Jill has a 9-inch tall cylinder that she is using to catch water leaking from a pipe. The water level in the cylinder is currently 2 inches deep and is increasing at a rate of $\frac{1}{4}$-inch per hour. How long will it be before the water overflows?

10-182. Determine whether or not the two triangles in each pair below are similar. If so, write a flowchart to show your reasoning. If not, explain why not.

a.

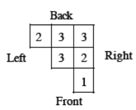


b.

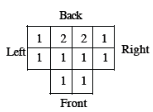


10-183. For each mat plan, create an isometric view of the solid.

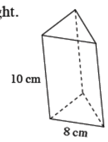
a.



b.



10-184. Calculate the total surface area and volume of the prism at right. The bases are equilateral triangles.



Lesson 10.3.5 Resource Page: Poker Solutions Page 1 of 2

There are $_{52}C_5 = 2,598,960$ possible distinct poker hands.

Royal flush: 10-J-Q-K-A, all the same suit.

There are 4 royal flushes, so P(royal flush) $= \frac{4}{2,598,960} \approx 0.000038\%$

Straight flush: 7-8-9-10-J, any five in a row, all the same suit (A can be used before 2 or after K).

There are 10 possible straights in each suit, so 40 possible straight flushes in all. You must subtract the 4 royal flushes, so there are 36 ways to get a straight flush.
P(straight flush) $= \frac{36}{2,598,960} \approx 0.0014\%$

Four of a kind: 2-2-2-2-X, four of a number or face card, and any other card.

There are 13 ways to draw four cards of the same number and there are 48 possibilities for the fifth card, so there are $13 \cdot 48 = 624$ possible four of a kind hands.
P(4 of a kind) $= \frac{624}{2,598,960} \approx 0.024\%$

Full house: 7-7-7-A-A, three of one kind and two of another.

There are $_4C_3$ ways to get three of a kind from four of each value and there are 13 values, so there are $13 \cdot {_4C_3}$ ways to get three of a kind. Then there are $_4C_2$ ways to get two of a kind from four of each value and there are 12 remaining values, so there are $12 \cdot {_4C_2}$ ways to get the two of a kind. In all, there are $13 \cdot {_4C_3} \cdot 12 \cdot {_4C_2} = 3744$ ways to get a full house.
P(full house) $= \frac{3744}{2,598,960} \approx 0.144\%$

Flush: any five cards of the same suit, not all consecutive.

There are $_{13}C_5$ ways to get 5 cards from 13 of the same suit and there are 4 suits. You must subtract the 36 straight flushes and 4 royal flushes, so there are $4 \cdot {_{13}C_5} - 40 = 5108$ ways to get a flush. Then P(flush) $= \frac{5108}{2,598,960} \approx 0.197\%$

Lesson 10.3.5 Resource Page: Poker Solutions Page 2 of 2

Straight: 3-4-5-6-7, any five in a row, a mixture of 2 or more suits.

Out of the numbers A through K, where A can be before 2 or after K, there are 10 possible straights. For each of these 10 possible straights, there are 4^5 possible combinations of suits, for a total of $10 \cdot 4^5 = 10,240$ possible straights. This number includes 36 straight flushes and 4 royal flushes, so the total number of poker hands classified as straights are $10 \cdot 4^5 - 40 = 10,200$. P(straight) $= \frac{10,200}{2,598,960} \approx 0.392\%$

Three of a kind: 8-8-8-J-A, three of a number or face card, the other two different.

There are ${}_4C_3$ ways to get three of a kind from four of each value and there are 13 values, so there are $13 \cdot {}_4C_3$ ways to get three of a kind. Then there are ${}_{12}C_2$ ways to get two differently valued cards from the remaining 12 values and, for each card, there are four possible suits, so there are a total of $13 \cdot {}_4C_3 \cdot {}_{12}C_2 \cdot 4 \cdot 4 = 54,912$ ways to get the hand three of a kind. P(three of a kind) $= \frac{54,912}{2,598,960} \approx 2.113\%$

Two pair: 9-9-5-5-2, pairs of two different numbers or face cards, with one other card.

