
6
CONGRUENT
TRIANGLES

# CHAPTER 6 Congruent Triangles

In Chapter 5, you completed your work with the measurement of triangles, so you can now find the missing side lengths and angles of a triangle when sufficient information is given. Earlier, you developed ways to determine if two triangles are similar, and can use the ratios of similarity to learn more about the sides and angles of similar figures. But what if two triangles are congruent? What information can congruent triangles provide? In this chapter, you will find ways to determine whether two triangles are congruent.

In addition, Section 6.2 offers several projects and activities that will help you synthesize your understanding and make connections between different concepts you have learned so far. You will consolidate what you know, apply it in new ways, and identify what you still need to learn.

## Guiding Question

Mathematically proficient students construct viable arguments and critique the reasoning of others.

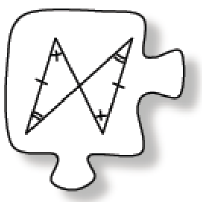
As you work through this chapter, ask yourself:

How can I use information to construct arguments, justify my conclusions and respond to the arguments presented by others?

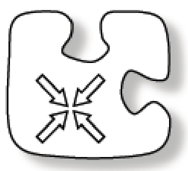
In this chapter, you will learn:

- The information that is needed in order to conclude that two triangles are congruent.
- The converse of a conditional statement and how to recognize whether or not the converse is true.
- How to organize a flowchart that concludes two triangles are congruent.

## Chapter Outline



**Section 6.1** This section turns the focus to congruent triangles. You will develop strategies to directly conclude that two triangles are congruent without first concluding that they are similar.



**Section 6.2** This section includes several big problems and activities to help you learn how different threads of geometry are connected and to help you assess what you know and what you still need to learn.

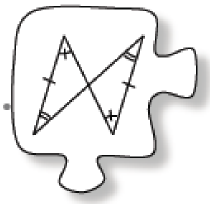
# Chapter 6 Teacher Guide

*Chapter 6 completes the first half of the course. Section 6.2 provides closure for Chapters 1 through 6 and offers practice with the major concepts studied so far.*

| Section | Lesson | Days | Lesson Objectives | Materials | Homework |
|---|---|---|---|---|---|
| 6.1 | 6.1.1 | 1 | Congruent Triangles | None | 6-4 to 6-10 |
| | 6.1.2 | 1 | Conditions for Triangle Congruence | • Lesson 6.1.2 Res. Pg. | 6-14 to 6-19 |
| | 6.1.3 | 1 | Congruence of Triangles Through Rigid Transformations | • Computer with projector | 6-23 to 6-28 |
| | 6.1.4 | 1 | Flowcharts for Congruence | None | 6-34 to 6-40 |
| | 6.1.5 | 1 | Converses | None | 6-46 to 6-52 |
| 6.2 | 6.2.1 | 1 | Angles on a Pool Table | • Lesson 6.2.1 Res. Pg. | 6-55 to 6-60 |
| | 6.2.2 | 1 | Investigating a Triangle | None | 6-62 to 6-67 |
| | 6.2.3 | 1 | Creating a Mathematical Model | • Modeling materials such as cardboard, string, glue, clay, etc. (opt.) | 6-73 to 6-78 |
| | 6.2.4 | 1 | Analyzing a Game | • *Hall* technology tool (opt.)<br>• Lesson 6.2.4 Res. Pg. (opt.)<br>• Paper and tape | 6-83 to 6-88 |
| | 6.2.5 | 1 | Using Transformations and Symmetry to Design Snowflakes | • Scissors<br>• Lesson 6.2.5 Res. Pg. | 6-94 to 6-99 |
| Chapter Closure | | Various Options | | | |

**Total: 10 days plus optional time for Chapter Closure and Assessment**

# 6.1.1 Are the triangles congruent?



## Congruent Triangles

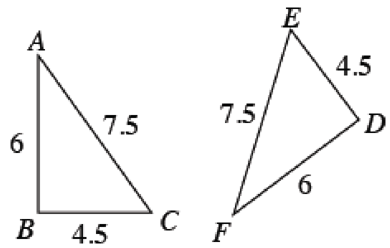
In Chapter 3, you learned how to identify similar triangles and used them to solve problems. But what can be learned when triangles are congruent? In today's lesson, you will practice identifying congruent triangles using what you know about similarity. As you search for congruent triangles in today's problems, focus on these questions:

What do I know about these triangles?

How can I show similarity?

What is the common ratio?

6-1. Examine the triangles at right.




a. Make a flowchart showing that these triangles are similar.

b. Are these triangles also congruent? Explain how you know.

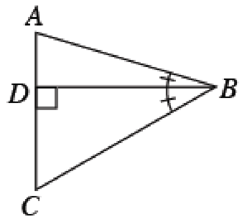
c. While the symbol for similar figures is "~", the symbol for congruent figures is "≅". How is the congruence symbol related to the similarity symbol? Why do you think mathematicians chose this symbol for congruence?

d. Luis wanted to write a statement to convey that these two triangles are congruent. He started with "$\Delta CAB$ …", but then got stuck because he did not know the symbol for congruence. Now that you know the symbol for congruence, complete Luis's statement for him.

6-2. The diagrams below are not drawn to scale. For each pair of triangles:

- Determine if the two triangles are congruent.
- If you find congruent triangles, write a congruence statement (such as $\Delta PQR \cong \Delta XYZ$).
- If the triangles are not congruent or if there is not enough information to determine congruence, then write "cannot be determined."

a. $\overline{AC}$ is a straight segment:

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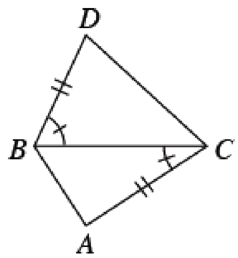


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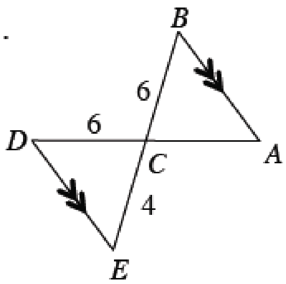
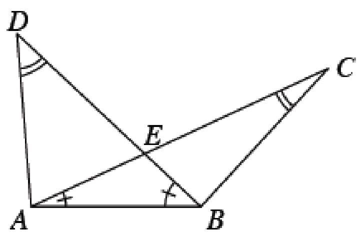
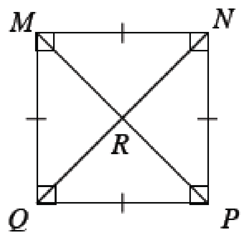



d.



6-3. Consider square *MNPQ* with diagonals intersecting at *R*, as shown at right.




a. How many triangles are there in this diagram? (Hint: There are more than 4!)

b. How many lines of symmetry does *MNPQ* have? On your paper, trace *MNPQ* and indicate the location of each line of symmetry with a dashed line.

c. Write as many triangle congruence statements as you can that involve triangles in this diagram. Be prepared to justify each congruence statement you write.

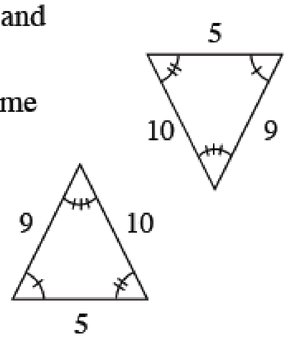
d. Write a similarity statement for two triangles in the diagram that are not congruent. Justify your similarity statement with a flowchart.

