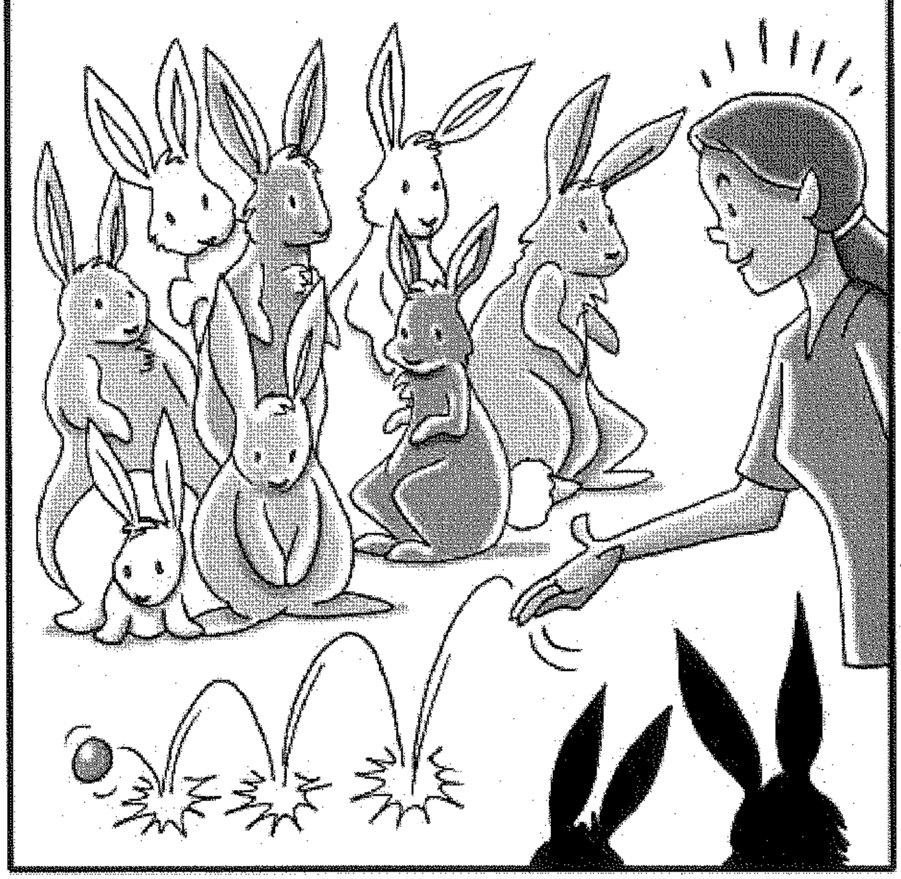
SEQUENCES 5



METHODS AND MEANINGS

MATH NOTES

The Elimination Method

When solving a system of equations, it may be easier to eliminate one of the variables by adding multiples of the two equations. This process is called **elimination**.

The first step is to rewrite the equations so that the x and y variables are lined up vertically. Next, decide what number to multiply each equation by, if necessary, in order to make the coefficients of either the x-terms or the y-terms add up to zero. Be sure that you can justify each step in the solution.

For example, consider the system at right.

$$\begin{aligned} 10y - 3x &= 14 \\ 4y + 2x &= -4 \end{aligned}$$

You can eliminate the x-terms by multiplying the top equation by 2 and the bottom equation by 3 and then adding the equations, as shown at right.

$$\begin{aligned} (10y - 3x = 14)\cdot 2 &\rightarrow 20y - 6x = 28 \\ (4y + 2x = -4)\cdot 3 &\rightarrow \underline{12y + 6x = -12} \\ \text{Add the resulting equations:} &\quad 32y \quad = 16 \\ \text{Divide:} &\quad y = 0.5 \end{aligned}$$

Finally, substitute 0.5 for y in either original equation:

$$\begin{aligned} 10(0.5) - 3x &= 14 \\ 5 - 3x &= 14 \\ -3x &= 9 \\ x &= -3 \end{aligned}$$

Thus, the solution to the original system is $(-3, 0.5)$.