There are ${}_{13}C_2$ ways to choose two values out of thirteen. Each of the two pairs chooses 2 suits from 4 or ${}_4C_2 \cdot {}_4C_2$. There are 11 values remaining for the 5th card, and each value has four suits. The total number of ways, then, to get the poker hand two pair is ${}_{13}C_2 \cdot ({}_4C_2)^2 \cdot 11 \cdot 4 = 123,552$. P(two pair) $= \frac{123,552}{2,598,960} \approx 4.754\%$

Two of a kind: A-A-7-8-J, any pair with three random others that do not match.

There are $13 \cdot {}_4C_2$ ways to get the pair. Then there are ${}_{12}C_3$ ways to choose the values of the three remaining cards, and for each card there are four possible suits. So, the total number of ways to get the poker hand two of a kind is $13 \cdot {}_4C_2 \cdot {}_{12}C_3 \cdot 4 \cdot 4 \cdot 4 = 1,098,240$. P(two of a kind) $= \frac{1,098,240}{2,598,960} \approx 42.257\%$

Bust: no matches, no runs of five in a row, different suits.

The probability of getting a bust is the sum of all the other probabilities subtracted from 1. P(bust) $= 1 - \frac{1,420,012}{2,598,960} \approx 45.362\%$

Chapter 10 Closure What have I learned?

Reflection and Synthesis

The activities below offer you a chance to reflect about what you have learned during this chapter. As you work, look for concepts that you feel very comfortable with, ideas that you would like to learn more about, and topics you need more help with. Look for connections between ideas as well as connections with material you learned previously.

① TEAM BRAINSTORM

What have you studied in this chapter? What ideas were important in what you learned? With your team, brainstorm a list. Be as detailed as you can. To help get you started, lists of Learning Log entries and Math Notes boxes are below.

What topics, ideas, and words that you learned *before* this chapter are connected to the new ideas in this chapter? Again, be as detailed as you can.

How long can you make your list? Challenge yourselves. Be prepared to share your team's ideas with the class.

Learning Log Entries

- Lesson 10.1.2 – Inscribed Angles
- Lesson 10.1.3 – Connections with Circles
- Lesson 10.2.1 – Independent or Mutually Exclusive?
- Lesson 10.2.3 – Probability Rules
- Lesson 10.3.4 – Counting Problems and Strategies

Math Notes

- Lesson 10.1.1 – Circle Vocabulary
- Lesson 10.1.2 – More Circle Vocabulary
- Lesson 10.1.3 – Inscribed Angle Theorem
- Lesson 10.1.4 – Intersecting Chords
- Lesson 10.1.5 – Points of Concurrency
- Lesson 10.2.1 – Mutually Exclusive
- Lesson 10.2.3 – Conditional Probability and Independence
- Lesson 10.3.1 – Fundamental Principle of Counting
- Lesson 10.3.2 – $n!$ and Permutations
- Lesson 10.3.3 – Combinations
- Lesson 10.3.5 – Definition of 0!

② MAKING CONNECTIONS

Below is a list of the vocabulary used in this chapter. Make sure that you are familiar with all of these words and know what they mean. Refer to the glossary or index for any words that you do not yet understand.

arc length	arc measure	association
center	center-radius form	central angle
chord	circle	circumcenter
circumference	circumscribed	combination
conditional probability	decision chart	diameter
factorial	Fundamental Principle of Counting	
independent events	inscribed	Inscribed Angle Theorem
intercepted arc	major arc	minor arc
Multiplication Rule	mutually exclusive	permutation
perpendicular	probability	radius
sample space	secant	semicircle
similar	tangent	two-way table
$x^2 + y^2 = r^2$	zero factorial	

Make a concept map showing all of the connections you can find among the key words and ideas listed above. To show a connection between two words, draw a line between them and explain the connection. A word can be connected to any other word as long as you can justify the connection.

While you are making your map, your team may think of related words or ideas that are not listed here. Be sure to include these ideas on your concept map.

③ PORTFOLIO: EVIDENCE OF MATHEMATICAL PROFICIENCY

Showcase your new knowledge of circles by describing all the new circle tools you have developed (such as lengths of two intersecting chords are related, and what is special about an angle inscribed in a semicircle). Be sure to include diagrams.

Choose one or two problems from Lesson 10.1.5 that you feel best showcases your understanding of circles and carefully copy your work, modifying and expanding it if needed. Make sure your explanation is clear and in detail. Remember you are not only showcasing your understanding of the mathematics, but you are also showcasing your ability to communicate your justifications.

