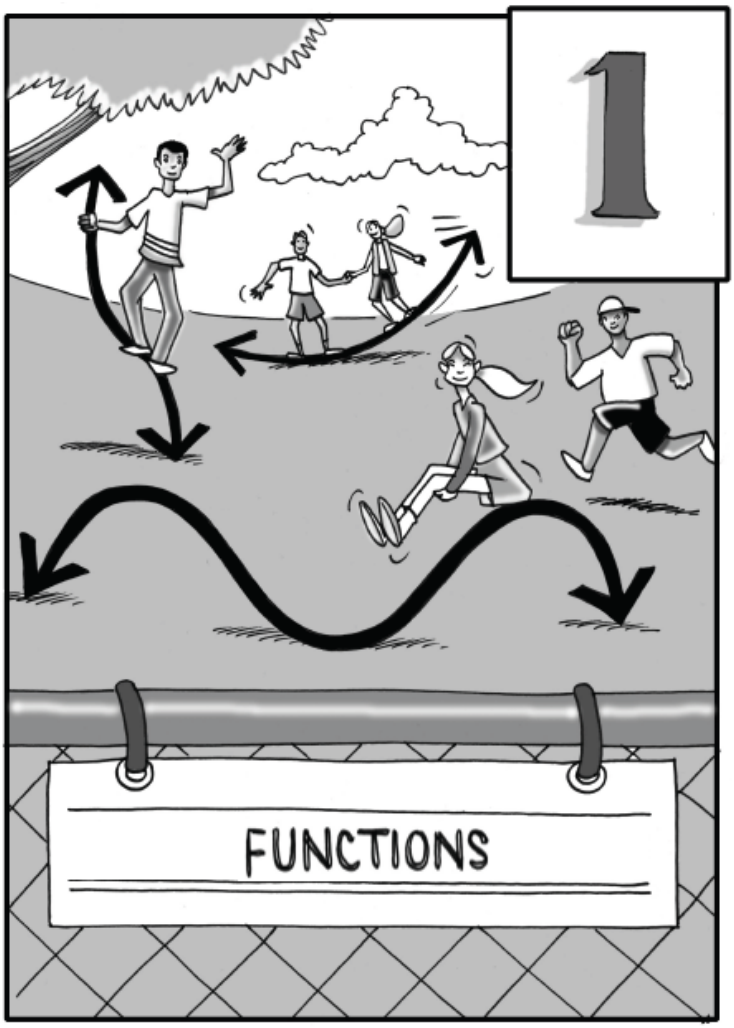

1
FUNCTIONS

Chapter 1 Functions

Welcome to algebra! In previous courses, you may have learned about relationships between two quantities that could be graphed with a straight line. In this chapter, you will explore nonlinear functions and learn how to describe a function completely. You will see the shapes and behaviors of several different nonlinear functions. This chapter also introduces you to sharing your mathematical knowledge with a study team as you work together to solve problems.

Guiding Question

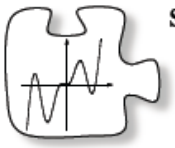
Mathematically proficient students model with mathematics.

As you work through this chapter, ask yourself:

Can I identify important quantities in situations and describe their relationships using graphs?

Chapter Outline

Section 1.1 This section starts with some function puzzles that you will solve with your team. Then you will encounter some real-life situations in which you will investigate the growth patterns, some of which are not linear. You will also investigate the family of quadratic functions to look at characteristics of their graphs.



Section 1.2 This section clarifies the description of non-linear graphs. You will make graphs of a variety of nonlinear functions. You will investigate what it means for a relationship to be a function, learn how to use function notation, and determine the domain and range of functions.

Chapter 1 Teacher Guide

Section	Lesson	Days	Lesson Title	Materials	Homework
1.1	1.1.1	1	Solving Puzzles in Teams	• Lesson 1.1.1A-C Res. Pgs. • Envelopes	1-4 to 1-8
	1.1.2	2	Investigating the Growth of Patterns	• Square tiles • Non-rolling beans • Stopwatch • Lesson 1.1.2A-D Res. Pgs. • 9" × 13" aluminum foil rectangular pans	1-13 to 1-17 and 1-18 to 1-22
	1.1.3	1	Investigating the Graphs of Quadratic Functions	• Poster graph paper and markers • Lesson 1.1.3 Res. Pg.	1-25 to 1-29
1.2	1.2.1	1-2	Describing a Graph	• Graph paper • Poster graph paper and markers • Lesson 1.2.1A-B Res. Pgs.	1-35 to 1-37 and 1-38 to 1-42
	1.2.2	1	Cube Root and Absolute Value Functions	• Student Learning Logs from problem 1-32 • Lesson 1.2.2 Res. Pg. (opt.)	1-47 to 1-51
	1.2.3	1	Function Machines	• Lesson 1.2.3 Res. Pg. (opt.)	1-57 to 1-61
	1.2.4	1	Functions	None	1-66 to 1-70
	1.2.5	1	Domain and Range	• Lesson 1.2.5 Res. Pg.	1-78 to 1-82
Chapter Closure		Various Options			

Total: 9-10 days plus optional time for Chapter Closure

1.1.1 How can I work with my team to figure it out?

Solving Puzzles in Teams

In previous courses, you have looked at patterns in tables, graphs, equations, and situations. In this course, you will not only continue your study of linear functions (which you have previously called "linear equations"), but will extend these patterns to new kinds of functions. Working with patterns will be the key to many of these functions. Note that we will define a function formally in Section 2 of this chapter. In today's lesson, you and your team will use clues in order to find patterns and solve puzzles.

1-1. TEAM SORT

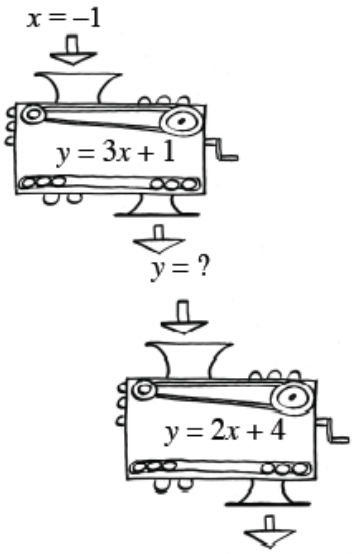
Your teacher will give you a card with a representation of a line (a table, graph, equation, or situation). Consider what you know about the line represented on your card. Then find the other students in your class who have a representation of the same line. These students will be your teammates, so you should sit together as instructed by your teacher. Be prepared to justify how you know your representation matches those of your teammates.

1-2. In this problem you will work with "function machines" like those pictured at right.

a. To help you work together in your new teams today, each member of your team has a specific job. Read the following Team Roles information on the next page.

b. When a value for x is put into the machine, a value for y comes out. That output then becomes the input for another machine. An example is shown at right.

What is the output from the second function machine? Explain.



Team Roles for Problem 1-2

If you had the "graph" card in problem 1-1, you are the:

Resource Manager:

- Get supplies for your team and make sure that your team cleans up.
  Return the linear functions cards from problem 1-1 to the teacher.
- Make sure that everyone has shared all of their ideas and help the team decide when it needs outside help.
  "Does anyone have another idea? Are we ready to ask a question?"
  Don't call the teacher over unless the entire team is stuck and out of ideas.
- Call the teacher over for team questions.
  "Are we ready to ask a question?"

If you had the "table" card in problem 1-1, you are the:

Facilitator:

- Get your team started by having someone read the task out loud.
- Check that everyone understands what to work on.
- Make sure that everyone understands your team's answer before you move on.
  "Does everyone understand how we got our answer?"

If you had the "equation" card in problem 1-1, you are the:

Recorder/Reporter:

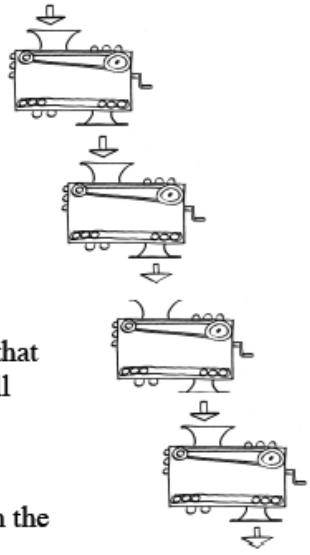
- Make sure that each team member can see the work the team is discussing.
  Place the resource page in the center of the table.
- Make sure that your team agrees about how to explain your ideas and each person has time to write their answer.
  "Does anyone need more time to write down our explanation?"
- Make sure that each member of your team is able to share ideas.
  "Is everybody ready to explain to the class how we got our answer?"

If you had the "situation" card in problem 1-1, you are the:

Task Manager:

- Make sure that no one talks outside your team.
- Help keep your team on task and talking about math.
- Listen for statements and reasons.
  "Why did you start that way?"
  "Will you say more about how you got that answer?"

1-3. Obtain the Lesson 1.1.1C Resource Page, which is a set of four function machines. Copies of the resource pages for this course can be downloaded at www.cpm.org/students/resources.htm. Your team's job is to use a specified input to get a particular output by putting those machines in order so that one machine's output becomes the next machine's input.



As you work, discuss what you know about the kind of output each machine produces to help you arrange the machines in an appropriate order.

The four functions are reprinted below:

$y = -2x + 34$ $y = \frac{-x}{3} - 10$

$y = -|3x|$ $y = (x-2)^2$

a. In what order should you stack the machines so that when 16 is dropped into the first machine, and all four machines have had their effect, the last machine's output is –6?

b. What order will result in a final output of 2 when the first input is 8?

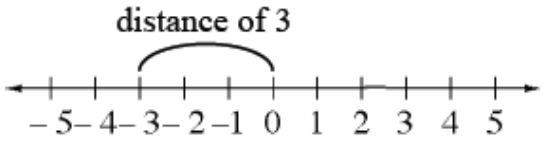
MATH NOTES

METHODS AND MEANINGS

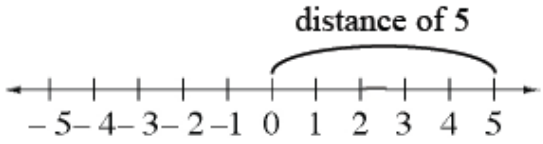
Definition of Absolute Value

Absolute value represents the numerical value of a number without regard to its sign. The symbol for absolute value is two vertical bars, | |. Absolute value can represent the distance on a number line between a number and zero. Since a distance is always positive, the absolute value is *always* either a positive value or zero. The absolute value of a number is *never* negative.

For example, the number –3 is 3 units away from 0, as shown on the number line at right. Therefore, the absolute value of –3 is 3. This is written $|-3| = 3$.

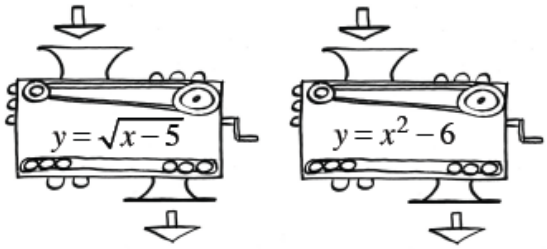


Likewise, the number 5 is 5 units away from 0. The absolute value of 5 is 5, written $|5| = 5$.



Review & Preview

1-4. Angelica is working with function machines. She has the two machines shown at right. She wants to put them in order so that the output of the first machine becomes the input of the second. She wants to use a beginning input of 6.



a. In what order must she put the machines to get a final output of 5?

b. Is it possible for her to find an input that will get a final output of –5? If so, show how she could do that. If not, explain why not.

1-5. Evaluate each absolute value expression. Review the Math Notes box in the lesson for the definition of absolute value.

a. $|54|$ b. $-\left|-7\frac{3}{5}\right|$ c. $|3|-|-1|$ d. $|2.2-5.13|$

1-6. Examine the tile pattern at right.

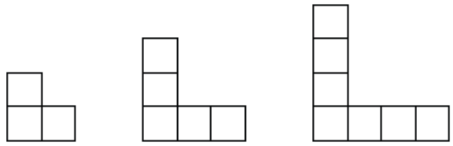


Figure 1 Figure 2 Figure 3

a. On your paper, sketch Figures 4 and 5.

b. How does the pattern grow? Explain how you know.

c. How many tiles will there be in Figure 0 (the figure before Figure 1)? Explain how you know.

1-7. Simplify each expression.

a. $-42+(-17)$ b. $8-(-9)$ c. $8(-9)$

d. $-42\div(-7)$ e. $-2(-3)(-4)$ f. $-18-7$

g. $(-5)^2$ h. -5^2 i. $\sqrt{49}$

1-8. For each equation below, find y if $x = 2$.

a. $y = 7-|x|$ b. $y = x^2-1$ c. $y = \sqrt{x+14}$

Team Roles for Problem 1-3

Resource Manager:

- Get supplies for your team and make sure that your team cleans up.
  Return the Lesson 1.1.1B Resource Page to the teacher.
  At the end of class, is your team's work area cleaned up and put away?
- Make sure that everyone has shared all of their ideas and help the team decide when it needs outside help.
  "Does anyone have another idea? Are we ready to ask a question?"
  Don't call the teacher over unless the entire team is stuck and out of ideas.
- Call the teacher over for team questions.

Facilitator:

- Get your team started by having someone read the task out loud.
- Check that everyone understands what to work on.
- Make sure that everyone understands your team's answer before you move on.
  "Does everyone understand why certain functions could not go last and others could not go first?"