Check your solution by evaluating for x and y in *both* of the *original* equations.

5-6. What if the data for Lenny and George (from problem 5-1) matched the data in each table below? Assuming that the growth of the rabbits multiplies as it did in problem 5-1, complete each of the following tables. Show your thinking or give a brief explanation of how you know what the missing entries are.

a.

Months	Rabbits
0	4
1	12
2	36
3	
4	

b.

Months	Rabbits
0	6
1	
2	24
3	
4	96

5-7. Solve the following systems of equations algebraically. Then graph each system to confirm your solution.

a. $x+y=3$
$x=3y-5$

b. $x-y=-5$
$y=-2x-4$

5-8. For the function $f(x)=\frac{6}{2x-3}$, find the value of each expression below.

a. $f(1)$ b. $f(0)$ c. $f(-3)$ d. $f(1.5)$

e. What value of x would make $f(x)=4$?

5-9. Benjamin is taking Algebra 1 and is stuck on the problem shown below. Examine his work so far and help him by showing and explaining the remaining steps.

Original problem: Simplify $(3a^{-2}b)^3$.

He knows that $(3a^{-2}b)^3=(3a^{-2}b)(3a^{-2}b)(3a^{-2}b)$. Now what?

5-10. Simplify each expression below. Assume that the denominator in part (b) is not equal to zero.

a. $(x^3)(x^{-2})$ b. $\frac{y^5}{y^{-2}}$ c. 4^{-1} d. $(4x^2)^3$

5-11. The equation of a line describes the relationship between the x- and y-coordinates of the points on the line.

a. Plot the points $(3,-1)$, $(3,2)$, and $(3,4)$ and draw the line that passes through them. State the coordinates of two more points on the line. Then answer this question: What will be true of the coordinates of any other point on this line? Now write an equation that says exactly the same thing. (Do not worry if it is very simple! If it accurately describes all the points on this line, it is correct.)

b. Plot the points $(5,-1)$, $(1,-1)$, and $(-3,-1)$. What is the equation of the line that goes through these points?

c. Choose any three points on the y-axis. What must be the equation of the line that goes through those points?

5-12. Jill is studying a strange bacterium. When she first looks at the bacteria, there are 1000 cells in her sample. The next day, there are 2000 cells. Intrigued, she comes back the next day to find that there are 4000 cells!

a. Should the graph of this situation be linear or curved?

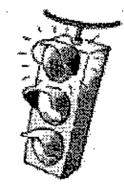
b. Create a table and graph for this situation. The inputs are the days that have passed after she first began to study the sample, and the outputs are the number of cells of bacteria.

5-13. Write each expression below in a simpler form.

a. $\frac{5^{723}}{5^{721}}$ b. $\frac{3^{300}}{3^{249}}$ c. $\left(\frac{3 \cdot 4^3}{3^{-2} \cdot 4^{-7}}\right)^0$ d. $\left(\frac{4 \times 10^3}{10^{-2}}\right)^2$

5-14. Jackie and Alexandra were working on homework together when Jackie said, *"I got $x = 5$ as the solution, but it looks like you got something different. Which solution is right?"*

$$(x+4)^2 - 2x - 5 = (x-1)^2$$
$$x^2 + 16 - 2x - 5 = x^2 + 1$$
$$16 - 2x - 5 = 1$$
$$11 - 2x = 1$$
$$-2x = -10$$
$$x = 5$$



"I think you made a mistake," said Alexa. Did Jackie make a mistake? Help Jackie figure out whether she made a mistake and, if she did, explain her mistake and show her how to solve the equation correctly. Jackie's work is shown above right.

5-15. Solve each of the following equations.

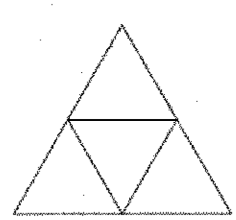
a. $\frac{m}{6} = \frac{15}{18}$ b. $\frac{x}{7} = \frac{a}{4}$

5-16. Write the equation of each line described below.

a. A line with slope –2 and y-intercept 7.

b. A line with slope $-\frac{3}{2}$ and x-intercept $(4, 0)$.