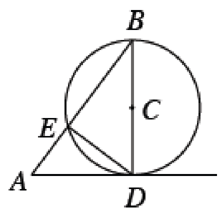
Showcase your understanding of counting methods by copying one of the solutions from Lesson 10.3.5, modifying and expanding it as needed. Again, make sure your explanation is clear and detailed.

④ WHAT HAVE I LEARNED?

Most of the problems in this section represent typical problems found in this chapter. They serve as a gauge for you. You can use them to determine which types of problems you can do well and which types of problems require further study and practice. Even if your teacher does not assign this section, it is a good idea to try these problems and find out for yourself what you know and what you still need to work on.

Solve each problem as completely as you can. The table at the end of the closure section has answers to these problems. It also tells you where you can find additional help and practice with problems like these.

CL 10-185. Copy the diagram at right onto your paper. Assume $\overline{AD}$ is tangent to $\odot C$ at D. Assume each part is a separate problem.

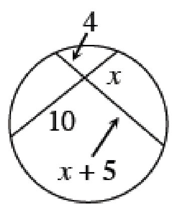


a. If $AD = 9$ cm and $AB = 15$ cm, what is the area of $\odot C$?

b. If the length of the radius of $\odot C$ is 10 cm and the $m\overset{\frown}{ED} = 30°$, what are $m\overset{\frown}{EB}$ and AD?

c. If $m\overset{\frown}{EB} = 86°$ and $BC = 7$ cm, find EB.

CL 10-186. A circle has two intersecting chords as shown in the diagram at right. Find the value of x.



CL 10-187. Eight friends go to the movies to celebrate their win in academic facts competition. They want to sit together in a row with a student on each aisle. (Assume the row is 8 seats wide including 2 aisle seats.)

a. If Kristen wants to sit in an aisle seat, how many ways can they all sit in the row?

b. If they sit down randomly, what is the probability they end up with five boys on the left and three girls on the right?

c. They decide to arrange themselves randomly by using the first letter of their last names. But two of the students' last names begin with K, and three begin with S. How many ways can they arrange themselves by using the first letter of their last name?

CL 10-188. At East College, 7776 students are in the freshman class, 6750 are sophomores, 6750 are juniors, and the rest are seniors. About 18% of students in each class are in the performing arts.

a. If there are 27,000 undergraduates at the school, what is the probability of being a senior in the performing arts?

b. Is being in a performing art independent of your class standing?

c. If a student is in the performing arts, what is the probability that he or she is a senior?

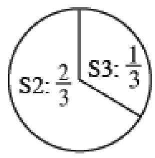
CL 10-189. Beethoven wrote nine symphonies and Mozart wrote 27 piano concertos.

a. If the local radio station KALG wants to play two pieces, a Beethoven symphony and then a Mozart concerto, in how many ways can this be done?

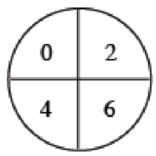
b. The station manager has decided that on each successive night (seven days a week), a Beethoven symphony will be played, followed by a Mozart concerto, followed by a Schubert string quartet (there are 15 of those). How long could this policy be continued before exactly the same program would have to be repeated?

CL 10-190. Antonio and Giancarlo are playing a board game that uses three spinners, which are shown in the diagrams at right. On each turn the player has to spin the first spinner. The first spinner determines which of the other two spinners the player also has to spin. The second spin determines how far the player gets to move his marker.

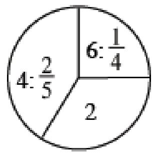
Spinner #1



Spinner #2



Spinner #3



a. What is the probability that Giancarlo will get to move his marker 4 or 6 spaces? Use a two-way table as needed.

b. What is the probability he will have to stay put?

c. What is the probability that he will get to move?

d. Antonio moved his marker 2 spaces. What is the probability that he spun S2 on the first spinner?

e. Explain your method for finding the probability in part (d) so that a student who was absent for today's work would understand conditional probability.

CL 10-191. Consider the solid represented by the mat plan at right.

3	1	0
0	1	1
0	2	3

Front

Right

a. Draw the front, right, and top view of this solid on graph paper.

b. Find the volume and surface area of this solid.

c. If this solid is enlarged by a linear scale factor of 3.5, what will be its new volume and surface area?