## METHODS AND MEANINGS

MATH NOTES

### Congruent Shapes

The information below is from Chapter 3 and is reprinted here for your convenience.



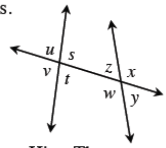

If two figures have the same shape and are the same size, they are **congruent**. Since the figures must have the same shape, they must be similar.

Two figures are congruent if there is a sequence of rigid transformations that carry one onto the other. Two figures are also congruent if they meet both the following conditions:

- The two figures are similar, and
- Their side lengths have a common ratio of 1.

Review & Preview

6-4. Use the diagram at right to answer the following questions.

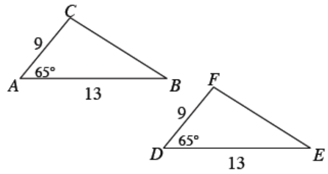



a. What type of angle pair is $\angle z$ and $\angle t$? That is, what is their geometric relationship?

b. What type of angle pair do $\angle s$ and $\angle v$ form?

c. Name all pairs of corresponding angles in the diagram. Hint: There are four different pairs.

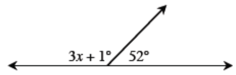
6-5. Examine the triangles at right.




a. Are these triangles similar? If so, use a flowchart to show how you know. If they are not similar, explain how you know.

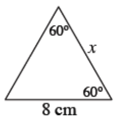
b. Are the triangles congruent? Explain your reasoning.

6-6. Using the diagram at right, write an equation and find $x$. Check your answer.

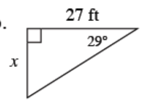



6-7. For each of the triangles below, find $x$.

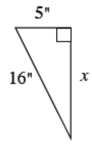
a.




b.




c.




6-8. Joey is in charge of selling cupcakes at the basketball games. The game is today at 5:00 p.m. At 3:00 p.m., Joey put some cupcakes into the oven and started working on his homework. He fell asleep and did not wake up until 3:55 p.m. What do you think happened?

Joey knows that the following four statements are facts:

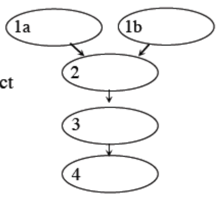
*i.* If the cupcakes are burned, then the fans that attend the varsity basketball games will not buy them.

*ii.* If cupcakes are in the oven for more than 50 minutes, they will burn.

*iii.* If the fans do not buy the cupcakes, then the team will not have enough money for new uniforms next year.

*iv.* The cupcakes are in the oven from 3:00 p.m. to 3:55 p.m.

Copy the flowchart at right and decide how to organize the facts into the ovals. You will need to fill in one of the ovals with your own logical conclusion. Be sure that your ovals are in the correct order and that the arrows really show connections between the ovals. What conclusion should your flowchart make?




6-9. Assume that 25% of the student body at your school is male and that 40% of the students walk to school. If a student from this school is selected at random, find the following probabilities.

a. P(student is female)

b. P(student is male and does not walk to school)

c. P(student walks to school or does not walk to school)

d. Identify the sample space in parts (b) and (c) above as a “union” or a “intersection.”

6-10. **Multiple Choice:** $\Delta ABC$ is defined by points $A(3, 2)$, $B(4, 9)$, and $C(6, 7)$. Which triangle below is the image of $\Delta ABC$ when it is rotated 90° counter-clockwise (↺) about the origin?

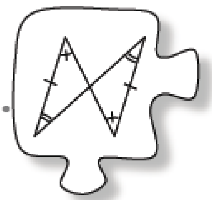
a. $A'(-2, 3)$, $B'(-9, 4)$, $C'(-7, 6)$

b. $A'(-3, 2)$, $B'(-4, 9)$, $C'(-6, 7)$

c. $A'(-2, 3)$, $B'(-7, 6)$, $C'(-9, 4)$

d. $A'(2, -3)$, $B'(9, -4)$, $C'(7, -6)$

e. None of these



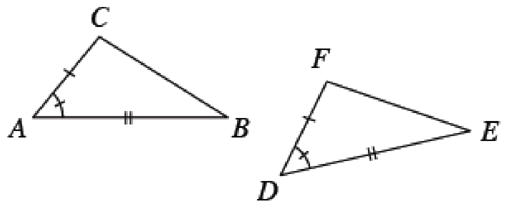
# 6.1.2 What information do I need?

## Conditions for Triangle Congruence

In Lesson 6.1.1, you identified congruent triangles by looking for similarity and a common side length ratio of 1. Must you go through this two-step process every time you want to argue that triangles are congruent? Are there shortcuts for establishing triangle congruence? Today you will investigate multiple triangle congruence conditions in order to quickly determine if two triangles are congruent.

6-11. Review your work from problem 6-5, which required you to determine if two triangles are congruent. Then work with your team to answer the questions below.

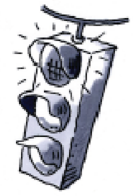
a. Derek wants to find general conditions that can help determine if triangles are congruent. To help, he draws the diagram at right to show the relationships between the triangles in problem 6-5.




If two triangles have the relationships shown in the diagram, do they have to be congruent? How do you know?

b. Write a theorem in the form of a conditional statement or arrow diagram based on this relationship. If you write a conditional statement, it should look like, "If two triangles …, then the triangles are congruent." What would be a good name (abbreviation) for this theorem?

6-12. TRIANGLE CONGRUENCE THEOREMS



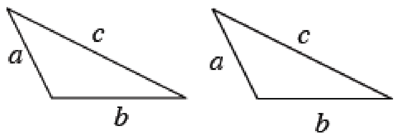
Derek wonders, "*What other types of information can determine that two triangles are congruent?*"

**Your Task:** Examine the pairs of triangles below to decide what other types of information force triangles to be congruent. Notice that since no measurements are given in the diagrams, you are considering the general case of each type of pairing. For each pair of triangles below that you can prove must be congruent, enter the appropriate triangle congruence theorem on your Lesson 6.1.2 Resource Page with an explanation defending your decision. An entry for SAS ≅ (the theorem you looked at problem 6-11) is already created on the resource page as an example. Be prepared to share your results with the class.

## Discussion Points

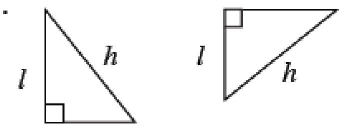
- For those triangles that must be congruent, how can you prove it?
- Which of the conditions below assure congruence?
- Which of the conditions below DO NOT assure congruence?

a.



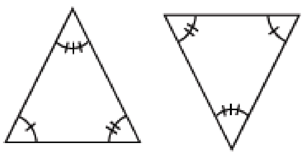

**Side-Side-Side: SSS**

b.



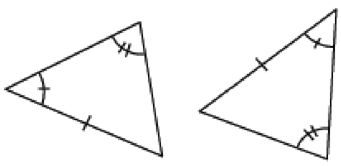

**Hypotenuse-Leg: HL**

c.



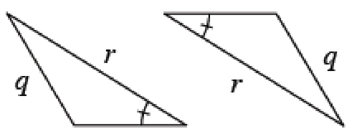
**Angle-Angle-Angle: AA**

d.



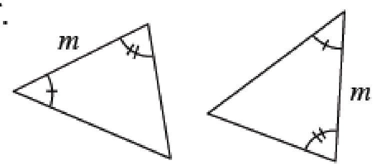
**Angle-Angle-Side: AAS**

e.