Recorder/Reporter:

- Make sure that each team member can see the work the team is discussing.
  Place this Resource Page in the center of the table.
  Place the four function cards in the center of the table.
- Make sure that your team agrees about how to explain your ideas and each person has time to write their answer.
  "Does anyone need more time to write down our explanation?"
- Make sure that each member of your team is able to share ideas.
  "Is everybody ready to explain to the class how we got our answer?"

Task Manager:

- Make sure that no one talks outside your team.
- Help keep your team on-task and talking about math.
- Listen for statements and reasons.
  "What are the outputs of this function?"
  "Which functions could not go last?"

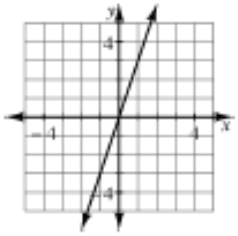
Team Sort

Table:

x	y
1	3
2	6
3	9
4	12

Equation: $y = 3x$

Graph:



Situation:

Figure 1 Figure 2

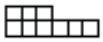


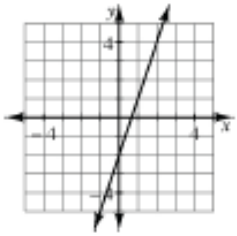
Figure 3

Table:

x	y
1	1
2	4
3	7
4	10

Equation: $y = 3x - 2$

Graph:



Situation:

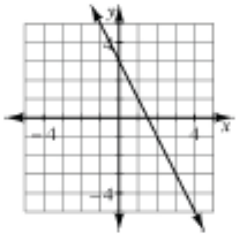
An elevator at Frump Tower climbs 3 floors per minute. After 1 minute, it is on the 1st floor.

Table:

x	y
5	–7
6	–9
7	–11
8	–13

Equation: $y = -2x + 3$

Graph:



Situation:

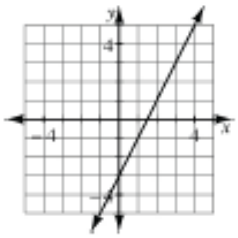
At 12 noon, the temperature was 3°F. Then the temperature fell steadily and reached –1°F at 2:00 PM.

Table:

x	y
2	1
3	3
4	5
5	7

Equation: $y = 2x - 3$

Graph:



Situation:

Figure 3 Figure 4

Figure 5

Lesson 1.1.1B Resource Page

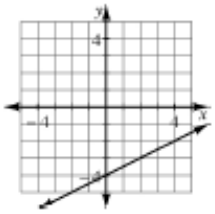
Team Sort

Table:

x	y
2	–3
3	–2.5
4	–2
5	–1.5

Equation: $y = \frac{1}{2}x - 4$

Graph:



Situation:

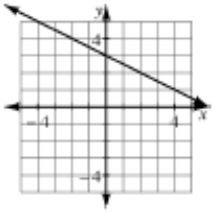
The temperature at midnight was 4°F below zero. It steadily grew 1°F warmer every 2 hours.

Table:

x	y
–3	4.5
–2	4
–1	3.5
0	3

Equation: $y = -\frac{1}{2}x + 3$

Graph:



Situation:

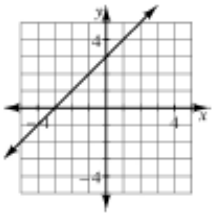
At noon, Carol had $3. She then bought a 50¢ soda every hour.

Table:

x	y
–3	0
–2	1
–1	2
0	3

Equation: $y = x + 3$

Graph:



Situation:

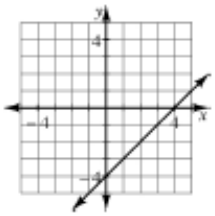
When a tree was planted, it was 3 feet tall. After 5 months of growing at a constant rate, it was 8 feet tall.

Table:

x	y
5	1
6	2
7	3
8	4

Equation: $y = x - 4$

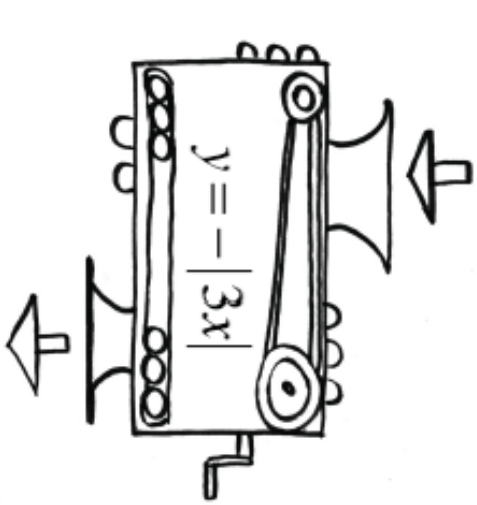
Graph:

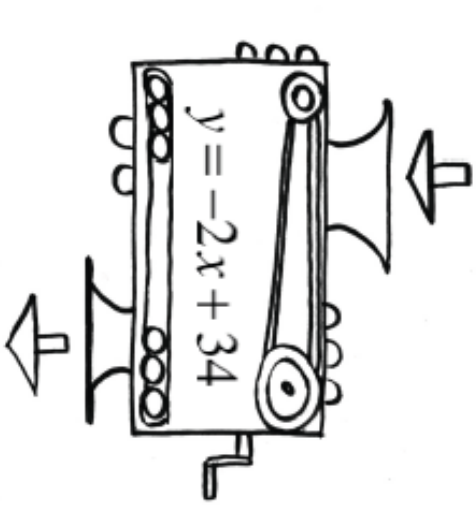


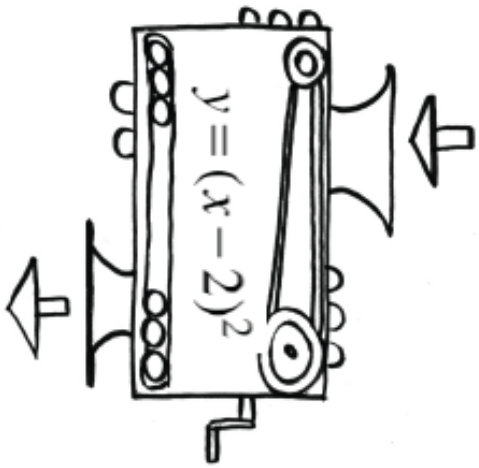
Situation:

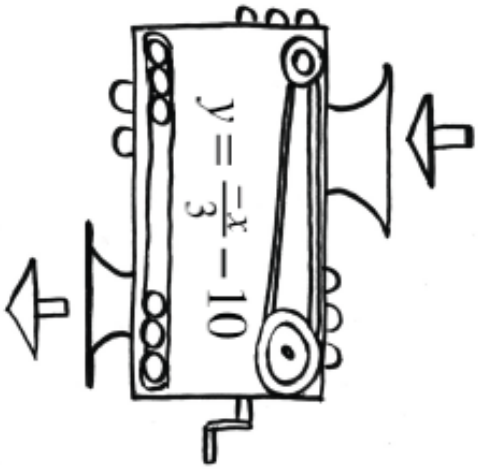
Joey is 4 miles south of his home. While walking north at a constant speed, he passes his house after 4 hours.

Lesson 1.1.1C Resource Page









1.1.2 How does it grow?

Investigating the Growth of Patterns

How a pattern grows is a major focus of this course. Understanding how something changes can help you make decisions and predict the future. For example, when the local health department needs to respond to an outbreak of an infectious disease, it makes a difference if the number of infected people increases by 1000 or by 10,000 people each day. And what if they learned that the number of infected people tripled each day? That might affect the way they respond to the disease.

Today you will work with your study team to analyze this and other situations that involve different types of functions. Your team will collect data about three different situations. After collecting the data, you will complete a table and make a graph for each situation. As you work together, ask each other the following questions to start and continue productive mathematical discussions.

What is the pattern? How is it changing? How can you describe it?

How does it grow (or get smaller)?

How can we organize the data?

1-9. DATA LABS

Today your team will collect and analyze data from three labs, which are described below. With your team, read and follow the directions for each lab carefully. Each description will tell you how much data to collect and how to collect it. Be sure to record your data in an organized way. Make sure that every team member understands what the data represents and how each pattern is changing.

Lab A: Hot Tub Design

Perry is designing a hot tub that he will locate behind his house. He has 36 square designer tiles that he will use to build a surface in his yard where he will place his hot tub. He wants to use all of the tiles, but he does not yet know how he will arrange them to form the base of the hot tub. If his hot tub will be rectangular, how many different rectangles with an area of 36 square tiles does he have to choose from?

Use the square tiles provided by your teacher to find as many rectangular configurations as you can. Remember to record the length and the width of each rectangle you find. Assume that Perry's yard is big enough to

Lab B: Local Crisis

Health officials in Parsnipville are concerned about the recent outbreak of the flu. While scientists are working hard to find a vaccine, the town leaders are turning to you to predict how many people will be sick over time. They hope to find a vaccine in a week. Here are the facts: The epidemic started on Day 0 when Velma and Stanley returned from their exotic jungle vacation with symptoms of the flu. Each day, a sick person infects two additional people. The town of Parsnipville has 3800 citizens.

Use the beans (or other material) provided by your teacher to represent the people infected with the flu. Start with two beans to represent Velma and Stanley. Then carefully add two beans to each existing bean to represent the growth of the disease. Collect (and record) data for how many people will be sick each day for a few days.

Lab C: Sign On the Dotted Line

Certain legal documents, such as those used when buying property, sometimes require up to 50 signatures! How long do you think that might take? To find out, collect data as one person of your team signs his or her first name. Have another team member use a stop watch to time how long, in seconds, it takes to neatly sign his or her first name 2, 3, 5, 7, and 10 times. Be sure to record the time it takes for each number of signatures. In order to collect good data, be sure to have your team member practice signing his or her first name a few times before you start. This is not a speed competition, but rather a way to collect typical data for one person's signature.

1-10. When you are working with your team to solve problems in this course, it will be important to work effectively with other people. Effective math conversations are a valuable part of the learning process throughout this course. Choose a member of your team to read these Collaborative Learning Expectations out loud.

COLLABORATIVE LEARNING EXPECTATIONS

Working with other students allows you to develop new ways of thinking about mathematics, helps you learn to communicate about math, and helps you understand ideas better by having to explain your thinking to others. The following expectations will help you get the most out of working together.

T Together, work to answer questions.

E Explain and give reasons.

A Ask questions and share ideas.

M Members of your team are your first resource.

S Smarter together than apart.

1-11. REPRESENTING DATA

In problem 1-9, you collected data for three different situations. Now your team will work together to find ways of representing the data. Obtain a Lesson 1.1.2D Resource Page from your teacher.

a. For each lab, complete the corresponding table on the resource page. Use patterns to complete your table for any values in the top row not already included in your data from problem 1-9. Some entries are started for you.

Lab A: Hot Tub Design

Width of Hot Tub (tiles)	1	2	3	4	6	9	12	18	36
Length of Hot Tub (tiles)	36								

Lab B: Local Crisis

Day	0	1	2	3	4	5	6	7
# of Infected People	2							

Lab C: Sign on the Dotted Line

# of Signatures	0	1	2	3	4	5	6	7	8	9	10

b. Now plot your data from each lab on the set of axes provided on the resource page. Note that some data points may not fit on the given axes. Then describe each graph. What does each graph look like? Should the points be connected? Be prepared to share your observations with the class.

c. For each graph, find the point where $x = 4$ and label it with its coordinate. Then explain what that point represents in each situation.

1-12. ANALYSIS

Graphs and tables not only represent data, but they also allow you to answer questions about the data. Use your tables and graphs on the resource page from problem 1-11 to answer the questions below.

a. Which data appears to be linear? That is, when graphed, which data forms a line? Explain why it makes sense for this situation to have a linear graph.

b. The town of Parsnipville will have a flu vaccine available on Day 7. Only people who have not yet gotten the flu will need to be vaccinated. Since the town has 3800 citizens, how many people will need the vaccine on that day? Is it easier to answer this question with your graph or with your table? Explain.

c. Now that Perry knows his options for the design of his hot tub, he wants to pick the hot tub that has the smallest perimeter. What do you recommend?

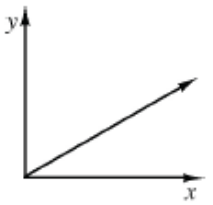
d. Why isn't there a point when $x = 0$ on your graph for Lab A? Could there be? Explain.

METHODS AND MEANINGS

MATH NOTES

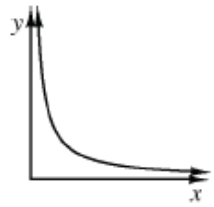
Families of Functions

There are several "families" of special functions that you will study in this course. One of these is called **direct variation** (also called **direct proportion**) which is a **linear** function. The data you gathered in the "Sign on the Dotted Line" lab (in problem 1-9) is an example of a linear function.



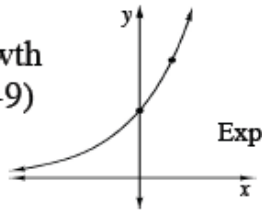
Linear

Another function is **inverse variation** (also called **inverse proportion**). The data collected in the "Hot Tub Design" lab (in problem 1-9) is an example of inverse variation.



Inverse Variation

You also observed an **exponential** function. The growth of infected people in the "Local Crisis" (in problem 1-9) was exponential.