CL 10-192. Consider the descriptions of the different shapes below. Which shapes *must* be a parallelogram? If a shape does not have to be a parallelogram, what other shapes could it be?

a. A quadrilateral with two pairs of parallel sides.

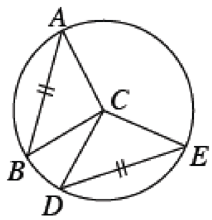
b. A quadrilateral with two pairs of congruent sides.

c. A quadrilateral with one pair of sides that is both congruent and parallel.

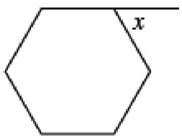
d. A quadrilateral with two diagonals that are perpendicular.

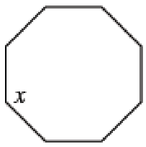
e. A quadrilateral with four congruent sides.

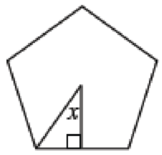
CL 10-193. In $\odot C$ at right, $\overline{AB} \cong \overline{DE}$.
Prove that $\angle ACB \cong \angle DCE$.



CL 10-194. Find the measure of x in each diagram below.
Assume each polygon is regular.

a.


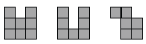
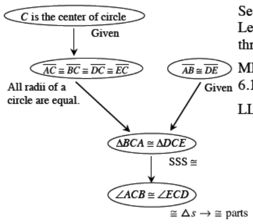
b.


c.


CL 10-195. Check your answers using the table at the end of the closure section. Which problems do you feel confident about? Which problems were hard? Use the table to make a list of topics you need help on and a list of topics you need to practice more.

Answers and Support for Closure Activity #4
What Have I Learned?