**Side-Side-Angle: SSA**

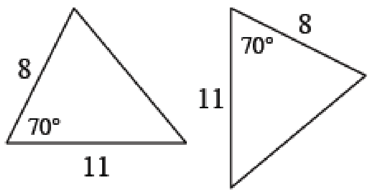
f.




**Angle-Side-Angle: ASA**

6-13. Use your triangle congruence theorems to determine if the following pairs of triangles must be congruent. Note: The diagrams are not necessarily drawn to scale.

a.

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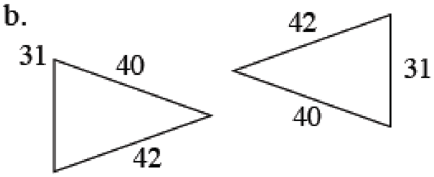


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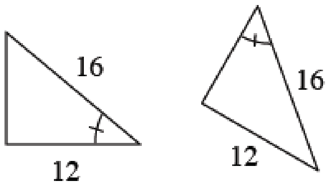


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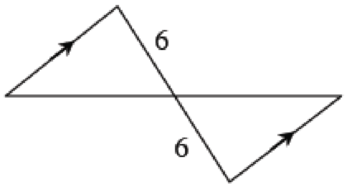


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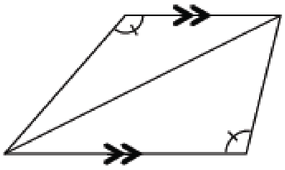
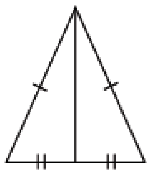
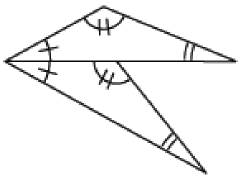


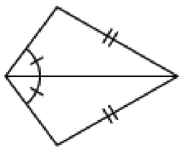
f.



g.

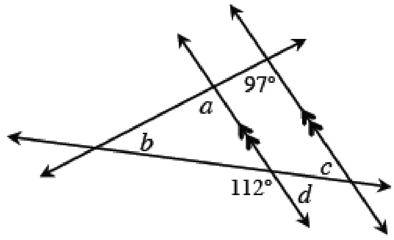


h.



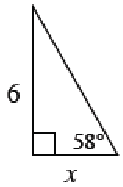
## Review & Preview

6-14. Examine the relationships that exist in the diagram at right. Find the measures of angles $a$, $b$, $c$, and $d$. Remember that you can find the angles in any order, depending on the angle relationships you use.

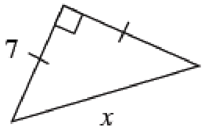



6-15. Examine the triangles below. For each one, solve for $x$ and name which tool you used. Show all work.

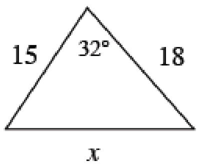
a.




b.



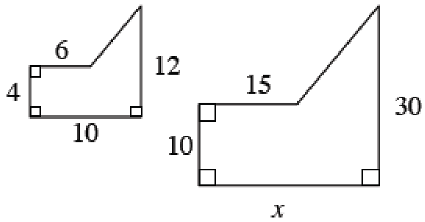

c.




6-16. The two shapes at right are similar.

a. Find the value of $x$. Show all work.

b. Find the area of each shape.




6-17. On graph paper, graph $\Delta ABC$ if its vertices are $A(-2, 7)$, $B(-5, 8)$, and $C(-3, 1)$.

a. Reflect $\Delta ABC$ across the $x$-axis to form $\Delta A'B'C'$. Name the coordinates of each new vertex.

b. Now rotate $\Delta A'B'C'$ from part (a) 180° about the origin (0, 0) to form $\Delta A''B''C''$. Name the coordinates of each new vertex.

c. Describe a single transformation that would change $\Delta ABC$ to $\Delta A''B''C''$.

6-18. Earl hates to take out the garbage and to wash the dishes, so he decided to make a deal with his parents: He will flip a coin once for each chore and will perform the chore if the coin lands on heads. What he doesn't know is that his parents are going to use a weighted coin that lands on heads 80% of the time!

a. What is the probability that Earl will have to do both chores?

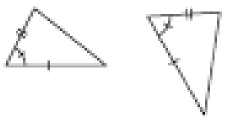
b. What is the probability that Earl will have to take out the garbage, but will not need to wash the dishes?

6-19. **Multiple Choice:** Which list of side lengths below could form a triangle?

a. 2, 6, 7  b. 3, 8, 13  c. 9, 4, 2  d. 10, 20, 30

Lesson 6.1.2 Resource Page

## Triangle Congruence Theorems

| | |
|---|---|
| **SAS $\cong$**<br>side-angle-side  | |
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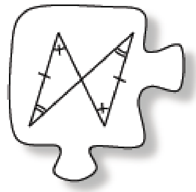
# 6.1.3 How can I prove it?

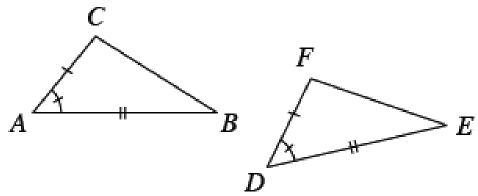


## Congruence of Triangles Through Rigid Transformations

In Lesson 6.1.2, you identified five conditions that guarantee triangle congruence. They are true because they require the triangles to be similar and because you know the ratio of the side lengths equals 1. However, the definition of congruence uses rigid transformations. So it is reasonable to assume these conditions can be proved without similarity. In this lesson, you will revisit each condition and develop new logic for why each one is true.

6-20. PROVING SAS TRIANGLE CONGRUENCE

A team is working together to try to prove SAS≅. Given the triangles shown at right, they want to prove that $\Delta ABC \cong \Delta DEF$.




Jurgen said, "*We know that congruent means that the two triangles have the same size and shape, so we have to be able to move $\Delta ABC$ right on top of $\Delta DEF$ using rigid motions, since they preserve lengths and angles. Does anyone see how we can do this?*"

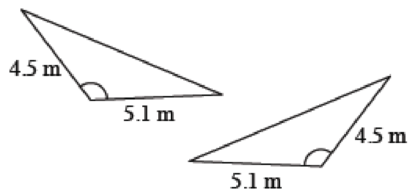
Carlos suggested, "*Well, we can certainly move point A on top of point D with a translation to get $\Delta A'B'C'$ with points $A' = D$, but nothing else matches.*"

Then Mary Sue added, "*Oh, then we can rotate $\Delta A'B'C'$ about point D to get $\Delta A''B''C''$ with the ray $A''B''$ pointing in the same direction as ray DE, and since points $A'' = D$, the two rays are the same. But does point $B''$ lie on top of point E then?*"

After a moment Emmy said, "*Sure. We know that rigid motions like translation and rotations preserve angles and lengths. So $AB = A'B' = A''B''$, right? And since we are assuming that $AB = DE$, then $A''B'' = DE$. Since point $A''$ is on point D, then point $B''$ must be at point E.*"

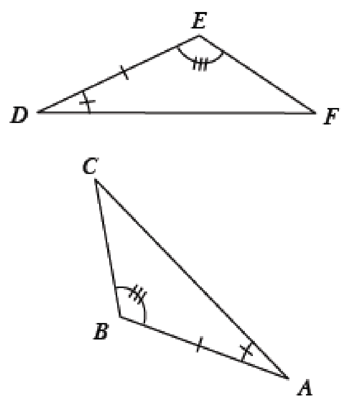
Jurgen added, "*And look, since $\Delta A''B''C''$ lies on top of $\Delta DEF$, they are congruent.*"

a. Use the SAS ~ tech tool and repeat their strategy to move $\Delta ABC$ onto $\Delta DEF$ to prove these triangles are congruent.




b. Emmy asks, "*What about these triangles? How can we make them coincide?*" Discuss how their previous strategy needs to be changed to show that these triangles are congruent using translations. Explain why this proves that all pairs of triangles with SAS ≅ must be congruent.