Exponential

Note that we will define and develop these and other functions later in the course, and formally introduce functions in Section 2 of this chapter.

Review & Preview

1-13. Consider the situation described below.

a. Meredith lives 24 blocks from her friend's house. If she travels 1 block every minute, how many minutes will it take her to reach her friend's house? What if she travels 2 blocks every minute? Show how you calculated each answer.

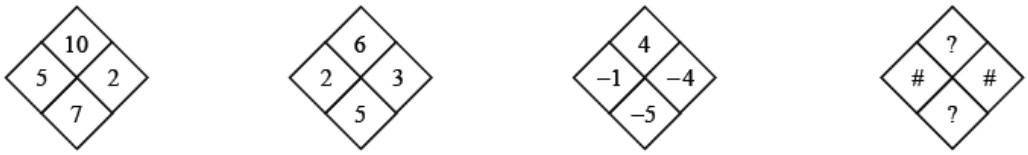
b. Copy and complete the table below to represent the amount of time it would take Meredith to get to her friend's house if she traveled at different rates.

1-14. Evaluate each expression if $r=-3$, $s=4$, and $t=7$.

a. $r^2+\sqrt{s}$ b. $\frac{t-r}{s}$ c. $2s^2+r-t$ d. $3(s-t)^2$

1-15. Finding and using a pattern is an important problem-solving skill you will use in algebra. The patterns in Diamond Problems will be used later in the course to solve other types of algebraic problems.

Look for a pattern in the first three diamonds below. For the fourth diamond, explain how you could find the missing numbers (?) if you know the two numbers (#).



Copy the Diamond Problems below onto your paper. Then use the pattern you discovered to complete each one.

a. 2, 5

b. −2, −1

c. 8, 6

d. 4, $\frac{1}{2}$

e. 12, −7

1-16. What value(s) of x will make each equation below true?

a. $x+5=5$ b. $2x-6=3x+1-x-7$

c. $3x+1=43$ d. $4x-1=4x+7$

1-17. Simplify each expression.

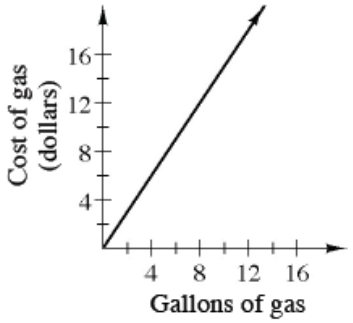
a. $\frac{2}{9}+\left(-\frac{1}{2}\right)$ b. $-\frac{6}{7}-\frac{3}{5}$ c. $\frac{9}{10}\left(-\frac{2}{3}\right)$ d. $\frac{1}{4}\div\frac{2}{7}$

1-18. In December of 2003, the average price for a gallon of regular gas in the United States was \$1.50.

a. At that time, what did it cost to buy 12 gallons of gas?

b. Gerald paid \$12.60 for a tank of gas. How many gallons did he buy?

c. At right is a graph of this situation. Predict how the line would change to represent the average cost of gas in December of 2005, when gas cost \$2.20 per gallon on average.



1-19. Solve each linear equation. Check your solutions.

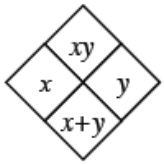
a. $-2x-3=3$ b. $7+2x=4x-3$ c. $6x-10=-8+3x$

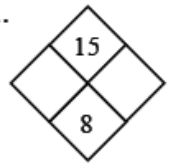
1-20. Evaluate the expressions below for the given values.

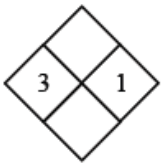
a. $-2x^2-3x+1$ for $x=-3$ b. $8-(3x-2)^2$ for $x=-2$

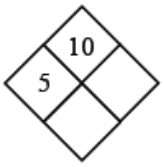
c. $\frac{-3}{k+2}$ for $k=-3$ d. $\frac{15m}{n+1}-m^2+n$ for $m=1$ and $n=2$

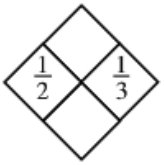
1-21. Copy and complete each of the Diamond Problems below. The pattern used in the Diamond Problems is shown at right.



a. 

b. 

c. 

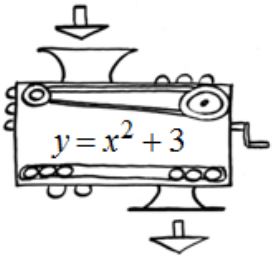
d. 

1-22. Function Machines

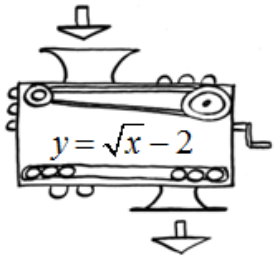
a. If an input of -9 is put into each of the machines at right, what is each output?

b. Eric wants to get an ouput of 0. Can he do this with each machine If so, how? If not, why not?

Relation A



Function B



Sign on the Dotted Line
Lab C

Certain legal documents, such as those used when buying property, sometimes require up to 50 signatures! How long do you think that might take? To find out, collect data as one person of your team signs his or her first name. Have another team member use a stop watch to time how long, in seconds, it takes to neatly sign his or her first name 2, 3, 5, 7, and 10 times. Be sure to record the time it takes for each number of signatures. In order to collect good data, be sure to have your team member practice signing his or her first name a few times before you start. This is not a speed competition, but rather a way to collect typical data for one person's signature.

Local Crisis
Lab B

Health officials in Parsnipville are concerned about the recent outbreak of the flu. While scientists are working hard to find a vaccine, the town leaders are turning to you to predict how many people will be sick over time. They hope to find a vaccine in a week. Here are the facts: The epidemic started on Day 0 when Velma and Stanley returned from their exotic vacation with symptoms of the flu. Each day, a sick person infects two additional people. The town of Parsnipville has 3800 citizens.

Use the beans (or other material) provided by your teacher to represent the people infected with the flu. Start with two beans to represent Velma and Stanley. Then carefully add two beans to each existing bean to represent the growth of the disease. Collect (and record) data for how many people will be sick each day for a few days.

Hot Tub Design
Lab A

Perry is designing a hot tub that he will locate behind his house. He has 36 square designer tiles to use for the bottom of his hot tub. He wants to use all of the tiles, but he does not yet know how he will arrange them to form the base of the hot tub. If his hot tub will be rectangular, how many different rectangles with an area of 36 square tiles does he have to choose from?

Use the square tiles provided to find as many rectangular configurations as you can. Remember to record the length and the width of each rectangle you find. Assume that Perry's yard is big enough to accommodate any rectangular design you create and that it matters which dimension is the width and which is the length.

Lesson 1.1.2B Resource Page

Team Roles for Problem 1-9

Resource Manager:

- Get supplies for your team and make sure that your team cleans up.
  Report missing lab supplies to the teacher.
  Assure your team has cleaned up the lab station before moving on to the next station.
  At the end of class, is your team's work area cleaned up and put away?
- Make sure that everyone has shared all of their ideas and help the team decide when it needs outside help.
  "Does anyone have another idea? Are we ready to ask a question?"
  Don't call the teacher over unless the entire team is stuck and out of ideas.
- Call the teacher over for team questions.

Facilitator:

- Get your team started by having someone read the task out loud.
  "Who wants to read the lab instructions out loud?"
- Check that everyone understands what to work on.
  "Who wants to do what part of this lab?"
  Are team members rotating tasks at the different lab stations?
- Make sure that everyone understands your team's answer before you move on.

Recorder/Reporter:

- Make sure that each team member can see the work the team is discussing.
  Place the lab instructions and these Team Roles where everybody in the team can see them.
- Make sure that your team agrees about how to explain your ideas and each person has time to write their answer.
  "Does everybody have a copy of the data for each lab?"
  "Does everybody have the data clearly labeled as to what numbers are what?"
  "Does anyone need more time to write down our explanation?"
- Make sure that each member of your team is able to share ideas.
  "Is everybody ready to explain to the class how we got our answer?"

Task Manager:

- Make sure that no one talks outside your team.
- Help keep your team on-task and talking about math.
  Are any team members talking to others instead of participating in the lab?
  Are the manipulatives being used for the mathematical task or for play?
  Are all team members participating in the lab? Is one person dominating?
- Listen for statements and reasons.

Lesson 1.1.2C Resource Page

Team Roles for Problems 1-11 and 1-12

Resource Manager:

- Get supplies for your team and make sure that your team cleans up.
  Obtain enough copies of the Resource Page for each team member.
- Make sure that everyone has shared all of their ideas and help the team decide when it needs outside help.
  "Does anyone have another idea? Are we ready to ask a question?"
  Don't call the teacher over unless the entire team is stuck and out of ideas
- Call the teacher over for team questions.

Facilitator:

- Get your team started by having someone read the task out loud.
  "Who wants to read the problem out loud?"
- Check that everyone understands what to work on.
  "Does everyone understand what we are supposed to do with the tables?"
- Make sure that everyone understands your team's answer before you move on.
  "Does everyone have the tables completed? The graphs?"
  "Let's compare graphs. Are they all similar?"

Recorder/Reporter:

- Make sure that each team member can see the work the team is discussing.
  Place the Team Roles where everybody in the team can see them.
  When talking about one student's paper, can the whole team see it?
- Make sure that your team agrees about how to explain your ideas and each person has time to write their answer.
  "Does everybody understand the pattern we used to make the table in Lab B?"
  "Can everybody explain how we completed the table for Lab C—the signatures?"
  "Does anyone need more time to write down our explanation?"
- Make sure that each member of your team is able to share ideas.
  "Is everybody ready to explain to the class how we got our answer?"

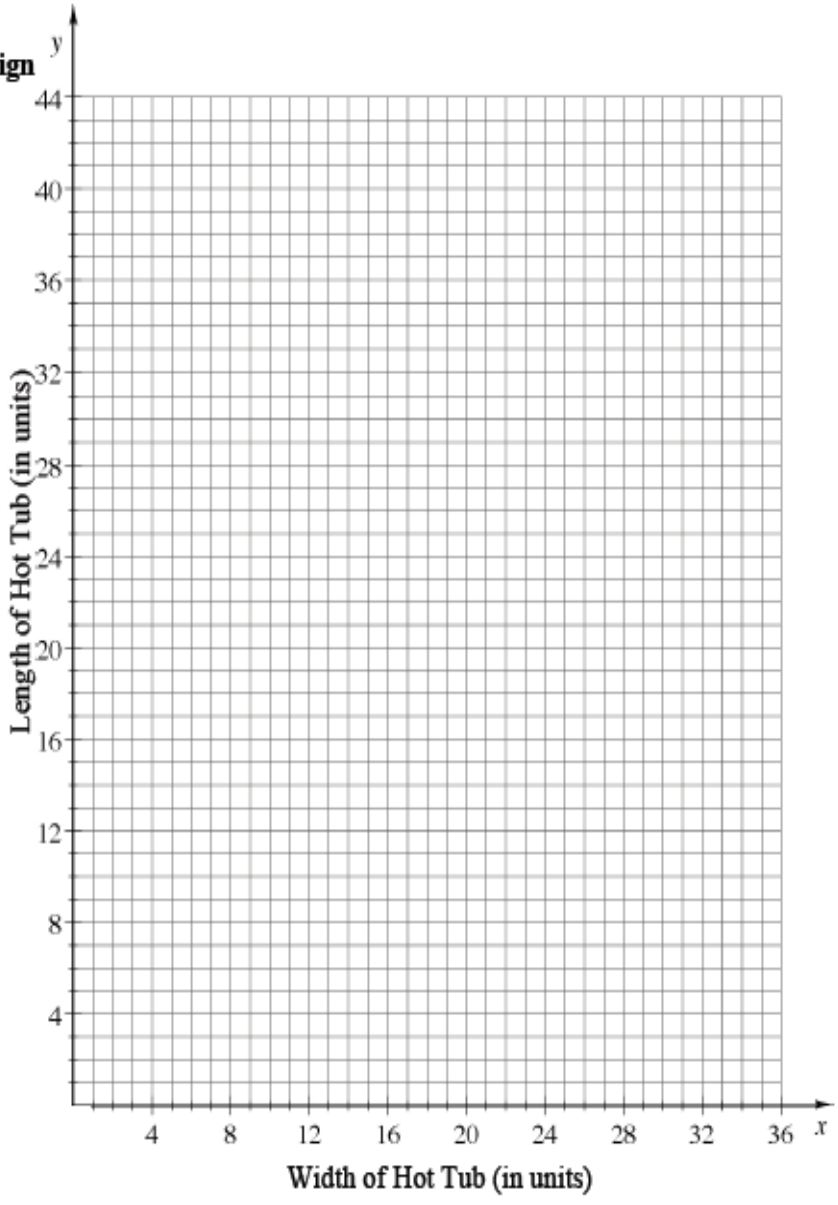
Task Manager:

- Make sure that no one talks outside your team.
- Help keep your team on-task and talking about math.
  Have all team members had a chance to contribute to the discussions and ideas?
  Is one team member doing all the talking? Is anybody not talking at all?
- Listen for statements and reasons.
  "How can we predict how many people will be infected?"
  "Why does the number of signatures grow like that?"