Note: MN = Math Note, LL = Learning Log

<table>
<tr><th>Problem</th><th>Solution</th><th>Need Help?</th><th>More Practice</th></tr>
<tr><td>CL 10-185.</td><td>a. 36π cm^2
b. $m\widehat{EB}=150°$,
$AD=20\tan 15° \approx 5.36$ cm
c. $14\sin 43° \approx 9.55$ cm</td><td>Lesson 8.3.2 and Section 10.1
MN: 5.1.2, 8.3.2, 10.1.1, 10.1.2, and 10.1.3
LL: 5.1.2, 8.3.2, 10.1.2, and 10.1.3</td><td>Problems 10-7, 10-18, 10-43, 10-54, 10-59, 10-68, 10-87, and 10-119</td></tr>
<tr><td>CL 10-186.</td><td>$4(x+5)=10x$
$x=\frac{20}{6}\approx 3.33$</td><td>Lesson 10.1.3
MN: 10.1.4
LL: 10.1.3</td><td>Problems 10-60, 10-78(c), and 10-107</td></tr>
<tr><td>CL 10-187.</td><td>a. $2\cdot 7!=10080$
b. $\frac{{}_5P_5\cdot{}_3P_3}{{}_8P_8}=\frac{720}{40320}\approx 1.8\%$
c. This is an anagram. $\frac{8!}{2!3!}=3360$</td><td>Section 10.3
MN: 10.3.1 and 10.3.2
LL: 10.2.3 and 10.3.4</td><td>Problems 10-114, 10-115, 10-127, 10-128, 10-132, 10-139, 10-145, and 10-154</td></tr>
<tr><td>CL 10-188.</td><td>a. About 18% of the 5724 seniors are in the performing arts, so P(senior and performing arts) = $\frac{(0.18)(5724)}{27,000}\approx\frac{1030}{27,000}\approx 3.8\%$.
b. They are independent. The probability of being in a performing art is always 18%; it does not change knowing the class standing of a student.
Or, make a two-way table and check if P(A given B) = P(A), for example, P(art given soph) = $\frac{1215}{6750}$ which equals P(art) = $\frac{1400+1215+1215+1030}{27,000}$.
c. Since they are independent, P(senior given performing arts) = P(senior) = $\frac{5724}{27000}=21.2\%$.</td><td>Section 10.2
MN: 10.2.1 and 10.2.3
LL: 10.2.1 and 10.2.3</td><td>Problems 10-85, 10-101, 10-116, and 10-130</td></tr>
<tr><td>CL 10-189.</td><td>a. 243
b. 3645 ways, 9.98 years</td><td>Section 10.3
MN: 10.3.1 and 10.3.2
LL: 10.2.3 and 10.3.4</td><td>Problems 10-114, 10-115, 10-127, 10-128, 10-132, 10-139, 10-145, and 10-154</td></tr>
<tr><td>CL 10-190.</td><td>a. Since the spinners are independent of each other, we can find entries in a two-way table by using P(A and B) = P(A) · P(B). See table below. $\frac{2}{12}+\frac{2}{12}+\frac{2}{15}+\frac{1}{12}\approx 63.3\%$
<table>
<tr><th></th><th>0 spaces</th><th>2 spaces</th><th>4 spaces</th><th>6 spaces</th><th></th></tr>
<tr><td>Spinner 2</td><td>$\frac{1}{4}\cdot\frac{2}{3}=\frac{2}{12}$</td><td>$\frac{2}{12}$</td><td>$\frac{2}{12}$</td><td>$\frac{2}{12}$</td><td>$\frac{2}{3}$</td></tr>
<tr><td>Spinner 3</td><td>0</td><td>$\frac{7}{20}\cdot\frac{1}{3}=\frac{7}{60}$</td><td>$\frac{2}{15}$</td><td>$\frac{1}{12}$</td><td>$\frac{1}{3}$</td></tr>
<tr><td></td><td>$\frac{2}{12}+0=\frac{2}{12}$</td><td></td><td></td><td></td><td></td></tr>
</table>
b. P(0 spaces) = $\frac{2}{12}\approx 16.7\%$
c. $1-\frac{2}{12}\approx 83.3\%$
d. P(Spinner 2 given 2 spaces) = $\frac{\frac{2}{12}}{\frac{2}{12}+\frac{7}{60}}\approx 58.8\%$
e. Answers will vary.</td><td>Section 10.2
MN: 10.2.3
LL: 10.2.1 and 10.2.3</td><td>Problems 10-102, 10-117, 10-131, 10-142, and 10-176</td></tr>
<tr><td>CL 10-191.</td><td>a. 
Front Right Top
b. V = 11 units^3, SA = 42 units^2
c. V = $11(3.5)^3 = 471.625$ units^3, $42(3.5)^2 = 514.5$ units^2</td><td>Lessons 9.1.1 and 9.1.2
MN: 9.1.3
LL: 9.1.1</td><td>Problems CL 9-111, 10-23, 10-105, and 10-183</td></tr>
<tr><td>CL 10-192.</td><td>Must be a parallelogram: (a), (c), and (e)
(b) could be a kite
(d) could be a kite</td><td>Lessons 7.3.1 and 7.3.3
MN: 7.2.3 and 8.1.2</td><td>Problems CL 7-156, CL 9-116, and 10-34</td></tr>
<tr><td>CL 10-193.</td><td></td><td>Section 3.2 and Lessons 6.1.1 through 6.1.4
MN: 3.2.2, 3.2.4, 6.1.4, and 7.1.3
LL: 3.2.2</td><td>Problems CL 3-123, CL 4-123, CL 5-140, CL 6-101, CL 7-155, CL 8-134, CL 9-117, 10-30, 10-36, and 10-47</td></tr>
<tr><td>CL 10-194.</td><td>a. 60°
b. 135°
c. 36°</td><td>Lessons 8.1.2, 8.1.3, and 8.1.4
Checkpoint 10
MN: 7.1.4 and 8.1.5
LL: 8.1.2, 8.1.3, and 8.1.4</td><td>Problems CL 8-138, CL 9-115, 10-22, 10-82, and 10-156</td></tr>
</table>

Arc length	Arc measure
Association	Center
Center-radius form	Central angle
Chord	Circle
Circumcenter	Circumference

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Circumscribed	Combination
Conditional probability	Decision chart
Diameter	Factorial
Fund. Princ. of Counting	Independent events
Inscribed	Inscribed Angle Thm.

Intercepted arc	Major arc
Minor arc	Multiplication Rule
Mutually exclusive	Permutation
Perpendicular	Probability
Radius	Sample space

Chapter 10 Closure Resource Page: Concept Map Cards Page 4 of 4

Secant	Semicircle
Similar	Tangent
Two-way table	$x^2 + y^2 = r^2$
Zero factorial	