6-21. PROVING ASA TRIANGLE CONGRUENCE




In problem 6-20 you proved SAS $\cong$ by finding a sequence of rigid transformations that would move one triangle onto another. A similar strategy can be used to prove the ASA $\cong$ condition. Suppose that $\Delta ABC$ and $\Delta DEF$ are triangles with $\overline{AB} \cong \overline{DE}$, $\angle A \cong \angle D$, and $\angle B \cong \angle E$ as shown in the diagram at right. Look at the work of a second team below as they work to prove that $\Delta ABC \cong \Delta DEF$ using rigid transformations.

a. Janet said, "*Like last time, we can translate and then rotate the triangle so that $A''$ lies on D and $B''$ lies on E. Except this time, we have to show that even though these points are on top of each other, the others (C and F) are also.*"

   She thinks for a bit and adds, "*I think $\angle E \cong \angle B''$.*" How can she be sure that $\angle E \cong \angle B''$?

b. Patrick points out, "*This means that the ray $B''C''$ is identical to ray EF. Using the same reasoning, I know that ray $A''C''$ is identical to ray DF.*" Is Patrick correct? Explain why or why not.

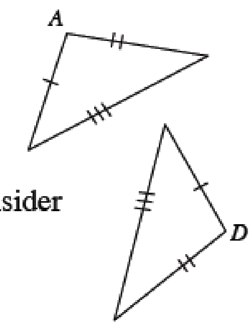
c. Aleesha then exclaimed, "*That's it! They must be congruent because this means that $C''$ is the same as F.*" Help her justify this conclusion.

d. Eddie asks, "*What if we need a reflection to have the triangles lie on top of each other?*" Does this affect the result? Explain why or why not.

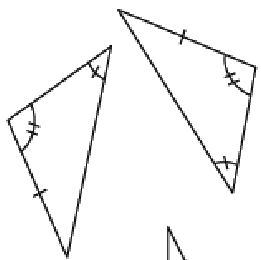
6-22. PROVING SSS, ASA, AND HL TRIANGLE CONGRUENCE

Interestingly, each of the other triangle congruence conditions can be shown to be true by either ASA ≅ or SAS ≅. Finish proving these three remaining conditions by answering the questions below.

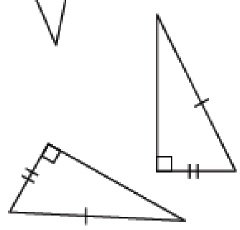
a. For the SSS ≅ condition, start with two triangles that have three pairs of congruent sides and explain why the triangles must be congruent. (Hint: Think about why the Law of Cosines guarantees that at least one of the pairs of angles has the same measure. To help, consider how you would calculate $m\angle A$ and $m\angle D$ at right.)




b. Explain why any pair of triangles with the AAS ≅ condition (two pairs of congruent angles and congruent sides that are not between them) can also be proved with the ASA ≅.



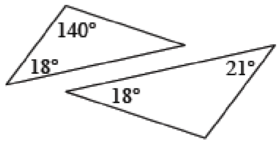
c. Finally, consider what you know about the side lengths of right triangles. How can you use SAS ≅ to prove the HL ≅ condition, that is, that right triangles with a pair of congruent legs and a pair of congruent hypotenuses must be congruent?



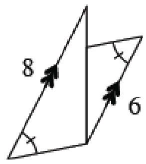
Review & Preview

6-23. For each pair of triangles below, decide if the pair is similar, congruent or neither. Justify your conclusion with a flowchart or the reasons why the triangles cannot be similar or congruent. Assume that the diagrams are not drawn to scale.

a.




b.




6-24. Use what you know about the slopes of parallel and perpendicular lines to find the equation of each line described below.

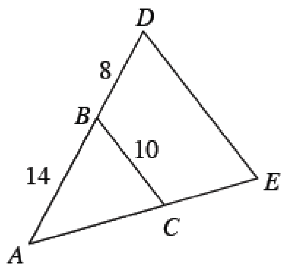
a. Find the equation of the line that goes through the point (2, –3) and is perpendicular to the line $y = -\frac{2}{5}x + 6$.

b. Find the equation of the line that is parallel to the line $-3x + 2y = 10$ and goes through the point (4, 7).

6-25. In the diagram at right, $\Delta ABC \sim \Delta ADE$.

a. Draw each triangle separately on your paper. Be sure to include all measurements in your diagrams.

b. Find the length of $\overline{DE}$.

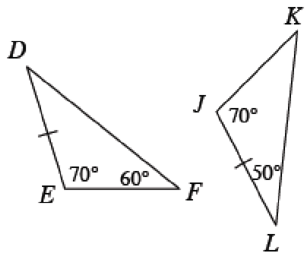



6-26. Examine the two triangles at right.

a. Are the triangles congruent? Justify your conclusion. If they are congruent, complete the congruence statement $\Delta DEF$ ____.

b. What series of transformation(s) are needed to change $\Delta DEF$ to $\Delta JKL$?

c. If $DE = 4$ units, find $KL$.



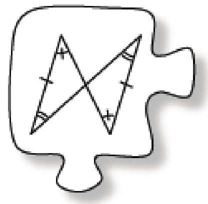

6-27. If $b + 2a = c$ and $2a + b = 10$, then what is $c$? Explain how you know.

6-28. There are 212 students enrolled in geometry at West Valley High School. 64 are freshman, and 112 are sophomores.

a. If a random geometry student is chosen, what is the chance (in percent) the student is a freshman or sophomore? Show how you can use the Addition Rule to answer this question. What was unusual about using the Addition Rule to answer this question?

b. 114 of the geometry students perform in band and 56 perform in chorus. There is a 75% chance that a geometry student performs in either band or chorus. What is the probability a geometry student performs both in band and in chorus?

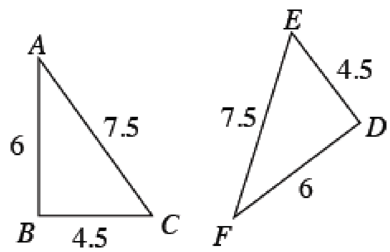
# 6.1.4 How can I organize my reasoning?



Flowcharts for Congruence

Now that you have shortcuts for establishing triangle congruence, how can you organize information in a flowchart to show that triangles are congruent? Consider this question as you work today with your team.

6-29. In problem 6-1, you determined that the triangles at right are congruent.



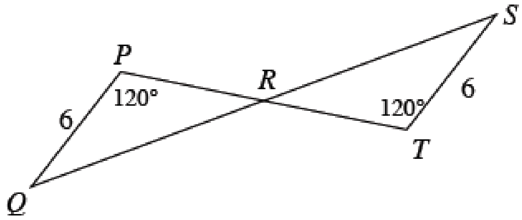

a. Which triangle congruence theorem shows that these triangles are congruent?

b. Make a flowchart showing your argument that these triangles are congruent.