Data Representations Sheet

Lab A: Hot Tub Design

Width of Hot Tub	Length of Hot Tub
1	
2	
3	
4	
6	
9	
12	
18	
36	

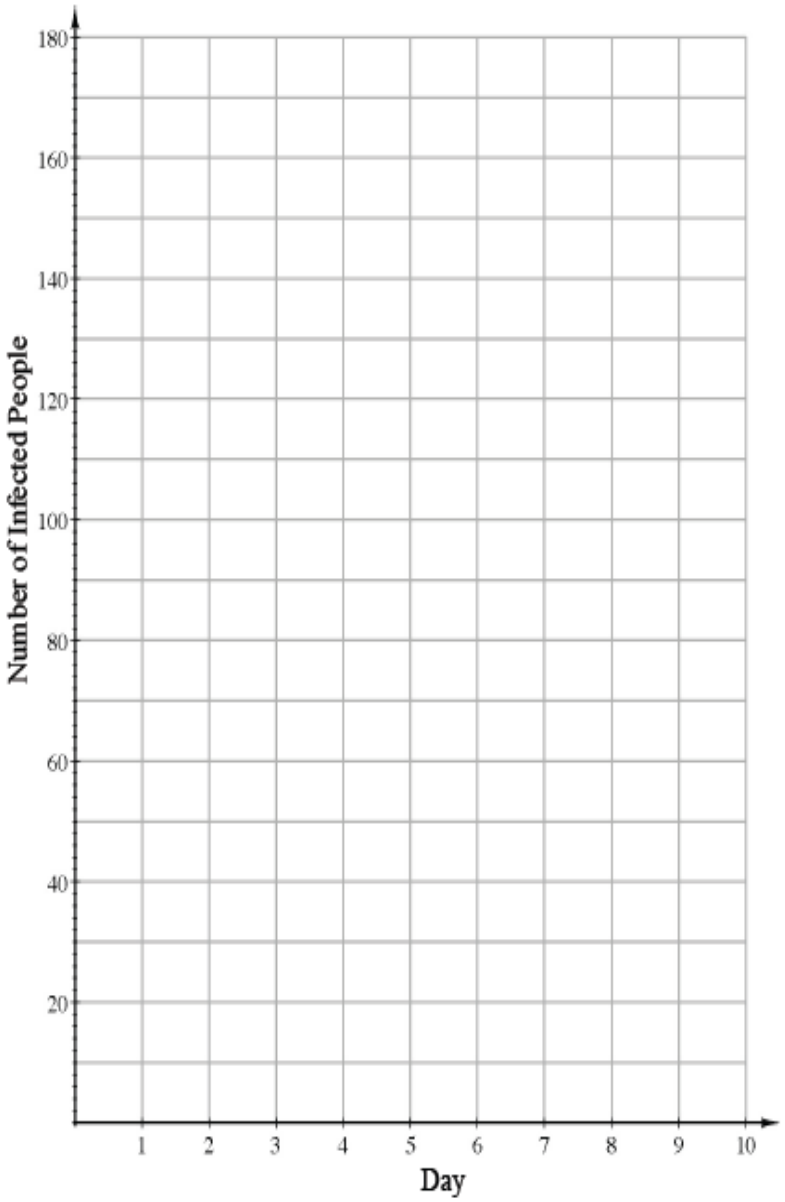


Describe the graph:

Data Representations Sheet

Lab B: Local Crisis

Day	Number of Infected People
1	
2	
3	
4	
5	
6	
7	

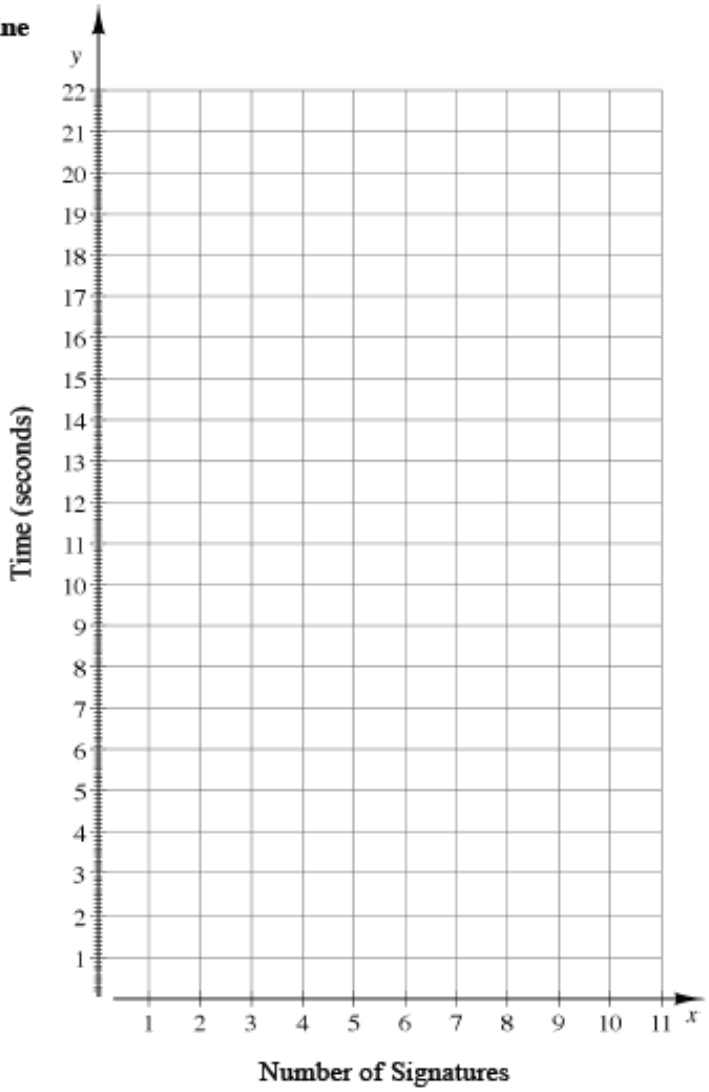


Describe the graph:

Data Representations Sheet

Lab C: Sign on the Dotted Line

Number of Signatures	Time (seconds)
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	



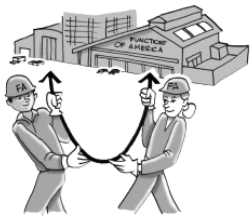
Describe the graph:

1.1.3 What do I know about a parabola?

Investigating the Graphs of Quadratic Functions

In the previous lesson, you observed linear, inverse variation, and exponential functions. In this lesson, you will study equations that create a family of functions called **quadratics**. The graph of a quadratic function has the shape of a **parabola**. You will learn all you can about their shape.

1-23. FUNCTIONS OF AMERICA



Congratulations! You have just been hired to work at a national corporation called Functions of America. Recently your company has had some growing pains, and your new boss has turned to your team for help. See her memo below.

MEMO

To: Your study team
From: Ms. Freda Function, CEO
Re: New product line

I have heard that while lines are very popular, there is a new craze in Europe to have non-linear designs. I recently visited Paris and Milan and discovered that we are behind the times!

Please investigate a new function called a quadratic function. A quadratic function can be written in the form $y = ax^2 + bx + c$. *Quadratic functions have the shape of a parabola.*

I'd like a full report at the end of today with any information your team can give me about its shape and equation. Spare no detail! I'd like to know everything you can tell me about how the equation for a quadratic function affects its shape. I'd also like to know about any special points on a parabola or any patterns that exist in its table.

Remember, the company is only as good as its employees! I need you to uncover the secrets that our competitors do not know.

Sincerely,
Ms. Function, CEO

Your Task: Your team will be assigned its own quadratic function to study. Investigate your team's function and be ready to describe everything you can about it by using its graph (which is in the shape of a parabola), equation, and table. Answer the questions below to get your investigation started. You may answer them in any order; however, do not limit yourselves to these questions!

- How would you describe the shape of your parabola? For example, would you describe your parabola as opening up or down? Do the sides of the parabola ever go straight up or down (vertically)? Why or why not? Is there anything else special about its shape?
- Does your parabola have any **lines of symmetry**? That is, can you fold the graph of your parabola so that each side of the fold exactly matches the other? If so, where would the fold be? Do you think this works for all parabolas? Why or why not? For more information on lines of symmetry, see the Math Notes box at the end of this lesson.
- Are there any special points on your parabola? Which points do you think are important to know?
- Are there x- and y-intercepts? What are they? Are there any intercepts that you expected but do not exist for your parabola?
- Is there a highest (maximum) or lowest (minimum) point on the graph of your parabola? If so, where is it? This point is called a **vertex**.

List of Quadratic Functions:

$y = x^2 - 2x - 8$ $y = -x^2 + 4$

$y = x^2 - 4x + 5$ $y = x^2 - 2x + 1$

$y = x^2 - 6x + 5$ $y = -x^2 + 3x + 4$

$y = -x^2 + 2x - 1$ $y = x^2 + 5x + 1$

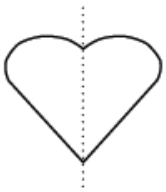
1-24. Prepare a poster for the CEO detailing your findings from your quadratic function investigation. Include any insights you and your teammates found. Explain your conclusions and justify your statements. Remember to include both a table and a complete graph of your parabola with all special points carefully labeled. Be thorough and complete.

METHODS AND MEANINGS

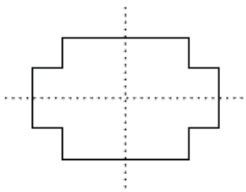
MATH NOTES

Lines of Symmetry

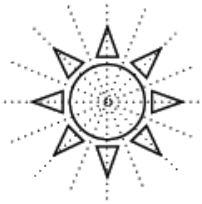
When a graph or picture can be folded so that both sides of the fold will perfectly match, it is said to have **reflective symmetry**. The line where the fold would be is called the **line of symmetry**. Some shapes have more than one line of symmetry. See the examples below.



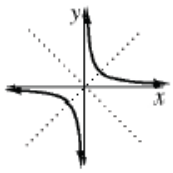
This shape has one line of symmetry.



This shape has two lines of symmetry.



This shape has eight lines of symmetry.



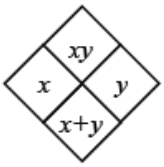
This graph has two lines of symmetry.

This shape has no lines of symmetry.

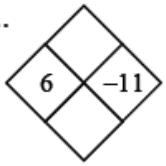
Review & Preview

1-25. Freda Function has another quadratic function for you to investigate! Graph the equation $y = x^2 + 3$ and then answer the questions from problem 1-23.

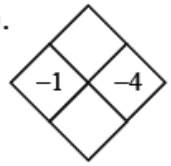
1-26. Copy these Diamond Problems and use the pattern you discovered earlier, shown at right, to complete each of them. Some of these may be challenging!



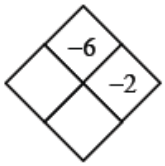
a.



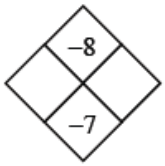
b.



c.



d.



1-27. Copy the figure at right onto your paper. Then draw any lines of symmetry.

1-28. Solve the equations below for x and check your solutions.

a. $-3 + 2x = -x + 6$

b. $5 - 3x = x + 1$

c. $-2x = 4x + 9$

d. $4x + 3 = x$

1-29. Mr. Guo is thinking of a number. When he takes the absolute value of his number, he gets 15. What could his number be? Is there more than one possible answer?

Lesson 1.1.3 Resource Page

Team Roles for Problems 1-23 and 1-24

Resource Manager:

- Get supplies for your team and make sure that your team cleans up.
  Ask the teacher if you have permission to trade markers with other teams.
  If any markers are dried up, let the teacher know. Don't just put them away.
  Assure your team has cleaned up from creating the posters.
- Make sure that everyone has shared all of their ideas and help the team decide when it needs outside help.
  Don't call the teacher over unless the entire team is stuck and out of ideas.
- Call the teacher over for team questions.

Facilitator:

- Get your team started by having someone read the task out loud.
  "Who wants to read this part of the problem out loud?"
- Check that everyone understands what to work on.
  "Did we describe all parts of the parabola?"
- Make sure that everyone understands your team's answer before you move on.
  "Can everybody explain how/why we got the descriptions we did?"
  "Does anyone disagree with any part or have another idea?"

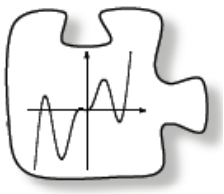
Recorder/Reporter:

- Make sure that each team member can see the work the team is discussing.
  Place the poster in the middle where everybody can see it.
- Make sure that your team agrees about how to explain your ideas and each person has time to write their answer.
  You have the important job of organizing the poster and presentation.
  "Does everybody about what we are going to put on the poster?"
  "Do we have time to make a draft copy of the poster?"
- Make sure that each member of your team is able to share ideas.
  "Is everybody ready to explain to the class how we got our answer?"

Task Manager:

- Make sure that no one talks outside your team.
- Help keep your team on-task and talking about math.
  You have the important job today of being very mindful of the remaining time.
  Are any team members talking to others instead of participating on the poster?
  Has everyone had a chance to write on the poster?
  Are all team members participating on the poster? Is one person dominating?
- Listen for statements and reasons.

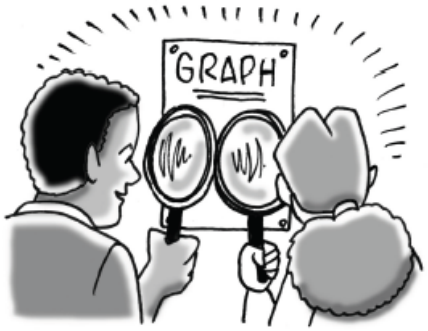
1.2.1 How can I describe a graph?