5-17. The dartboard shown at right is in the shape of an equilateral triangle. It has a smaller equilateral triangle in the center, which was made by joining the midpoints of the three edges. If a dart hits the board at random, what is the probability that:



a. The dart hits the center triangle?

b. The dart misses the center triangle but hits the board?

MATH NOTES

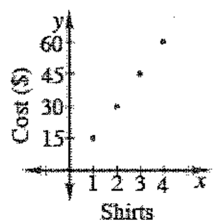
METHODS AND MEANINGS

Continuous and Discrete Graphs

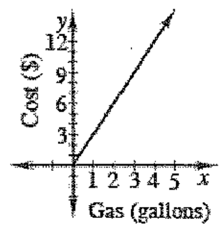
When the points on a graph are connected, and it *makes sense* to connect them, the graph is said to be **continuous**. If the graph is not continuous, and is just a sequence of separate points, the graph is called **discrete**.

For example, the graph below left represents the cost of buying x shirts, and it is discrete because you can only buy whole numbers of shirts. The graph farthest right represents the cost of buying x gallons of gasoline, and it is continuous because you can buy any (non-negative) amount of gasoline.

Discrete Graph



Continuous Graph



Review & Preview

5-22. Solve each system of equations below.

a. $y = 3x + 1$
$x + 2y = -5$

b. $2x + 3y = 9$
$x - 2y = 1$

5-23. Solve each equation for the indicated variable.

a. $t = an + b$ (for b)

b. $\frac{y}{3} - a = b$ (for y)

c. $m = \frac{y}{x}$ (for y)

d. $m = \frac{y}{x}$ (for x)

5-24. Simplify each expression below.

a. $\frac{6x^2y^3}{3xy}$

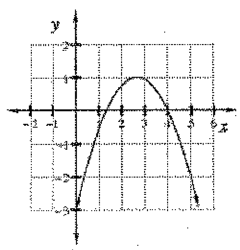
b. $(-mn)^3$

c. $(mn)^{-3}$

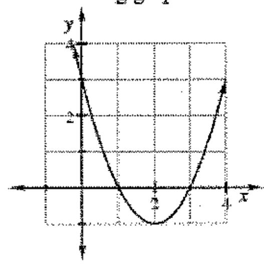
d. $\frac{3.2\times10^{-2}}{8\times10^3}$

5-25. Determine the domain and range of each of the following graphs.

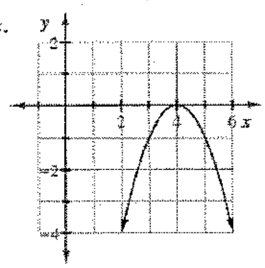
a.



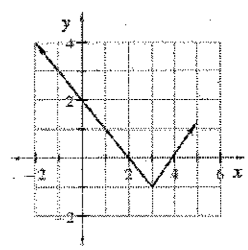
b.



c.

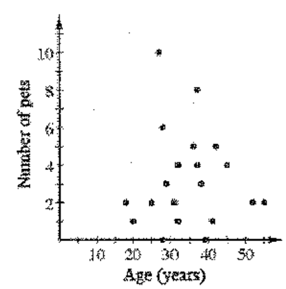


d.

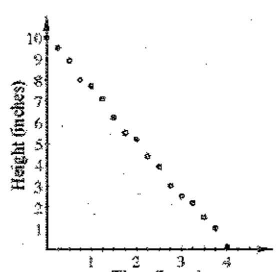


5-26. The graph at right compares the age and the number of pets for a certain population.

Describe the association for this population.



5-27. At an aunt's wedding, Nicolas collected data about an ice sculpture that was about to completely melt. A graph of his data is shown at right.



a. Calculate the equation of a line of best fit.

b. Based on your equation, how tall was the ice sculpture one hour before Nicolas started measuring?

Review & Preview

5-34. DeShawna and her team gathered data for their ball and recorded it in the table shown at right.