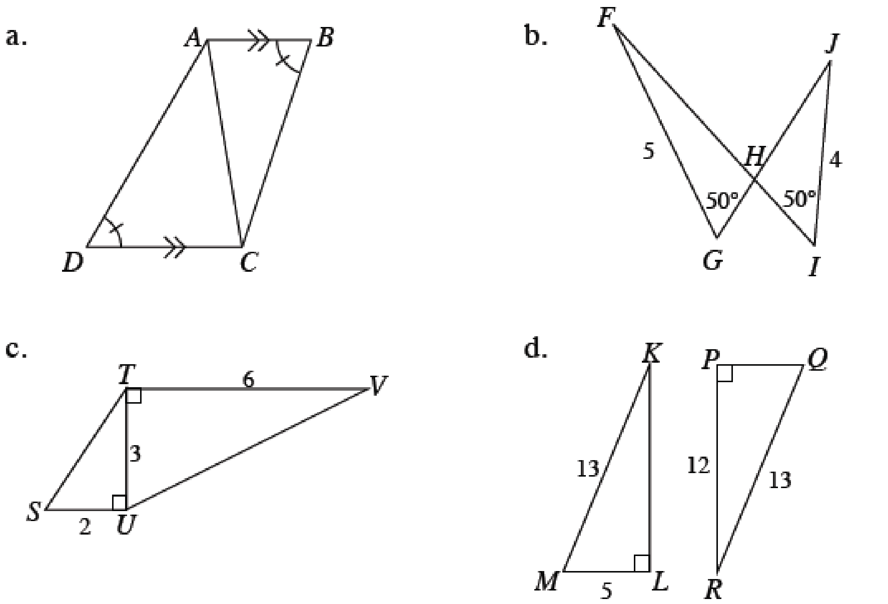
c. How is a flowchart showing congruence different from a flowchart showing similarity? List every difference you can find.

6-30. In Don's congruence flowchart for problem 6-29, one of the ovals said "$\frac{AB}{FD}=1$". In Phil's flowchart, one of the ovals said, "$AB = FD$". Discuss with your team whether these ovals say the same thing. Can equality statements like Phil's always be used in congruence flowcharts?

6-31. Make a flowchart showing that the triangles below are congruent.



6-32. In each diagram below, determine whether the triangles are congruent, similar but not congruent, or not similar. If you claim the triangles are similar or congruent, make a flowchart justifying your answer.



6-33. Suppose you are working on a problem involving the two triangles $\Delta UVW$ and $\Delta XYZ$ and you know that $\Delta UVW \cong \Delta XYZ$. What can you conclude about the sides and angles of $\Delta UVW$ and $\Delta XYZ$? Write down every equation involving side lengths or angle measures that must be true.

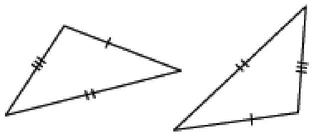
# Methods and Meanings

MATH NOTES

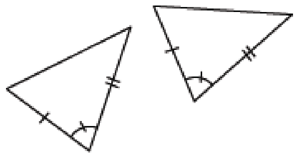
## Triangle Congruence Conditions

To show that triangles are congruent, you can show that the triangles are similar and that the common ratio between side lengths is 1 or you can use rigid motions (transformations). However, you can also use certain combinations of congruent, corresponding parts as shortcuts to determine if triangles are congruent. These combinations, called **triangle congruence conditions**, are:

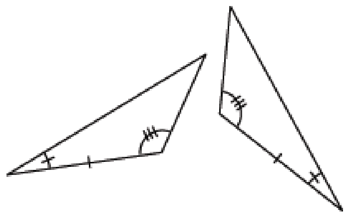
**SSS ≅** (Pronounced "side–side–side")
If all three pairs of corresponding sides have equal lengths, then the triangles are congruent.



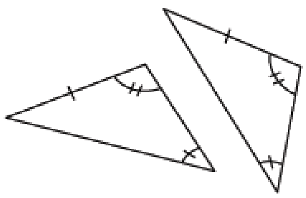
**SAS ≅** (Pronounced "side–angle–side")
If two pairs of corresponding sides have equal lengths *and* the angles between them (the included angle) are equal, then the triangles are congruent.



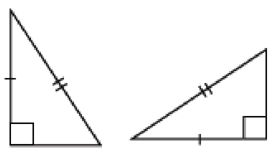
**ASA ≅** (Pronounced "angle–side–angle") If two angles and the side between them in a triangle are congruent to the corresponding angles and side in another triangle, then the triangles are congruent.



**AAS ≅** (Pronounced "angle–angle–side")
If two pairs of corresponding angles *and* a pair of corresponding sides that are not between them have equal measures, then the triangles are congruent.

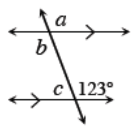


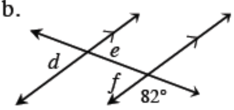
**HL ≅** (Pronounced "hypotenuse–leg")
If the hypotenuse and a leg of one right triangle have the same lengths as the hypotenuse and a leg of another right triangle, the triangles are congruent.

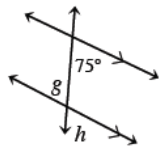


## Review & Preview

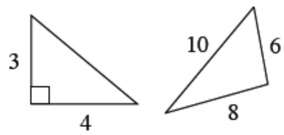
6-34. Use your theorems about parallel lines and the angles formed with a third line to find the measures of the labeled angles below. Show each step you use and be sure to justify each one with an angle theorem from your Angle Relationships Toolkit. Be sure to write down your reasoning in the order that you find the angles.

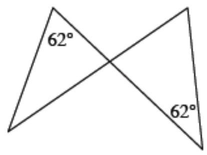
a.



b.



c.



6-35. For each pair of triangles below, decide if the pair is similar, congruent or neither. Justify your conclusion using a flowchart. Assume that the diagrams are not drawn to scale.

a.


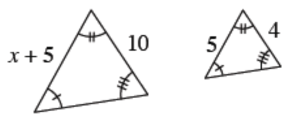

b.



6-36. Solve the systems of equations below using any method, if possible. Show all work and check your solution. If there is no solution, explain why not.

a. $y = 2x + 8$
$3x + 2y = -12$

b. $2x - 5y = 4$
$5y - 2x = 10$

6-37. Examine the triangles at right. Solve for $x$.




6-38. Solve the problem below using any method. Show all work.

Angle $A$ of $\Delta ABC$ measures 5° more than 3 times the measure of angle $B$. Angle $C$ measures 20° less than angle $B$. Find the measure of angles $A$, $B$, and $C$.

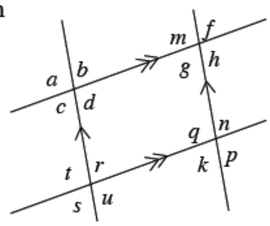
6-39. Kendra has programmed her cell phone to randomly show one of six photos when she turns it on. Two of the photos are of her parents, one is of her niece, and three are of her boyfriend, Bruce. Today she will need to turn her phone on twice: once before school and again after school.

a. Choose a model (such as a tree diagram or generic area model) to represent this situation.

b. What is the probability that both photos will be of her boyfriend?

c. What is the probability that neither photo will be of her niece?