Describing a Graph

What does it mean to describe the graph of a function completely? Today you will graph and investigate a new function: $y = \sqrt{x}$.

1-30. DESCRIBING A GRAPH



Your teacher will assign your team one of the functions below. On graph paper, graph your function for x-values between –4 and 10.

When your team is convinced that your graph is correct, discuss all the ways you can describe this graph. Obtain one or two "seed questions" from your teacher (on the Lesson 1.2.1A Resource Page) to get you started with describing the graph. Then write as many summary statements about the graph as you can.

$y = \sqrt{x}$

$y = \sqrt{x} + 1$

$y = \sqrt{x+2} - 1$

$y = \sqrt{x-1} + 3$

$y = -\sqrt{x}$

$y = -\sqrt{x} - 2$

1-31. PRESENT YOUR FINDINGS

With your team, prepare to present your findings to the rest of the class. Your presentation should contain not only the graph of your function but also all of your observations and summary statements from problem 1-30. Be thorough and complete. Remember that a main goal of this activity is to determine what items a "complete description" of a graph must contain, so be sure to include everything you can. Remember to give reasons for all statements that you make.

1-32. LEARNING LOG

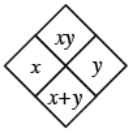
Throughout this course, you will be asked to reflect about your understanding of mathematical concepts in a Learning Log. Writing about your understanding will help you consolidate ideas, develop new ways to describe mathematical ideas, and recognize gaps in your understanding. It is important to write each entry of the Learning Log in your own words so that later you can use your Learning Log as a resource to refresh your memory. Your teacher will tell you where to write your Learning Log entries. Remember to label each entry with a title and a date so that it can be referred to later.

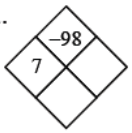
In this first Learning Log entry, as a class, create a list of all the ways to describe a graph from the presentations given by each team. Then, next to each description, create a question that will prompt you to look for this quality in the graphs of other functions you encounter.

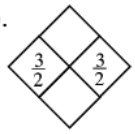
Once your class's list is complete, copy the questions into your first entry in your Learning Log. Title this entry "Graph Investigation Questions" and include today's date.

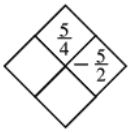
Review & Preview

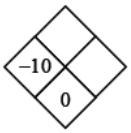
1-33. Copy these Diamond Problems and use the pattern you discovered earlier, shown at right, to complete each of them. Some of these may be challenging!



a. 

b. 

c. 

d. 

1-34. Evaluate the following absolute value expressions.

a. $|-100|-98$

b. $5|2-8|$

c. $|-13|+|0|$

d. $14-|-10+3|$

1-35. The solution to the equation $x^3 = 64$ is called the **cube root** of 64. The idea is similar to the idea of a square root, except that the value must be cubed (multiplied by itself three times) to become 64. One way to write the cube root of 64 is using the notation $\sqrt[3]{64}$. Use this information to evaluate each of the following expressions.

a. $\sqrt[3]{64}$ b. $\sqrt[4]{16}$ c. $\sqrt[3]{-8}$ d. $\sqrt[3]{125}$

1-36. Solve the following linear equations.

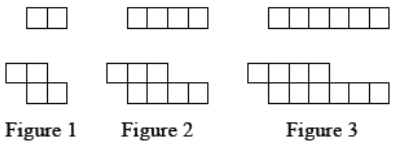
a. $8x+1=-x-1$

b. $-4x-3=3x-4-7x$

c. $4-5x=1+6x$

d. $7-x+3=9x+10$

1-37. Examine the tile pattern shown at right.



a. On graph paper, draw Figure 0 and Figure 4.

b. How many tiles will Figure 10 have? How do you know?

1-38. Chari performed a series of jumps on a trampoline. Her coach measured the height of each jump. The coach's data was recorded in the table at right.

Jump Number	Height (feet)
1	0.5
2	0.9
3	1.6
4	2.9
5	5.2

a. Make a graph of the data.

b. Fully describe the graph.

c. If this pattern continues, what are a reasonable maximum and minimum for the graph?

d. Which family of functions could model this data? Review the Lesson 1.1.2 Math Note if you need help.

1-39. Use the idea of cube root from problem 1-35 to evaluate the following expressions.

a. $\sqrt[3]{1}$ b. $\sqrt[3]{0}$ c. $\sqrt[3]{2^3}$ d. $\sqrt[3]{7^3}$

1-40. Solve the equations below for x and check your solutions.

a. $-3+2x=-x+6$ b. $5-3x=x+1$

c. $-2x=4x+9$ d. $4x+3=x$

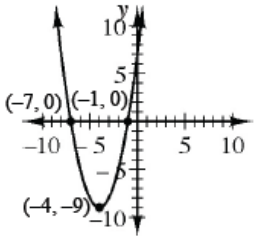
1-41. Find y in each equation if $x = 16$.

a. $y=3+\sqrt[3]{x-8}$

b. $y=\sqrt{x}-7$

c. $y=4+|9-x|$

1-42. Use your graph investigation questions from problem 1-32 to describe the graph of the quadratic equation $y=x^2+8x+7$, shown at right.



Lesson 1.2.1A Resource Page

Seed Questions

Does this graph look like any other graphs you have seen? If so, how? If not, describe the shape of the graph. Remember to give reasons for your statements.

Do the y-values grow at a constant rate? If not, how do they grow? Do they grow faster as x gets bigger? Remember to give reasons for your statements.

What happens to y as x gets bigger? What happens to y as x gets smaller? Justify your conclusions.

Does this graph have any symmetry? If so, where? Remember to give reasons for your statements.

Can all numbers go into this function? Why or why not? Can any number be an output? Remember to justify your conclusions.

What special point(s) does your graph have? Is there a highest or a lowest point? Remember to give reasons for your statements. Is there a starting point or stopping point?

What is the x-intercept, if any? What is the y-intercept?

What is the maximum value of this function? What is the minimum value?

Lesson 1.2.1B Resource Page

Team Roles for Problem 1-30

Facilitator:

- Make sure your team understands the entire task before you begin.
  "Does everyone know what to do?"
  "Does anyone need help?"
- Keep your team together. Make sure everyone's ideas are heard.
  "What did you notice?"
  "What else can we say about this graph?"
  "Do we all agree?"

Recorder/Reporter:

- Your poster needs to contain all the summary statements for this investigation. Be prepared to help your team members find a way to describe their ideas in clear statements.
  "What are you trying to say?"
  "Can a picture or diagram help?"
  "How can we write that?"
- Be sure that reasons are given for each statement.
  "How do you know that is true?"

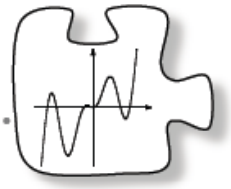
Resource Manager:

- You are responsible to get materials and help for your team. Your teacher may call you over to get more information during this investigation. Be sure that all questions are team questions. Don't let your team stay stuck!
  "Is everyone stuck? Should I call the teacher?"
  "What team question can we ask the teacher?"
  "Are we sure that no one here can answer the question?"

Task Manager:

- You need to make sure that your team is accomplishing the task effectively and efficiently. Make sure that all talking is *within* your team and is helping you to accomplish the task. Eliminate side conversations.
  "How does that information help us?"
  "Okay, let's get back to work!"
  "We need to finish this part in 5 minutes so we have time for…"
  "How can we have more than one person work on the poster at a time?"

1.2.2 What is the difference?



Cube Root and Absolute Value Functions

You have drawn graphs of different families of functions: lines, quadratic functions, and square roots. As a class you saw an inverse proportional graph (in the hot tub design problem) and an exponential graph (in the flu outbreak problem). Today you will explore the graphs of two additional functions, cube root and absolute value.

1-43. CUBE ROOTS

The solution of the equation $b^3 = 8$ is called the **cube root** of 8. The idea of a cube root is similar to the idea of a square root, except that the cube root of 8 must be cubed (multiplied by itself three times) to become 8. One way to write the cube root of 8 is using the notation $\sqrt[3]{8}$.

a. How can this notation be used to write the solution of $t^3 = 30$?

b. What is the $(\sqrt{64})^2$? What is $(\sqrt{19})^2$?

c. Can you extend this logic to find $(\sqrt[3]{30})^3$?

d. Use your calculator to obtain a decimal estimate for the solution in part (a).

1-44. Lydia wants to know what the graph of the cube root function, $y = \sqrt[3]{x}$, looks like.

a. Help Lydia by making an $x \to y$ table. What values of x could you choose (between –150 and 150) to make all of the y-values in your table integers? Everyone should take a few moments on his or her own to think about how to create some values for the table.

Then, with your team put as many integer x-values between –150 and 150 in your table as you can.

b. Create a graph of $y = \sqrt[3]{x}$. Scale the x-axis from –150 to 150. Refer to your list of graph investigation questions from Lesson 1.2.1 to help you completely describe the graph. Be as detailed as you can.

1-45. Riley is impressed with Lydia's graph of a cube root function. He wants to impress her in return by describing an absolute value graph.

a. Graph $y = |x|$. Describe for Riley what the graph looks like. Refer to your list of graph investigation questions from Lesson 1.2.1 to help you completely describe the graph. Be as detailed as you can.

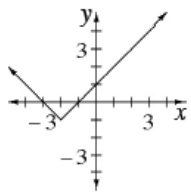
b. Are there any values of x that you cannot use in the equation above? If so, what are they? Are there any values for y that you will never get when you evaluate the equation above? If so, what are they?

1-46. Fully describe and graph the function $y = |2x - 1|$.

Review & Preview

1-47. Use your list of graph investigation questions from your Learning Log to answer questions about the graph shown at right.

$y = |x+2|-1$



1-48. Calculate the value of each expression below.

a. $|-4|-3$ b. $|6-11+3|$ c. $-9-|-2|$ d. $5|6|-2$

1-49. Throughout this book, key problems have been selected as "checkpoints." Each checkpoint problem is marked with an icon like the one at left. These checkpoint problems are provided so that you can check to be sure you are building skills at the expected level. When you have trouble with checkpoint problems, refer to the review materials and practice problems that are available in the "Checkpoint Materials" section at the back of your book.

This problem is a checkpoint for solving linear equations without parentheses. It will be referred to as Checkpoint 1.

Solve each equation.

a. $3x+7=-x-1$ b. $1-2x+5=4x-3$

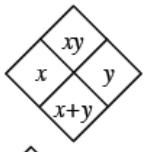
c. $4x-2+x=-2+2x$ d. $3x-4+1=-2x-5+5x$

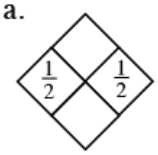
Check your answers by referring to the Checkpoint 1 materials located at the back of your book.

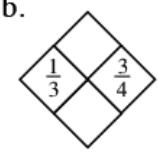
If you needed help solving these problems correctly, then you need more practice. Review the Checkpoint 1 materials and try the practice problems. Also consider getting help outside of class time. From this point on, you will be expected to do problems like this one quickly and easily.

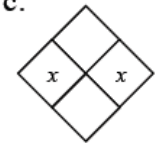
1-50. Graph the points (–3, 4) and (1, 1). If you drew a line through the points, name 3 other points that would be on the line. How did you find them?

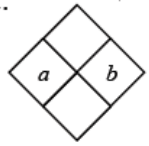
1-51. Copy and complete each of the Diamond Problems below. The pattern used in the Diamond Problems is shown at right.