Drop Height	Rebound Height
150 cm	124 cm
70 cm	59 cm
120 cm	100 cm
100 cm	83 cm
110 cm	92 cm
40 cm	33 cm

a. What is the rebound ratio for their ball?

b. Predict how high DeShawna's ball will rebound if it is dropped from 275 cm. Look at the precision of DeShawna's measurements in the table. Round your calculation to a reasonable number of decimal places.

c. Suppose the ball is dropped and you notice that its rebound height is 60 cm. From what height was the ball dropped? Use an appropriate precision for your answer.

d. Suppose the ball is dropped from a window 200 meters up the Empire State Building. What would you predict the rebound height to be after the first bounce?

e. How high would the ball in part (d) rebound after the second bounce? After the third bounce?

5-35. Look back at the data given in problem 5-18 that describes the rebound ratio for an official tennis ball. Suppose you drop such a tennis ball from an initial height of 10 feet.

a. How high would it rebound after the first bounce?

b. How high would it rebound after the second bounce?

c. How high would it rebound after the fifth bounce?

5-36. Solve the following systems of equations algebraically and then confirm your solutions by graphing.

a. $y = 3x - 2$
$4x + 2y = 6$

b. $x = y - 4$
$2x - y = -5$

5-37. Lona received a stamp collection from her grandmother. The collection is in a leather book and currently has 120 stamps. Lona joined a stamp club, which sends her 12 new stamps each month. The stamp book holds a maximum of 500 stamps.

a. Complete the table at right.

Month	Stamps
0	120
1	132
2	
3	
4	
5	

b. How many stamps will Lona have in one year from now?

c. Write an equation using function notation to represent the total number of stamps that Lona has in her collection after n months. Let the total be represented by $t(n)$.

d. Solve your equation from part (c) for n when $t(n) = 500$. Will Lona be able to fill her book exactly with no stamps remaining? How do you know? When will the book be filled?

5-38. Use slope to determine whether the points $A(3,5)$, $B(-2,6)$, and $C(-5,7)$ are on the same line. Justify your conclusion algebraically.

5-39. Serena wanted to examine the graphs of the equations below on her graphing calculator. Rewrite each of the equations in **y-form** (when the equation is solved for y) so that she can enter them into the calculator.

a. $5 - (y - 2) = 3x$

b. $5(x + y) = -2$

5-44. Find the slope of the line you would get if you graphed each sequence listed below and connected the points.

a. 5, 8, 11, 14, ... b. 3, 9, 15, ...

c. 26, 21, 16, ... d. 7, 8.5, 10, ...

5-45. For the line passing through the points $(-2,1)$ and $(2,-11)$,

a. Calculate the slope of the line.

b. Find an equation of the line.

5-46. Allie is making 8 dozen chocolate-chip muffins for the Food Fair at school. The recipe she is using makes 3 dozen muffins. If the original recipe calls for 16 ounces of chocolate chips, how many ounces of chocolate chips does she need for her new amount? (Allie buys her chocolate chips in bulk and can measure them to the nearest ounce.)

5-47. The area of a square is 225 square centimeters.

a. Make a diagram and determine the length of each side.

b. Use the Pythagorean theorem to find the length of its diagonal.

5-48. Refer to sequences (c) and (i) in problem 5-41.

a. How are these two sequences similar?

b. The numbers in the sequence in part (e) from problem 5-41 are called **Fibonacci numbers**. They are named after an Italian mathematician who discovered the sequence while studying how fast rabbits could breed. What is different about this sequence than the other three you discovered?

5-49. Chelsea dropped a bouncy ball off the roof while Nery recorded its rebound height. The table at right shows their data. Note that the 0 in the "Bounce" column represents the starting height.

Bounce	Rebound Height
0	800 cm
1	475 cm
2	290 cm
3	175 cm
4	100 cm
5	60 cm

a. To what family does the function belong? Explain how you know.

b. Show the data as a sequence. Is the sequence arithmetic, geometric, quadratic, or something else? Justify your answer.

5-50. For the function $f(x)=\sqrt{3x-2}$, find the value of each expression below.

a. $f(1)$ b. $f(9)$ c. $f(4)$ d. $f(0)$

e. What value of x makes $f(x)=6$?