6-40. **Multiple Choice:** Given the diagram at right, which of the statements below is not necessarily true?

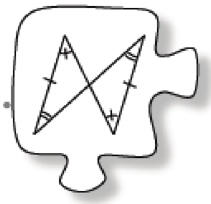



a. $a = d$ b. $d + r = 180°$

c. $u = n$ d. $t = m$

e. These all must be true.

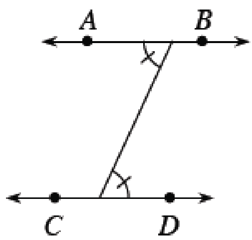
# 6.1.5 What is the relationship?



## Converses

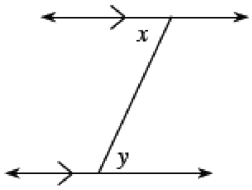
So far in this chapter, you have completed several problems in which you were given certain information and had to determine whether triangles were congruent. But what if you already know triangles are congruent? What information can you conclude then? Thinking this way requires you to reverse your triangle congruence theorems. Today you will look more generally at what happens when you reverse a theorem.

6-41. Jorge is working with the diagram at right, and concludes that $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$. He writes the following conditional statement to justify his reasoning:




*If alternate interior angles are equal, then lines are parallel.*

a. Margaret is working with a different diagram, shown at right. She concludes that $x = y$. Write a conditional statement or arrow diagram that justifies her reasoning.




b. How are Jorge's and Margaret's statements related? How are they different?

c. Conditional statements that have this relationship are called **converses**. Write the converse of the conditional statement below.

*If lines are parallel, then corresponding angles are equal.*

6-42. In problem 6-41, you learned that each conditional statement has a converse. Do you think that all converses true? Consider the arrow diagram of a familiar theorem below.

*Triangles congruent* → *corresponding sides are congruent.*

a. Is this arrow diagram true?

b. Write the converse of this arrow diagram as an arrow diagram or as a conditional statement. Is this converse true? Justify your answer.

c. Now consider another true congruence conjecture below. Write its converse and decide if it is true. If it is true, prove it. If it is not always true, explain why not.

*Triangles congruent* → *corresponding angles are congruent.*

d. Write the converse of the arrow diagram below. Is this converse true? Justify your answer.

*A shape is a rectangle* → *the area of the shape is* $b \cdot h$.

6-43. CRAZY CONVERSES

For each of these problems below, make up a conditional statement or arrow diagram that meets the stated conditions. You must use a different example each time, and none of your examples can be about math!

a. A true statement whose converse is true.

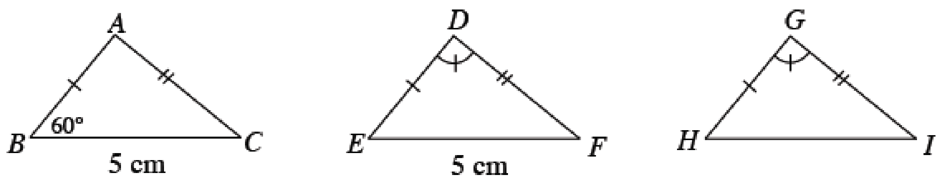
b. A true statement whose converse is false.

c. A false statement whose converse is true.

d. A false statement whose converse is false.

6-44. INFORMATION OVERLOAD

Raj is solving a problem about three triangles. He is trying to find the measure of $\angle H$ and the length of $\overline{HI}$. Raj summarizes the relationships he has found so far in the diagrams below:




a. Help Raj out! Assuming everything marked in the diagram is true, find $m\angle H$ and the length of $\overline{HI}$. Make sure to justify all your claims – do not make assumptions based on how the diagram looks!

b. Raj is still confused. Write a careful explanation of the reasoning you used to find the values in part (a). Whenever possible, use arrow diagrams or conditional statements in explaining your reasoning.

6-45. LEARNING LOG

Write an entry in your Learning Log about the converse relationship. Explain what a converse is, and give an example of a conditional statement and its converse. Also discuss the relationship between the truth of a statement and its converse. Title this entry "Converses" and label it with today's date.

## METHODS AND MEANINGS

MATH NOTES

### Converses

When conditional statements (also called "If …, then …" statements) are written backwards so that the condition (the "if" part) is switched with the conclusion (the "then" part), the new statement is called a **converse**. For example, examine the theorem and its converse below:

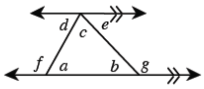
**Theorem:** If two parallel lines are cut by a transversal, then pairs of corresponding angles are equal.

**Converse:** If two corresponding angles formed when two lines are cut by a transversal are equal, then the lines cut by the transversal are parallel.

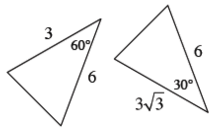
Since the second statement is a reversal of the first, is called its converse. Note that just because a theorem is true does not mean that its converse must be true. For example, if the conditional statement, "If the dog has a meaty bone, then the dog is happy," is true, but its converse, "If the dog is happy, then the dog has a meaty bone," is not necessarily true. The dog could be happy for other reasons, such as going for a walk.

Review & Preview

6-46. Copy the diagram at right onto your paper. If $a = 53°$ and $g = 125°$, find the measures of each labeled angle. Explain how you find each angle, citing definitions and theorems from your toolkit that support your steps. Remember that you can find the angles in any order, depending on the angle relationships you use.




6-47. As Samone looked at the triangles at right, she said, "*I think these triangles are congruent.*" Her teammate, Darla, said, "*But they don't look the same. How can you tell?*" Samone smiled and said, "*Never trust the picture! Look at the angles and the sides. The measures are all the same.*"




a. Solve for the missing side of each triangle. How do they compare?

b. Are you convinced that Samone is correct? Explain.

6-48. Write a converse for each conditional statement below. Then, assuming the original statement is true, decide if the converse must be true or not.

a. If it rains, then the ground is wet.

b. If a polygon is a square, then it is a rectangle.

c. If a polygon is a rectangle, then it has four 90° angles.

d. If the shape has three angles, it is a triangle.

e. If two lines intersect, then vertical angles are congruent.

6-49. On graph paper, graph the line $y = -\frac{3}{2}x + 6$. Name the $x$- and $y$-intercepts.

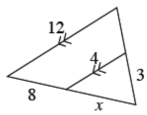
6-50. This problem is a checkpoint for solving proportional equations and similar figures. It will be referred to as Checkpoint 6.

a. $\frac{7-y}{5} = \frac{3}{4}$

b. $\frac{3}{y} = \frac{6}{y-2}$

c. Sam grew $1\frac{3}{4}$ inches in $4\frac{1}{2}$ months. If he continued at the same rate, how much would he grow in one year?

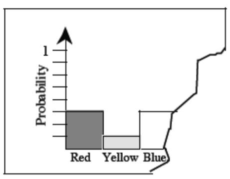
d. Use the figure at right to solve for $x$.




Check your answers by referring to the Checkpoint 6 materials located at the back of your book.

If you needed help solving these problems correctly, then you need more practice. Review the Checkpoint 6 materials and try the practice problems. Also, consider getting help outside of class time. From this point on, you will be expected to do problems like these quickly and easily.

6-51. Donnell has a bar graph which shows the probability of a colored section coming up on a spinner, but part of the graph has been ripped off.