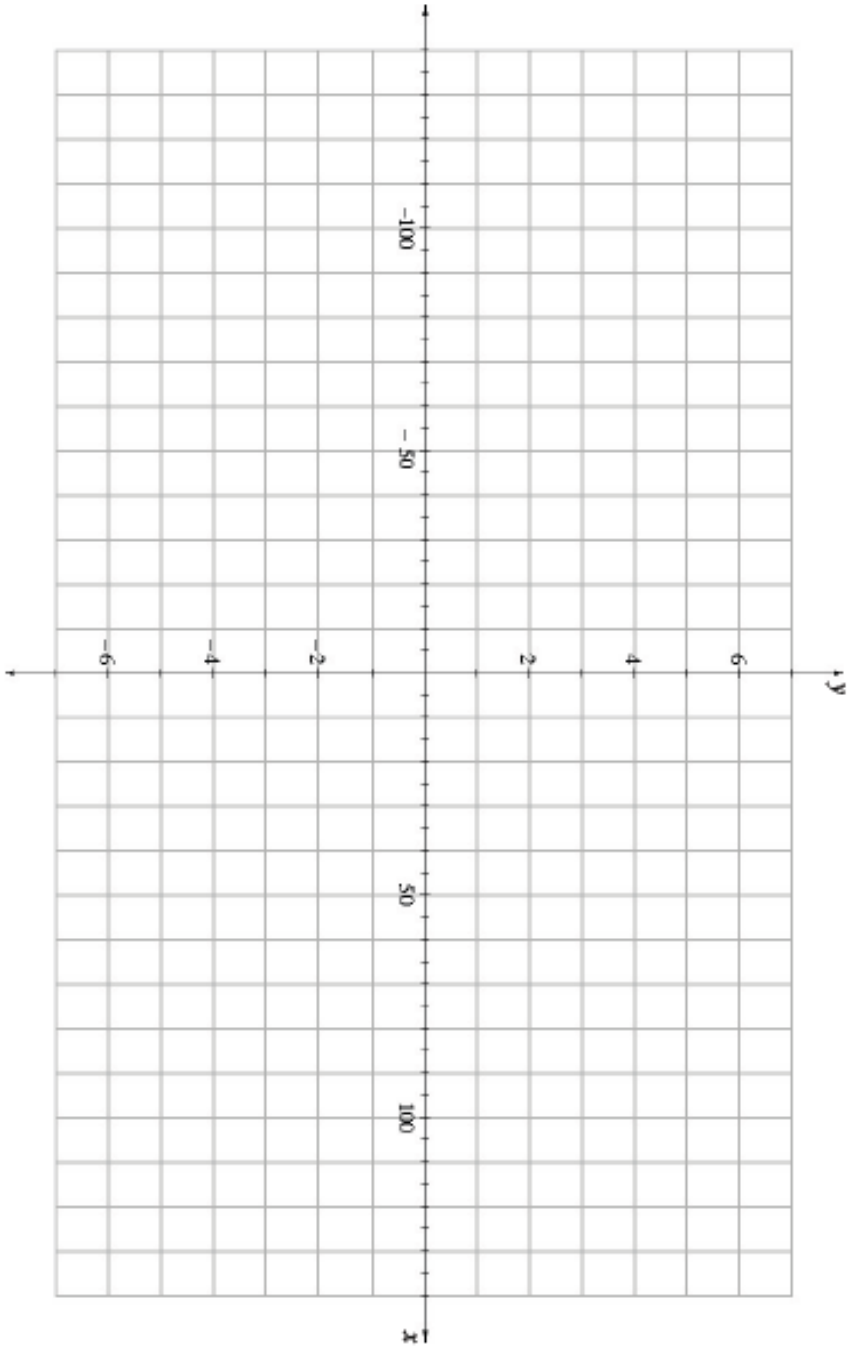
a. 

b. 

c. 

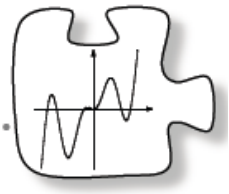
d. 

Lesson 1.2.2 Resource Page
Problem 1-44



y	x

1.2.3 What is the function?



Function Machines

In the next few lessons you will add to your list of what you can ask about a graph of a function. Throughout this chapter, you have used functions between two variables (like $y = -x^2 + 3x + 4$) to make graphs and find information. Today you will look more closely at how equations that relate two variables help establish a function between the variables. You will also learn a new notation to help represent these functions.

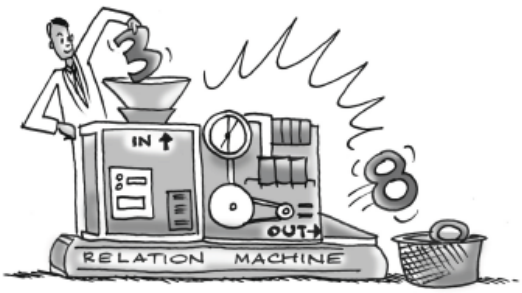
1-52. ARE WE RELATED?

Examine the table of input (x) and output (y) values below. Is there a relationship between the input and output values? If so, write an equation for this relationship. In what family is this function?

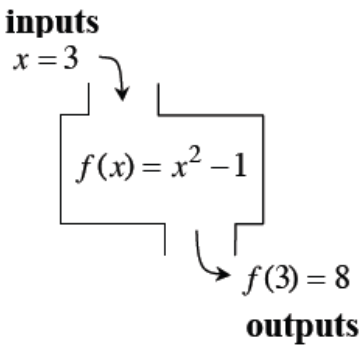
x	–3	–2	–1	0	1	2	3
y	8	3	0	–1	0	3	8

1-53. FUNCTION MACHINES

A function works like a machine, as shown in the diagram below. A function is given a name that can be a letter, such as f or g. The notation $f(x)$ represents the output when x is processed by the machine. (Note: $f(x)$ is read, "f of x.") When x is put into the machine, $f(x)$, the value of a function for a specific x-value, comes out. In this notation, $f(x)$ replaces y.



Numbers are put into the function machine (in this case, $f(x) = x^2 - 1$) one at a time, and then the function performs the operation(s) on each input to determine each output. For example, when $x = 3$ is put into the function $f(x) = x^2 - 1$, the function squares it and then subtracts 1 to get the output, which is 8. The notation $f(3) = 8$ shows that the function named f connects the input (3) with the output (8).



a. Find the output for $f(x) = x^2 - 1$ when the input is $x = 4$; that is, find $f(4)$.

b. Now find $f(-1)$ and $f(10)$.

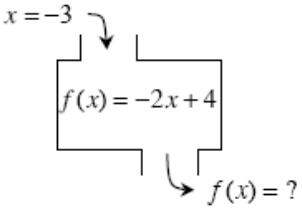
c. If the output of this function is 24, what was the input? That is, if $f(x) = 24$, then what is x? Is there more than one possible input?

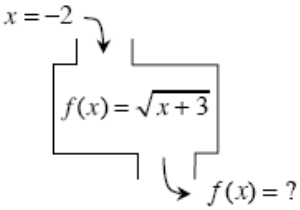
1-54. Find the relationship between x and $f(x)$ in the table below and write the equation.

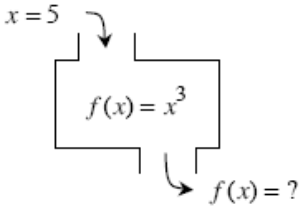
x	9	1	100	4	49		0	25	20
$f(x)$		1			7	4		5	

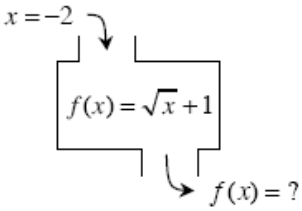
$f(x) =$ _____

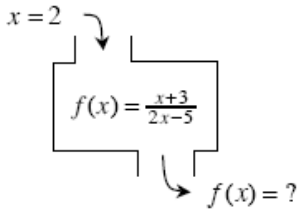
1-55. Find the corresponding outputs or inputs for the following functions. If there is no possible output for the given input, explain why not.

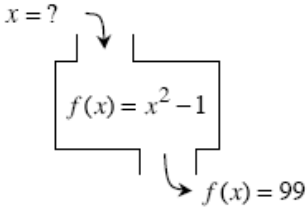
a. 

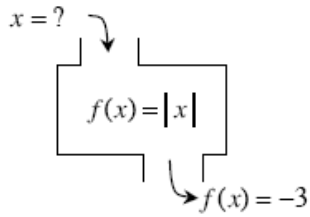
b. 

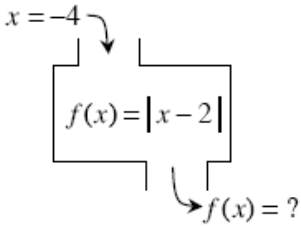
c. 

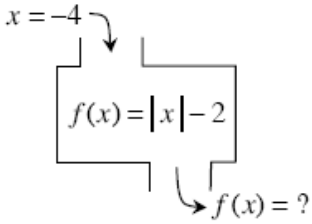
d. 

e. 

f. 

g. 

h. 

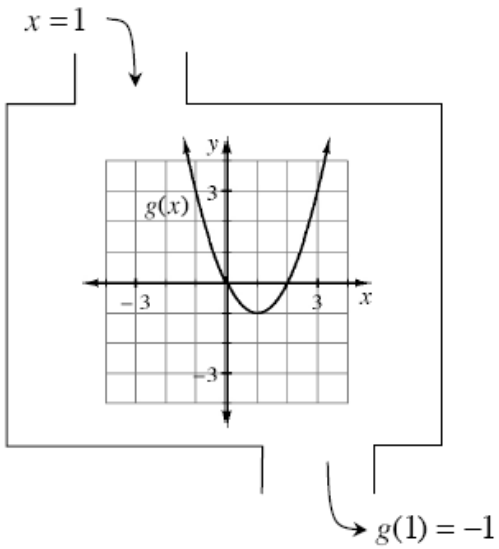
i. 

1-56. Examine the function defined at right. Notice that $g(1) = -1$; that is, when x is 1, the output (y or $g(1)$) is -1.

a. What is the output of the function when the input is 2? That is, find $g(2)$.

b. Likewise, what are $g(-1)$ and $g(0)$?

c. What is the input of this function when the output is 1? In other words, find x when $g(x) = 1$. Is there more than one possible solution?



Review & Preview

1-57. If $f(x) = x^2$, then $f(4) = 4^2 = 16$. Find:

a. $f(1)$ b. $f(-3)$ c. $f(t)$

1-58. Evaluate each expression.

a. $\sqrt[3]{27}$ b. $\sqrt{144}$ c. $\sqrt{3^2}$ d. $\sqrt[4]{2^4}$

1-59. Graph and fully describe the function $y = \sqrt[3]{x} - 2$.

1-60. A line passes through the points A(–3, –2) and B(2, 1). Does it also pass through the point C(5, 3)? Justify your conclusion.

1-61. Find the following absolute values.

a. $|0.75|$ b. $|-99|$

c. $|4 - 2 \cdot 3|$ d. $|\pi|$

Lesson 1.2.3 Resource Page

General Team Roles

Resource Manager:

- Get supplies for your team and make sure that your team cleans up.
- Make sure that everyone has shared all of their ideas and help the team decide when it needs outside help.
- Call the teacher over for team questions.

Facilitator:

- Get your team started by having someone read the task out loud.
- Check that everyone understands what to work on.
- Make sure that everyone understands your team's answer before you move on.

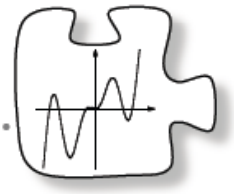
Recorder/Reporter:

- Make sure that each team member can see the work the team is discussing.
- Make sure that your team agrees about how to explain your ideas and each person has time to write their answer.
- Make sure that each member of your team is able to share ideas.

Task Manager:

- Make sure that no one talks outside your team.
- Help keep your team on task and talking about math.
- Listen for statements and reasons.

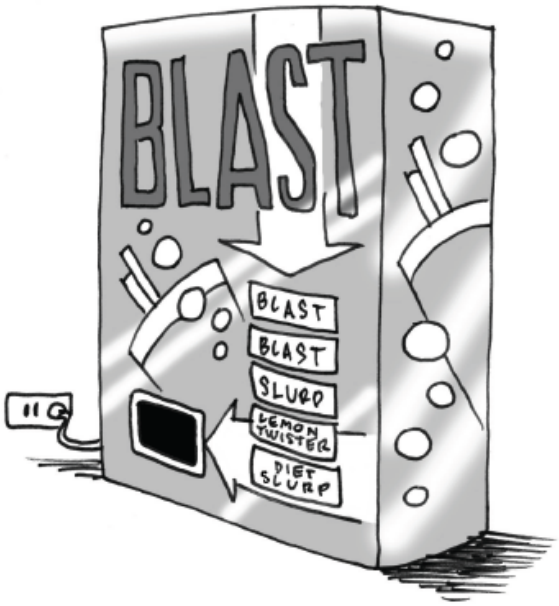
1.2.4 Can I predict the output?



Functions

You have studied relationships between input and output values. But what happens when your relationship gives you unpredictable results? That is, what happens when you cannot predict the output for a given input? Today you will study this situation and will be introduced to the quality that makes a relationship a *function*.

1-62. THE COLA MACHINE



The cola machine at your school offers several types of soda. There are two buttons for your favorite drink, *Blast*, while the other drinks (*Slurp*, *Lemon Twister*, and *Diet Slurp)* each have one button.

a. Describe the input and output of this soda machine.

b. While buying a soda, Ms. Whitney pushed the button for *Lemon Twister* and got a can of *Lemon Twister*. Later she went back to the same machine, but this time pushing the *Lemon Twister* button got her a can of *Blast*. Is the machine functioning consistently? Why or why not?

c. When Brandi pushed the top button for *Blast* she received a can of *Blast*. Her friend, Miguel, decided to be different and pushed the second button for *Blast*. He, too, received a can of *Blast*. Is the machine functioning consistently? Why or why not?

d. When Loutfi pushed a button for *Slurp*, he received a can of *Lemon Twister*! Later, Tayeisha also pushed the *Slurp* button and received a can of *Lemon Twister*. Still later, Tayeisha noticed that everyone else who pushed the *Slurp* button received a *Lemon Twister*. Is the machine functioning consistently? Explain why or why not.

1-63. FUNCTIONS

a. In a relationship like the soda machine, we want the outcome to be consistent and predictable. When it is, we say that the machine is functioning properly.

Examine each of the tables and graphs below that show different inputs and their outputs. Decide if the graph or table could be describing a soda machine that is "functioning properly." Explain your reasoning.

i.