5-51. Simplify each expression below.

a. $y+0.03y$ b. $z-0.2z$ c. $x+0.002x$

5-52. A tank contains 8000 liters of water. Each day, half of the water in the tank is removed. How much water will be in the tank at the end of:

a. The 4th day?

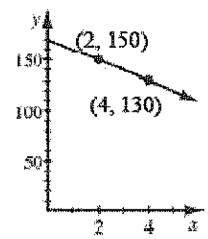
b. The 8th day?

5-53. Solve each system.

a. $y+3x=-10$
$5x-y=2$

b. $6x=7-2y$
$4x+y=4$

5-54. Draw a slope triangle and use it to find the equation of the line shown in the graph at right.



5-55. This problem is a checkpoint for laws of exponents and scientific notation. It will be referred to as Checkpoint 5A.

Simplify each expression. In parts (e) through (f) write the final answer in scientific notation.

a. $4^2 \cdot 4^5$

b. $(5^0)^3$

c. $x^{-5} \cdot x^3$

d. $(x^{-1} \cdot y^2)^3$

e. $(8 \times 10^5) \cdot (1.6 \times 10^{-2})$

f. $\frac{4 \times 10^3}{5 \times 10^5}$

Check your answers by referring to the Checkpoint 5A materials located at the back of your book.

If you needed help solving these problems correctly, then you need more practice. Review the Checkpoint 5A materials and try the practice problems. Also, consider getting help outside of class time. From this point on, you will be expected to do problems like these quickly and easily.

Review & Preview

5-65. Determine whether 447 is a term of each sequence below. If so, which term is it?

a. $t(n)=5n-3$

b. $t(n)=24-5n$

c. $t(n)=-6+3(n-1)$

d. $t(n)=14-3n$

e. $t(n)=-8-7(n-1)$

5-66. Choose one of the sequences in problem 5-65 for which you determined that 447 is *not* a term. Write a clear explanation describing how you can be sure that 447 is not a term of the sequence.

5-67. Find the sequence generator for each sequence listed below. Write an equation for the n^{th} term in each sequence below, keeping in mind that the first term of each sequence is $t(1)$.

a. $4, 7, 10, 13, \ldots$

b. $3, 8, 13, \ldots$

c. $24, 19, 14, \ldots$

d. $7, 9.5, 12, \ldots$

5-68. Great Amusements Park has been raising its ticket prices every year, as shown in the table at right.

Year	Price
0	\$50
1	\$55
2	\$60.50
3	\$66.55

a. Describe how the ticket prices are growing.

b. What will the price of admission be in year 6?

5-69. Solve the system at right for m and b.

$$1239=94m+b$$
$$810=61m+b$$

5-70. Write an equation or system of equations and solve the problem below.

The French club sold rose bouquets and chocolate hearts for Valentine's Day. The roses sold for \$5 and the hearts sold for \$3. The number of bouquets sold was 15 more than the number of hearts sold. If the club collected a total of \$339, how many of each gift was sold.

Review & Preview

5-76. Avery and Collin were trying to challenge each other with equations for sequences. Avery was looking at an explicit equation that Collin wrote.

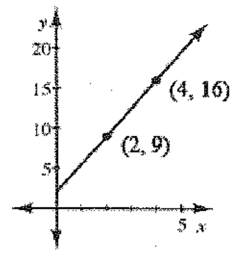
$$t(n) = 4.5n - 8$$

a. Write the first 4 terms for the sequence.

b. What would Avery do to write the 15^{th} term of this sequence?

5-77. Write both an explicit equation and a recursive equation for the sequence: $5, 8, 11, 15, 18, \ldots$

5-78. Draw a slope triangle and use it to find the equation of the line shown in the graph at right.



5-79. Find the following products.

a. $(4x+5)(4x-5)$

b. $(4x+5)^2$

5-80. Write an equation or system of equations to solve the problem below.

Apollo and Zeus are both on diets. Apollo currently weighs 105 kg and is gaining 2 kg per month. Zeus currently weighs 130 kg and is losing 3 kg per month. In how many months will they weigh the same?