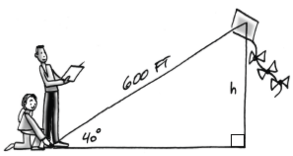

a. What is the probability of spinning red?

b. What is the probability of spinning yellow?

c. What is the probability of spinning blue?

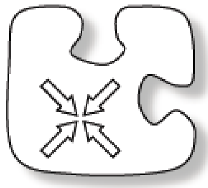
d. If there is only one color missing from the graph, namely green, what is the probability of spinning green? Why?

6-52.




One measurement used in judging kite-flying competitions is the size of the angle formed by the kite string and the ground. This angle can be used to find the height of the kite. Suppose the length of the string is 600 feet and the angle at which the kite is flying measures 40°. Calculate the height, $h$, of the kite.

# 6.2.1 How can I use it? What is the connection?



## Angles on a Pool Table

The activities in this section review several big topics you have studied so far. Work with your team to decide which combination of tools you will need for each problem. As you work together, think about which skills and tools you are comfortable using and which ones you need more practice with.

As you work on this activity, keep in mind the following questions:

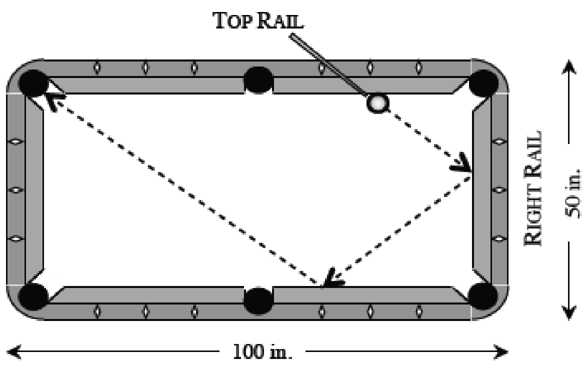
What mathematical concepts did you use to solve this problem?

What do you still want to know more about?

What connections did you find?

6-53. TAKE IT TO THE BANK

Ricky just watched his favorite pool player, Montana Mike, make a double bank shot in a trick-shot competition. Montana bounced a ball off two rails (sides) of the table and sank it in the corner pocket. "*That doesn't look too hard,*" Ricky says, "*I just need to know where to put the ball and in which direction to hit it.*"




A diagram of Montana's shot is shown at right. The playing area of a tournament pool table is 50 inches by 100 inches. Along its rails, a pool table is marked with a diamond every 12.5 inches. Montana started the shot with the ball against the top rail and the ball hit the bottom rail three diamonds from the right rail.

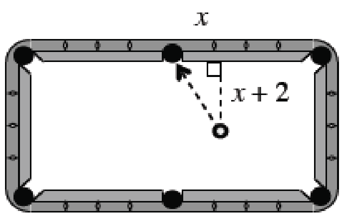
**Your Task:** Figure out where on the top rail Ricky needs to place his ball and where he needs to aim to repeat Montana Mike's bank shot. Write instructions that tell Ricky how to use the diamonds on the table to place his ball correctly, and at what angle from the rail to hit the ball.

6-54. EXTENSIONS

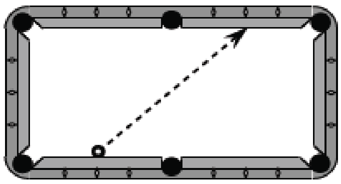
The algebraic and geometric tools you have developed so far will enable you to answer many questions about the path of a ball on a pool table. Work with your team to analyze the situations below.

a. Ricky decided he wants to alter Montana's shot so that it hits the right rail exactly at its midpoint. Where would Ricky need to place the ball along the top rail so that his shot bounces off the right rail, then the bottom rail, and enters the upper left pocket? At what angle with the top rail would he need to hit the ball?

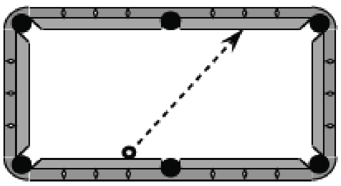
b. During another shot, Ricky noticed that Montana hit the ball as shown in the diagram at right. He estimated that the ball traveled 18 inches before it entered the pocket. Before the shot, the announcers noted that the distance of the ball to the top rail was 2 inches more than the distance along the top rail, as shown in the diagram. Where was the ball located before Montana hit it?




c. Ricky wants to predict how Montana's next shot will end. The ball is placed at the second diamond from the left along the bottom rail, as shown at right. Montana is aiming to hit the ball toward the second diamond from the right along the top rail. Assuming he hits the ball very hard so that the ball continues traveling indefinitely, will the ball ever reach a pocket? If so, show the path of the ball. If not, explain how you know.



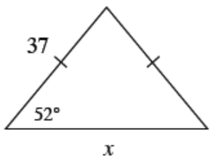
d. After analyzing the path in part (c), Montana decided to start his ball from the third diamond from the left along the bottom rail, as shown at right. He is planning to aim at the same diamond as he did in part (c). If he hits the ball sufficiently hard, will the ball eventually reach a pocket? If so, show the path of the ball. If not, explain how you know.



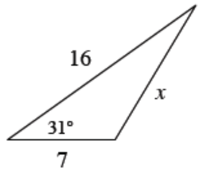
## Review & Preview

6-55. Use your triangle tools to solve for $x$ in the triangles below.

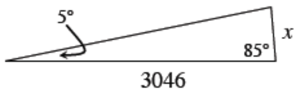
a.




b.




c.

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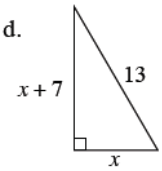
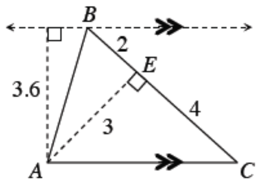


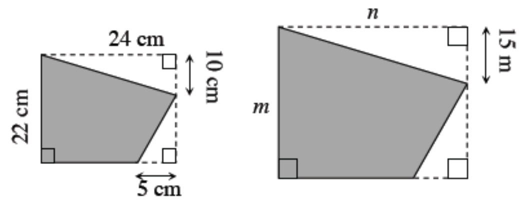

6-56. Penelope measured several sides and heights of $\Delta ABC$, as shown in the diagram at right. Find the area of $\Delta ABC$ *twice*, using two different methods.



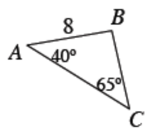

6-57. The shaded figures at right are similar.

a. Solve for $m$ and $n$.

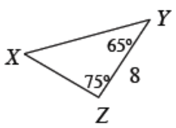
b. Find the area and perimeter of each figure.



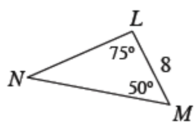

6-58. Decide if each triangle below is congruent to $\Delta ABC$ at right, similar but not congruent to $\Delta ABC$, or neither. Justify each answer. If you decide that they are congruent, organize your reasoning into a flowchart.



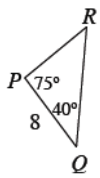

a.




b.




c.




6-59. On graph paper, graph the line $y = 3x + 1$.

a. What is the slope angle of the line? That is, what is the acute angle the line makes with the $x$-axis?

b. Find the equation of a new line that has a slope angle of 45° and passes through the point $(0, 3)$. Assume that the slope is positive.

c. Find the intersection of these two lines using any method. Write your solution as a point in the form $(x, y)$.