Button Number	1	1	2	4	2	3
Type of Candy	Stix	Stix	M&Ns	M&Ns	Duds	Duds

ii.

x	7	–2	0	4	9	–3	6
$f(x)$	6	–3	4	2	10	–3	0

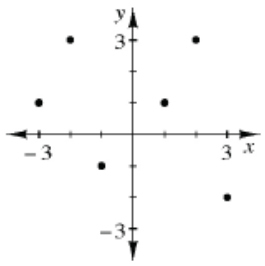
iii.

x	3	–1	2	0	1	2	9
y	4	–5	9	7	4	–8	2

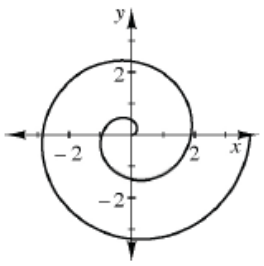
b. A relationship between inputs and outputs is called a **function** if the inputs and outputs behave like a soda machine that is functioning properly. Discuss with your team what it means for a relationship between inputs and outputs to be a **function**.

c. Examine each of the tables and graphs below. Compare the inputs and outputs and decide if the graph or table could be a **function**. Explain your reasoning.

iv.



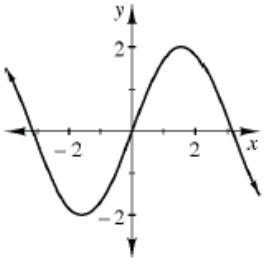
v.



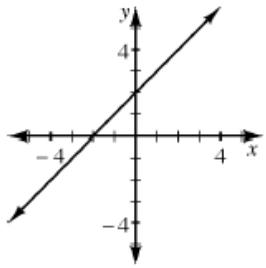
vi.

x	$h(x)$
–8	11
4	3
11	–8
6	3
–8	11

vii.



1-64. Jade noticed that the line graphed at right is a function. "Hey – I think *all* lines are functions!" she exclaimed. Is she correct? Support your claim with a diagram.



1-65. LEARNING LOGS

Throughout this course, you will be asked to reflect about your understanding of mathematical concepts in a Learning Log. Writing about your understanding will help you consolidate ideas, develop new ways to describe mathematical ideas, and recognize gaps in your understanding. It is important to write each entry of the Learning Log in your own words so that later you can use your Learning Log as a resource to refresh your memory. Your teacher will tell you where to write your Learning Log entries. Remember to label each entry with a title and a date so that it can be referred to later.

In this Learning Log entry, describe what it means for a relationship to be a function. Think of a type of machine that you use on a regular basis and describe how it also operates as a function. Title this entry "Functions" and include today's date.

1-66. If $g(x)=\sqrt{x-7}$, find $g(8)$, $g(32)$, and $g(80)$.

1-67. Solve each equation below. Check each solution.

a. $6-x-3=10$

b. $100x+300=200$

c. $\frac{1}{3}x+4=x-2$

d. $36-2x=-x+2$

1-68. Find $f(-4)$ for each function below.

a. $f(x)=|x-3|$

b. $f(x)=-5|x|$

c. $f(x)=|x+1|$

d. $f(x)=|x+3|-6$

1-69. Graph and fully describe the function $f(x)=-x^2+3$. Graph values of x from -3 to 3.

1-70. Find the corresponding inputs or outputs for the following functions. If there is no solution, explain why not. Be careful: In some cases, there may be no solution or more than one possible solution.

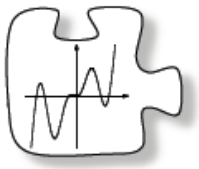
a. $x=8$ → $f(x)=|x|$ → $f(8)=?$

b. $x=?$ → $f(x)=3-\sqrt{x}$ → $f(x)=2$

c. $k=-6$ → $f(k)=\frac{k}{2}+1$ → $f(-6)=?$

d. $x=3$ → $f(x)=\sqrt{x-5}$ → $f(3)=?$

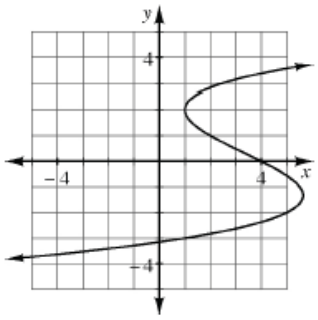
1.2.5 What can go in? What can come out?



Domain and Range

You have many characteristics that you can describe about graphs and functions using the questions that you began developing in Lesson 1.2.1. You also have learned how to tell if a relationship is a function. Today you will complete your focus on functions by describing the inputs and outputs of functions.

1-71. Examine the graph of the relationship $h(x)$ at right. Use it to estimate:



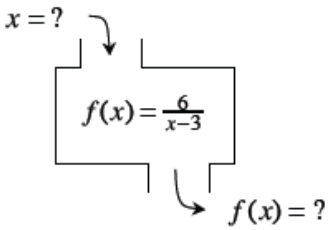
a. $h(4)$

b. $h(1)$

c. $h(-4)$

d. Is this relationship a function? Why or why not?

1-72. Examine the function shown at right.



a. Find $f(-3)$, $f(0)$, and $f(2)$.

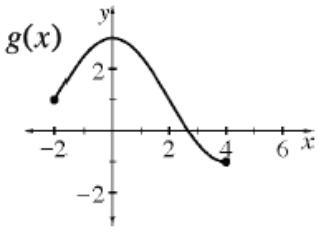
b. Find $f(3)$. What happened?

c. Are there any other inputs that cannot be evaluated by this function? In other words, are there any other values that x cannot be? Explain how you know.

d. The set (collection) of numbers that can be used for x in a function and still get an output is called the **domain** of the function. The domain is a description or list of all the possible x-values for the function. Describe the domain of $f(x) = \frac{6}{x-3}$.

e. What other types of functions have you looked at thus far in this course or in previous courses that have limited domains? How were they limited?

1-73. Now examine $g(x)$ graphed at right.



a. Is $g(x)$ a function? How can you tell?

b. Which x-values have points on the graph? That is, what is the domain of $g(x)$?

c. What are the possible outputs for $g(x)$? This is called the **range** of the function.

d. Ricky thinks the range of $g(x)$ is: $-1, 0, 1, 2,$ and 3. Is he correct? Why or why not?

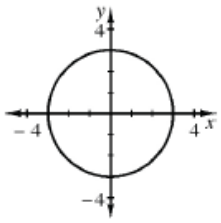
e. What other functions have you worked with previously in this course or previous courses that have limited ranges? How were they limited?

1-74. FINDING DOMAIN AND RANGE

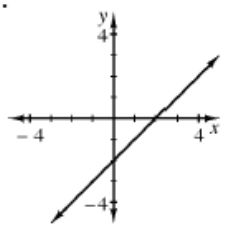
The domain and range are good descriptors of a function because they help you know what numbers can go into and come out of a function. The domain and range can also help you set up useful axes when graphing and help you describe special points on a graph (such as a missing point or the lowest point).

Work with your team to describe in words the domain and range of each relationship below. Then state whether or not the relationship is also a function.

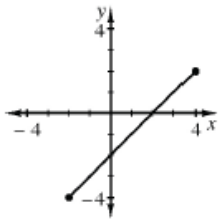
a.



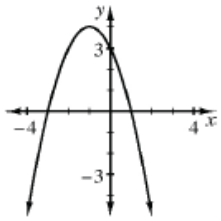
b.



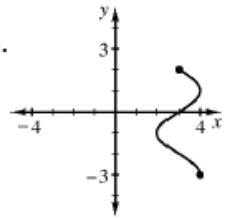
c.



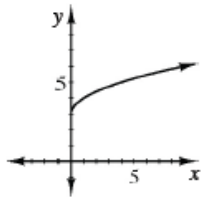
d.



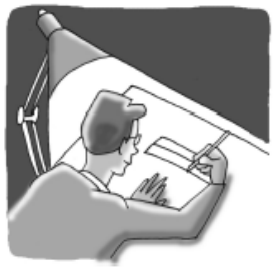
e.



f.



1-75. Chiu loves tables! He has decided to make the table below for a function $f(x)$ to help him find its domain and range.



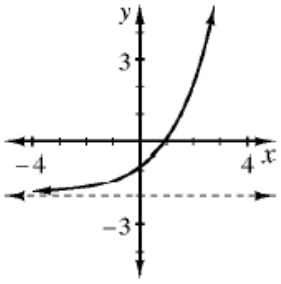
x	–3	–2	–1	0	1	2	3
$f(x)$	5	0	–3	–4	–3	0	5

a. From his table, can you tell what the domain of $f(x)$ is? Why or why not?

b. From the table, can you tell the range of $f(x)$? Why or why not?

c. Is using a table an effective way to determine the domain and range of a function?

1-76. Daniel is thinking about the function shown at right.



a. He noticed that the curve continues to the left and to the right. What is the domain of this function?

b. He found out that the dotted line is a line that the graph gets closer and closer to as x gets very, very large. (Another name for this dotted line is **asymptote**.) How should Daniel describe the range?

1-77. TEAM CHALLENGE

Sketch the graph of a function that has a domain of all numbers greater than or equal to –2 and a range of all numbers less than or equal to 3. Is there more than one possible answer?

METHODS AND MEANINGS

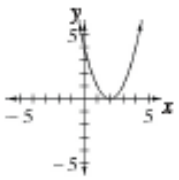
MATH NOTES

Functions

A relationship between inputs and outputs is a **function** if there is no more than one output for each input. We often write a function as y = some expression involving x, where x is the input and y is the output. The following is an example of a function.

$$y=(x-2)^2$$

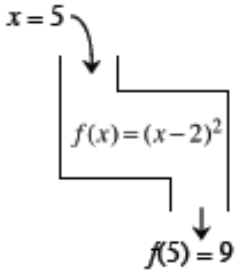
x	–2	–1	0	1	2	3	4	5
y	16	9	4	1	0	1	4	9



In the example above the value of y depends on x, so y is also called the **dependent variable** and x is called the **independent variable**.

Another way to write a function is with the notation "$f(x)$ =" instead of "y =". The function named "f" has output $f(x)$. The input is x.

In the example at right, $f(5)=9$. The input is 5 and the output is 9. You read this as, "f of 5 equals 9."

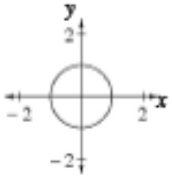


The set of all inputs for which there is an output is called the **domain**. The set of all possible outputs is called the **range**. In the example above, notice that you can input any x-value into the equation and get an output. The domain of this function is "all real numbers" because any number can be an input. But the outputs are all greater than or equal to zero. The range is $y \geq 0$.

$x^2+y^2=1$ is not a function because there are two y-values (outputs) for some x-values, as shown below.

$$x^2+y^2=1$$

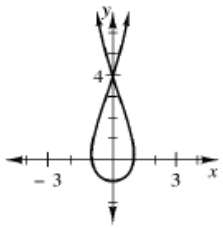
x	–1	0	0	1
y	0	–1	1	0



Review & Preview

1-78. Which of the relationships below are functions? If a relationship is not a function, give a reason to support your conclusion.

a.



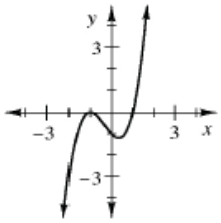
b.

x	y
–3	19
5	19
19	0
0	–3

c.

x	7	–2	0	7	4
y	10	0	10	3	0

d.



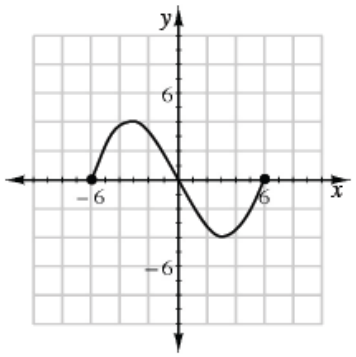
1-79. Find the x- and y-intercepts for the graphs of the relationships in problem 1-78.

1-80. Find the inputs for the following functions with the given outputs. If there is no possible input for the given output, explain why not.

a. $x = ?$ → $f(x) = 3x - 7$ → $f(x) = -1$

b. $x = ?$ → $f(x) = \sqrt{2x - 6}$ → $f(x) = 10$

1-81. Use the relationship graphed at right to answer the questions below.



a. Is the relationship a function?