5-81. Solve each system.

a. $6x - 2y = 10$
$3x - 2 = y$

b. $3x - 9y = 3$
$2x = 16 - y$

Review & Preview

5-85. Identify the following sequences as linear, exponential, or other. For the linear and exponential sequences, identify the rate of change and whether it is a constant that is added or multiplied.

a. $12, 144, 1728, \ldots$

b. $0, 5, 10, 15, 20, 25, \ldots$

c. $0, 4, 16, 36, 64, \ldots$

d. $1.5, 2.25, 3.375, 5.0625, \ldots$

5-86. Solve the system of equations at right.

$$y = -x - 2$$
$$5x - 3y = 22$$

5-87. Write the first five terms of each sequence.

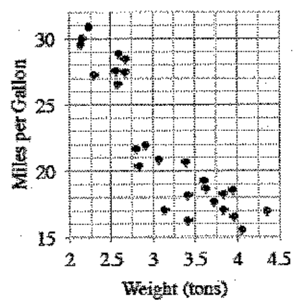
a. $t(1) = -3$, $t(n+1) = -2 \cdot t(n)$

b. $t(1) = 8$, $t(n+1) = t(n) - 5$

c. $t(1) = 2$, $t(n+1) = (t(n))^{-1}$

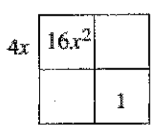
5-88. The graph at right compares the gas mileage to the weight of numerous vehicles.

Describe the association between these two quantities.

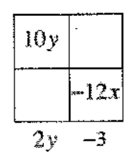


5-89. Find the missing areas and dimensions for each generic rectangle below. Then write each area as a sum and as a product.

a.



b.



5-90. This problem is a checkpoint for writing the equation of a line. It will be referred to as Checkpoint 5B.

Use the given information to find an equation of the line.

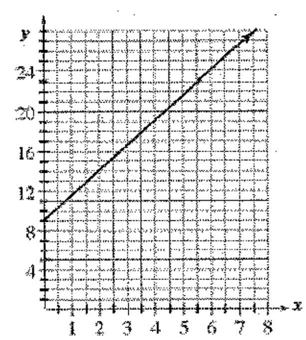
a. Slope 2 and passing through $(10,17)$.

b. Passing through $(1,-4)$ and $(-2,5)$.

c.

x	-6	-3	0	3	6
y	-6	-4	-2	0	2

d.



Check your answers by referring to the Checkpoint 5B materials located at the back of your book.

If you needed help solving these problems correctly, then you need more practice. Review the Checkpoint 5B materials and try the practice problems. Also, consider getting help outside of class time. From this point on, you will be expected to do problems like these quickly and easily.

MATH NOTES

METHODS AND MEANINGS

Types of Sequences

An **arithmetic sequence** is a sequence with an addition (or subtraction) **sequence generator**. The number added to each term to get the next term is called the **common difference.**.

A **geometric sequence** is a sequence with a multiplication (or division) generator. The number multiplied by each term to get the next term is called the **common ratio** or the **multiplier**.

A multiplier can also be used to increase or decrease by a given percentage. For example, the multiplier for an increase of 7% is 1.07. The multiplier for a decrease of 7% is 0.93.

A **recursive sequence** is a sequence in which each term depends on the term(s) before it. The equation of a recursive sequence requires at least one term to be specified. A recursive sequence can be arithmetic, geometric, or neither.

For example, the sequence $-1, 2, 5, 26, 677, \ldots$ can be defined by the **recursive equation:**

$$t(1) = -1, \quad t(n+1) = \left(t(n)\right)^2 + 1$$

An alternative notation for the equation of the sequence above is:

$$a_1 = -1, \quad a_{n+1} = (a_n)^2 + 1$$

Review & Preview

5-101. For each table below, find the missing entries and write an equation.

a.

Month (x)	0	1	2	3	4	5	6
Population (y)	2	8	32				

b.