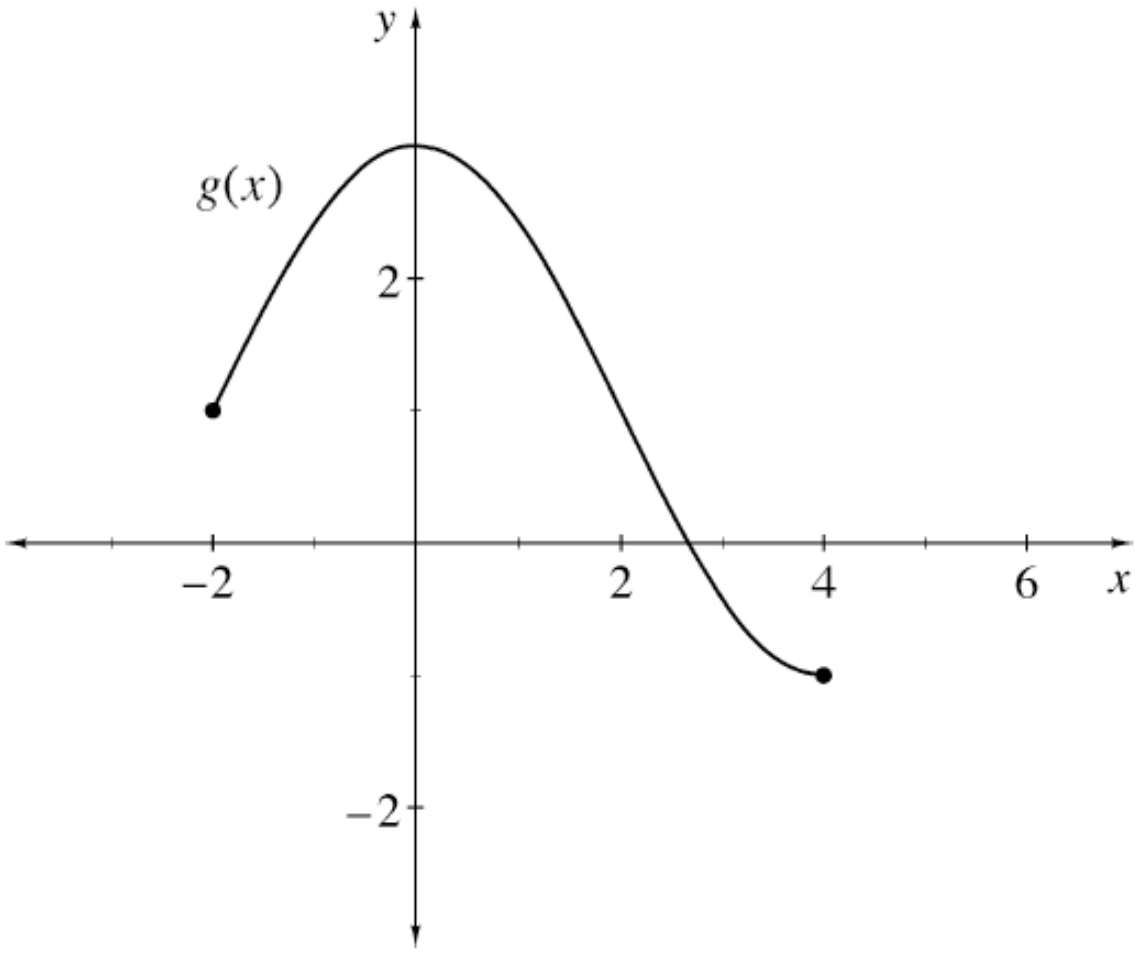
b. What is the domain?

c. What is the range?

1-82. What value(s) of x will make each equation true?

a. $\sqrt[3]{x} = -2$ b. $\sqrt{x} = 12$ c. $|x + 1| = 4$

Lesson 1.2.5 Resource Page
Problem 1-73



Chapter 1 Closure What have I learned?

Reflection and Synthesis

The activities below offer you a chance to reflect about what you have learned during this chapter. As you work, look for concepts that you feel very comfortable with, ideas that you would like to learn more about, and topics you need more help with. Look for connections between ideas as well as connections with material you learned previously.

① TEAM BRAINSTORM

What have you studied in this chapter? What ideas were important in what you learned? With your team, brainstorm a list. Be as detailed as you can. To help get you started, a list of Learning Log entries and Math Notes boxes are below.

What topics, ideas, and words that you learned *before* this chapter are connected to the new ideas in this chapter? Again, be as detailed as you can.

How long can you make your list? Challenge yourselves. Be prepared to share your team's ideas with the class.

Learning Log Entries

- Lesson 1.2.1 – Graph Investigation Questions
- Lesson 1.2.4 – Functions

Math Notes

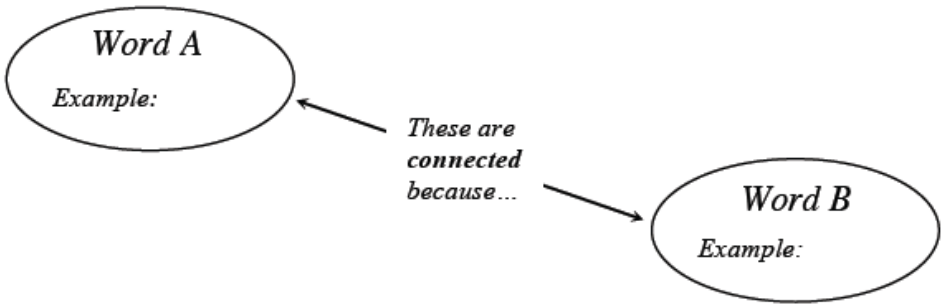
- Lesson 1.1.1 – Definition of Absolute Value
- Lesson 1.1.2 – Families of Functions
- Lesson 1.1.3 – Lines of Symmetry
- Lesson 1.2.5 – Functions

② MAKING CONNECTIONS

The following is a list of the vocabulary used in this chapter. Make sure that you are familiar with all of these words and know what they mean. Refer to the glossary or index for any words that you do not yet understand.

domain	function	graph
input	maximum	output
range	minimum	equation
parabola	x-intercept	$x \rightarrow y$ table
y-intercept function	line of symmetry	quadratic
absolute value		

Make a concept map showing all of the connections you can find among the key words and ideas listed above. To show a connection between two words, draw a line between them and explain the connection, as shown in the model below. A word can be connected to any other word as long as you can justify the connection. For each key word or idea, provide an example or sketch that shows the idea.



Your teacher may provide you with vocabulary cards to help you get started. If you use the cards to plan your concept map, be sure either to re-draw your concept map on your paper or to glue the vocabulary cards to a poster with all of the connections explained for others to see and understand.

While you are making your map, your team may think of related words or ideas that are not listed above. Be sure to include these ideas on your concept map.

③ PORTFOLIO: EVIDENCE OF MATHEMATICAL PROFICIENCY

Your teacher may have instructed you to take a photograph of the poster you made for Lesson 1.1.3 as evidence of your early understanding about describing functions. If so, include the photograph in your portfolio.

Your teacher will give you instructions for how to showcase your current understanding of describing a function. Part of this showcase will be to choose a function or two and make a graph and table of it. Make a list of the questions you will ask to fully investigate the function. Your teacher may give you the Chapter 1 Closure Resource Page: Investigations Graphic Organizer to help you organize your work. The purpose of the portfolio is to give you an opportunity to show what you know about fully investigating a function. Make sure you do your best work, and include as much detail as you can.

④ WHAT HAVE I LEARNED?

Most of the problems in this section represent typical problems found in this chapter. They serve as a gauge for you. You can use them to determine which types of problems you can do well and which types of problems require further study and practice. Even if your teacher does not assign this section, it is a good idea to try these problems and find out for yourself what you know and what you still need to work on.

Solve each problem as completely as you can. The table at the end of the closure section has answers to these problems. It also tells you where you can find additional help and practice with problems like these.

CL 1-83. Use the Order of Operations to simplify the following expressions.

a. $5-2\cdot 3^2$ b. $(-2)^2$

c. $18\div 3\cdot 6$ d. -2^2

e. $(5-3)(5+3)$ f. $24\cdot\frac{1}{4}\div -2$

g. Why are your answers for parts (b) and (d) different?

CL 1-84. Copy the pattern below onto graph paper. Draw the 1st and 5th figures on your paper.

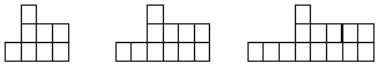


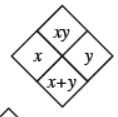
Figure 2 Figure 3 Figure 4

a. How many tiles are in each figure?

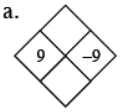
b. Describe how the pattern is changing.

c. How many tiles would the 6th figure have? The 10th figure?

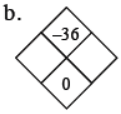
CL 1-85. Copy and complete each of the Diamond Problems below. The pattern used in the Diamond Problems is shown at right.



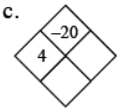
a.



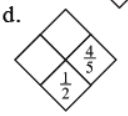
b.



c.



d.



CL 1-86. Graph and fully describe the function $y=2\sqrt{x-1}+3$.

CL 1-87. Solve each equation. Check your solution.

a. $3x-1=4x+8-x$ b. $-10+5x=7x-4$

c. $28-6x+4=30-3x$ d. $4x-1=9x-1-5x$

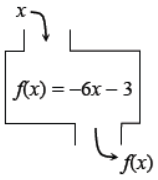
CL 1-88. Find $f(4)$ for each function below.

a. $f(x)=-|x-7|+3$ b. $f(x)=\frac{\sqrt{x+12}}{4}$ c. $f(x)=2-\sqrt[3]{x+23}$

CL 1-89. Evaluate each expression.

a. $2\div|3-4|$ b. $11|-6|+15$ c. $-19+\sqrt[3]{-8}$ d. $-11-\sqrt{16}$

CL 1-90. Use the function machine shown at right to answer the following questions.



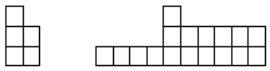
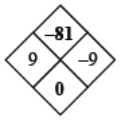
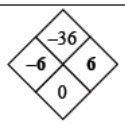
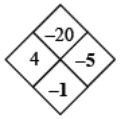
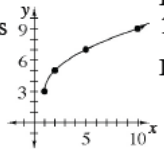
a. If the input is –8, what is the output?

b. If the output was 21, what was the input?

CL 1-91. Check your answers using the table at the end of the closure section. Which problems do you feel confident about? Which problems were hard? Use the table to make a list of topics you need help with and a list of topics you need to practice more.

Answers and Support for Closure Activity #4
What Have I Learned?

MN = Math Note, LL = Learning Log

Problem	Solution	Need Help?	More Practice
CL 1-83.	a. –13 b. 4 c. 36 d. –4 e. 16 f. –3 g. They are different because $(-2)^2 = (-2)(-2) = 4$, while $-2^2 = -(2 \cdot 2) = -4$.	Problems 1-7 and 1-14	Problems 1-20 and 1-28
CL 1-84.	 Figure 1 Figure 5 a. 5, 8, 11, 14, 17 b. Each figure has three more tiles than the one before it. c. The 6th figure would have 20 tiles. The 10th figure would have 32 tiles.	Lesson 1.1.2	Problems 1-6 and 1-37
CL 1-85.	a.  b.  c.  d. 	Problem 1-15	Problems 1-21, 1-26, 1-33, and 1-51
CL 1-86.	The graph is half of a parabola on its side. As x increases, y increases. The starting point is (1, 3).  Domain: $x \geq 1$ Range: $y \geq 3$	Lessons 1.1.3, 1.2.1, and 1.2.2 LL: 1.2.1	Problems 1-25, 1-30, 1-42, 1-47, 1-59, and 1-69
CL 1-87.	a. no solution b. $x = -3$ c. $x = \frac{2}{3}$ d. all real numbers	Checkpoint 1	Problems 1-16, 1-19, 1-36, 1-40, 1-49, and 1-67
CL 1-88.	a. 0 b. 1 c. –1	Lesson 1.2.3	Problems 1-57, 1-66, and 1-67
CL 1-89.	a. 2 b. 81 c. –21 d. –15	Lesson 1.2.2 MN: 1.1.1	Problems 1-5, 1-34, 1-35, 1-39, 1-48, 1-58, and 1-60
CL 1-90.	a. 45 b. –4	Lesson 1.2.3 MN: 1.2.5	Problems 1-71 and 1-81

Chapter 1 Closure Resource Page: Investigations Graphic Organizer Page 1 of 2

When I **investigate a function**, I answer each question below with a statement and justification.

Below is my investigation of the function _____

Question 1: _____

Statement 1: _____

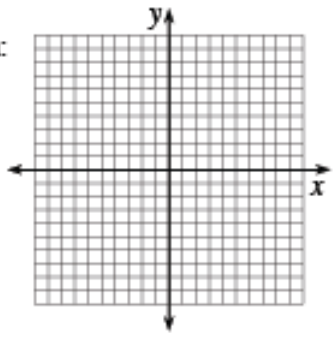
_____

Justification:

Table:

x							
y							

Graph:



Equation:

Other:

Question 2: _____

Statement 2: _____

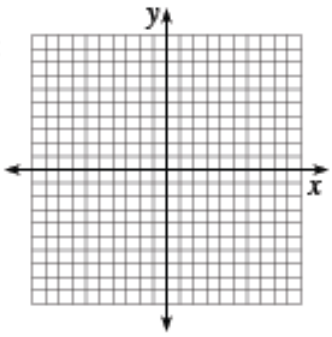
_____

Justification:

Table:

x							
y							

Graph:



Equation:

Other:

Question 3: _____

Statement 3: _____

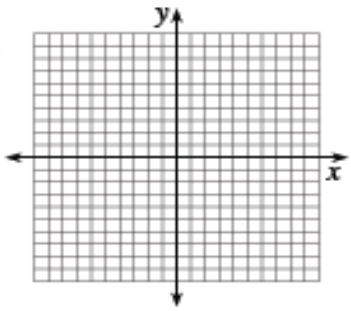
_____

Justification:

Table:

x						
y						

Graph:



Equation:

Other:

Question 4: _____

Statement 4: _____

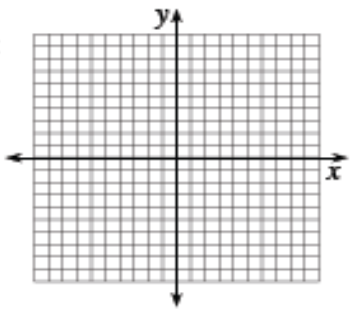
_____

Justification:

Table:

x						
y						

Graph:



Equation:

Other:

domain	function
graph	input
maximum	output
range	minimum
equation	parabola

x-intercept	$x \rightarrow y$ table
y-intercept	line of symmetry
quadratic function	absolute value
cube root	