Year (x)	0	1	2	3	4	5	6
Population (y)	5	6	7.2				

5-102. Convert each percent increase or decrease into a multiplier.

a. 3% increase

b. 25% decrease

c. 13% decrease

d. 2.08% increase

5-103. Mr. C is such a mean teacher! The next time Mathias gets in trouble, Mr. C has designed a special detention for him. Mathias will have to go out into the hall and stand exactly 100 meters away from the exit door and pause for a minute. Then he is allowed to walk exactly halfway to the door and pause for another minute. Then he can again walk exactly half the remaining distance to the door and pause again, and so on. Mr. C says that when Mathias reaches the door he can leave, *unless* he breaks the rules and goes more than halfway, even by a tiny amount. When can Mathias leave? Prove your answer using multiple representations.

5-104. Simplify each expression.

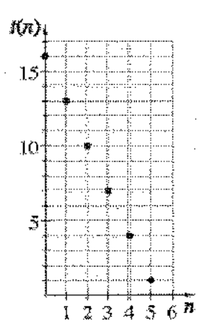
a. $(2m^3)(4m^2)$

b. $\frac{6y^5}{3y^2}$

c. $\frac{-4y^2}{6y^7}$

d. $(-2x^2)^3$

5-105. For this problem, refer to the sequences graphed below.

#1


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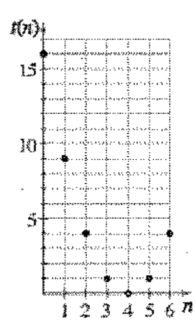

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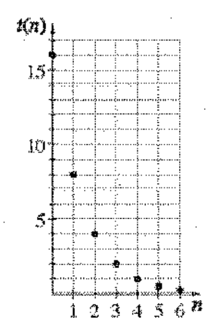


a. Identify each sequence as arithmetic, geometric, or neither.

b. If it is arithmetic or geometric, describe the sequence generator.

5-106. Read the Math Notes box in this lesson for information about an alternative notation for sequences and write the first 5 terms of these sequences.

a. $a_n = 2n - 5$

b. $a_1 = 3$
$a_{n+1} = -2 \cdot a_n$

5-107. Solve each equation.

a. $(x+2)(x+3) = x^2 - 10$

b. $\frac{1}{2}x + \frac{1}{3}x - 7 = \frac{5}{6}x$

c. $|2x - 1| = 9$

d. $\frac{x+1}{3} = \frac{x}{2}$

5-108. For each sequence defined recursively, write the first 5 terms and then define it explicitly.

a. $t(1) = 12$
$t(n+1) = t(n) - 5$

b. $a_1 = 32$
$a_{n+1} = \frac{1}{2}a_n$

5-109. **Multiple Choice:** Which line below is parallel to $y = -\frac{2}{3}x + 5$?

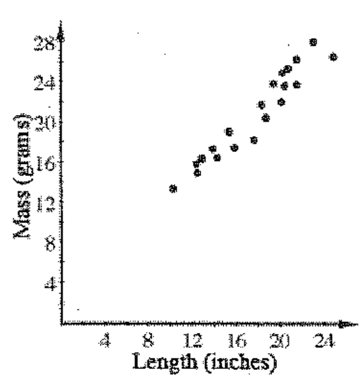
a. $2x - 3y = 6$

b. $2x + 3y = 6$

c. $3x - 2y = 6$

d. $3x + 2y = 6$

5-110. The graph at right shows a comparison of the length of several gold chain necklaces (including the clasp) to the total mass.



a. Write an equation for the line of best fit.

b. Based on your equation, what would you expect to be the mass of a 26 inch chain?

5-117. Is it possible for the sequence $t(n)=5\cdot 2^n$ to have a term with the value of 200? If so, which term is it? If not, justify why not.

5-118. Is it possible for the function $f(x)=5\cdot 2^x$ to have an output of 200? If so, what input gives this output? If not, justify why not.

5-119. Consider the following sequences as you complete parts (a) through (c) below.

Sequence 1	Sequence 2	Sequence 3
2, 6, ...	24, 12, ...	1, 5, ...

a. Assuming that the sequences above are arithmetic with $t(1)$ as the first term, find the next four terms for each sequence. For each sequence, write an explanation of what you did to get the next term and write an equation for $t(n)$.

b. Would your terms be different if the sequences were geometric? Find the next four terms for each sequence if they are geometric. For each sequence, write an explanation of what you did to get the next term and write an equation for $t(n)$.

c. Create a totally different type of sequence for each pair of values shown above, based on your own equation. Write your equation clearly (using words or algebra) so that someone else will be able to find the next three terms that you want.

5-120. For the function $g(x)=x^3+x^2-6x$, find the value of each expression below.

a. $g(1)$ b. $g(-1)$ c. $g(-2)$ d. $g(10)$

e. Find at least one value of x for which $g(x)=0$.

f. If $f(x)=x^2-x+3$, find $g(x)-f(x)$.

5-121. Write equations to solve each of the following problems.

a. When the Gleo Retro (a trendy commuter car) is brand new, it costs $23,500. Each year it loses 15% of its value. What will the car be worth when it is 15 years old?

b. Each year the population of Algeland increases by 12%. The population is currently 14,365,112. What will the population be 20 years from now?

5-122. An arithmetic sequence has $t(8)=1056$ and $t(13)=116$. Write an equation for the sequence. What is $t(5)$?

5-123. Describe the domain of each function or sequence below.

a. The function $f(x)=3x-5$. b. The sequence $t(n)=3n-5$.

c. The function $f(x)=\frac{5}{x}$. d. The sequence $t(n)=\frac{5}{n}$.

CL 5-124. Determine if the following sequences are arithmetic, geometric, or neither:

a. $-7, -3, 1, 5, 9, \ldots$

b. $-64, -16, -4, -1, \ldots$

c. $1, 0, 1, 4, 9, \ldots$

d. $0, 2, 4, \ldots$

CL 5-125. Find an equation to represent each table as a sequence.

a.

n	$t(n)$
1	4
2	1
3	–2
4	

b.

n	$t(n)$
1	6
2	7.2
3	8.64
4	

CL 5-126. Solve the following systems algebraically.

a. $x+2y=17$
$x-y=2$

b. $4x+5y=11$
$2x+6y=16$

c. $4x-3y=-10$
$x=\frac{1}{4}y-1$

d. $2x+y=-2x+5$
$3x+2y=2x+3y$

CL 5-127. Solve each equation after first rewriting it in a simpler equivalent form.

a. $3(2x-1)+12=4x-3$

b. $\frac{3x}{7}+\frac{2}{7}=2$

c. $\frac{x-3}{x}=\frac{3}{5}$

d. $4x(x-2)=(2x+1)(2x-3)$

CL 5-128. Simplify each expression.

a. $(-3x)^2$

b. $(3x)^{-2}$

c. $\frac{2(3x)^2}{3x^3}$

d. $\frac{2(3x)^2}{(3x)^{-2}}$

CL 5-129. Create multiple representations of each line described below.

a. A line with slope 4 and y-intercept -6.

b. A line with slope $\frac{3}{2}$ that passes through the point $(5,7)$.

CL 5-130. Create an explicit equation for each recursively-defined sequence below.

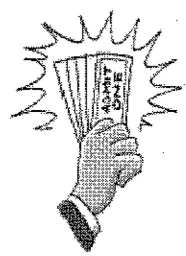
a. $a_1=17,\ a_{n+1}=a_n-7$

b. $t(1)=3,\ t(n+1)=5\cdot t(n)$

CL 5-131. Use a graph to describe the domain and range of each function or sequence below.

a. The function $f(x) = (x-2)^2$. b. The sequence $t(n) = 3n - 5$.

CL 5-132. When a family with two adults and three children bought tickets for an amusement park, they paid a total of \$56.50. The next family in line, with four children and one adult, paid \$49.50. Find the adult and child ticket prices by writin and solving a system of equations.



CL 5-133. Check your answers using the table at the end of this section. Which problems do you feel confident about? Which problems were hard? Have you worked on problems like these in previous math classes? Use the table to make a list of topics you need to learn more about, and a list of topics you just need to practice more.