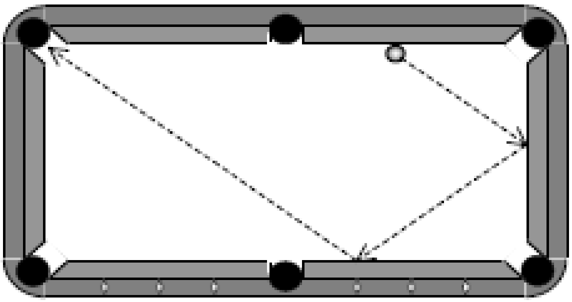
6-60. **Multiple Choice:** Listed below are the measures of several different angles. Which angle is obtuse?

a. 0° b. 52° c. 210° d. 91°

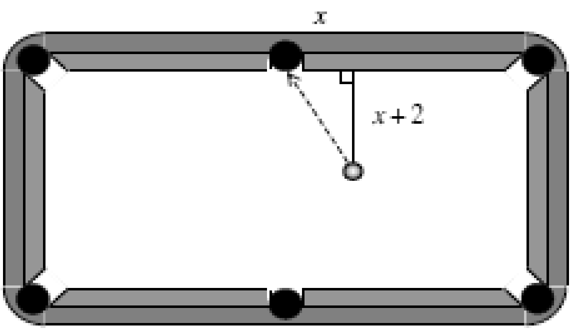
Lesson 6.2.1 Resource Page

## Take It To The Bank

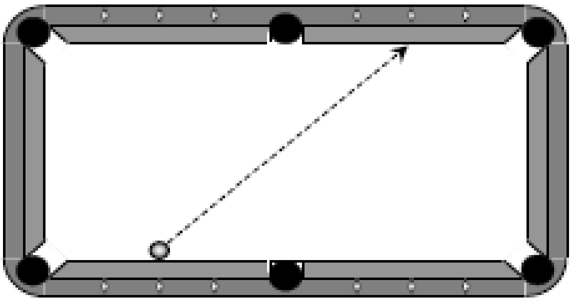
6-53.



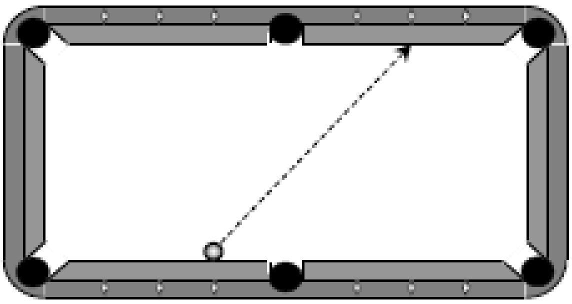
6-54 part (a).



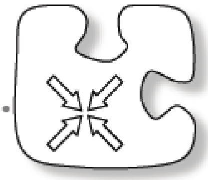

6-54 part (b).



6-54 part (c).



# 6.2.2 How can I use it? What is the connection?



## Investigating a Triangle

The activities in this section review several big topics you have studied so far. Work with your team to decide which combination of tools you will need for each problem. As you work together, think about which skills and tools you are comfortable using and which ones you need more practice with.

As you work on this activity, keep in mind the following questions:

What mathematical concepts did you use to solve this problem?

What do you still want to know more about?

What connections did you find?

6-61. GETTING TO KNOW YOUR TRIANGLE

a. If you were asked to give every possible measurement of a triangle, what measurements could you include?

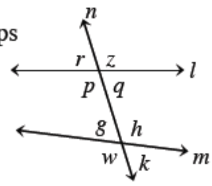
b. Consider a triangle on a coordinate grid with the following vertices:

$A(2, 3), B(32, 15), C(12, 27)$

**Your Task:** On graph paper, graph $\Delta ABC$ and find all of its measurements. Be sure to find every measurement you listed in part (a) of this problem and show all of your calculations. With your team, be prepared to present your method for finding the area.

## Review & Preview

6-62. Examine the diagram at right. Based on the information below, what statement can you make about the relationships between the lines? Be sure to justify each conclusion. Remember: each part below is a separate problem.



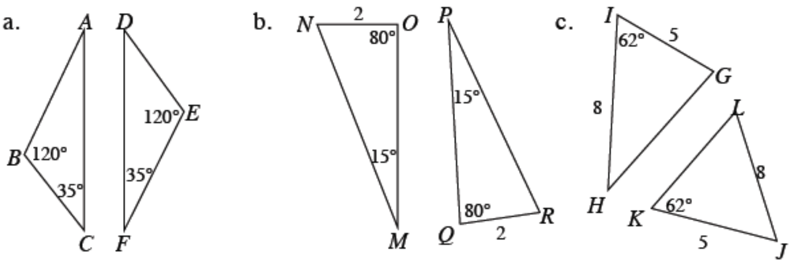

a. $p = h$

b. $w = k$

c. $r = q$

d. $z + k = 160°$

6-63. Determine if each pair of triangles below are congruent, similar but not congruent, or if they are neither. If they are congruent, organize your reasoning into a flowchart. Remember that the triangles below may not be drawn to scale.



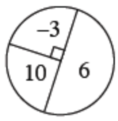

6-64. Write a converse for each conditional statement below. Then, assuming the original statement is true, decide if the converse must be true or not.

a. If the cat knocks over the lamp, then it will run away frightened.

b. If a 6-sided die is rolled, then the chances of getting a 3 are $\frac{1}{6}$.

c. If a triangle has a 90° angle, then it is a right triangle.

6-65. For the spinner at right, find the expected value of one spin.




6-66. **Multiple Choice:** Which equation below correctly represents the relationship of the sides given in the diagram at right?


a. $3x - 2 + 2x + 17 = 360°$

b. $3x - 2 + 2x + 17 = 180°$

c. $3x - 2 + 2x + 17 = 90°$

d. $3x - 2 = 2x + 17$

6-67. On graph paper, plot and connect the points to form quadrilateral $WXYZ$ if its vertices are $W(3, 7)$, $X(3, 4)$, $Y(9, 1)$, and $Z(5, 6)$.

a. What is the shape of quadrilateral $WXYZ$? Justify your conclusion.

b. Find the perimeter of quadrilateral $WXYZ$.

c. If quadrilateral $WXYZ$ is reflected using the transformation function $(x \to -x, y \to y)$ to form quadrilateral $W'X'Y'Z'$, then where is $Y'$?

d. Rotate quadrilateral $WXYZ$ about the origin 90° clockwise (↻) to form quadrilateral $W''X''Y''Z''$. What is the slope of $\overline{W''Z''}$?

# 6.2.3 How can I use it? What is the connection?

## Creating a Mathematical Model

The activities in this section review several big topics you have studied so far. Work with your team to decide which combination of tools you will need for each problem. As you work together, think about which skills and tools you are comfortable using and which ones you need more practice with.

As you work on this activity, keep in mind the following questions:

What mathematical concepts did you use to solve this problem?

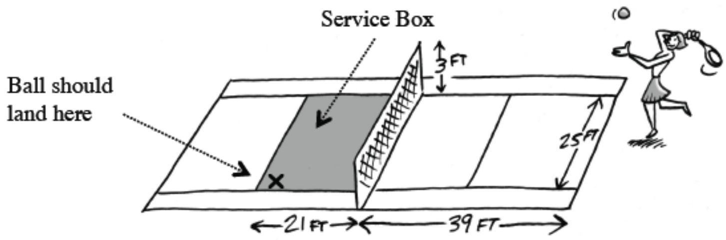
What do you still want to know more about?

What connections did you find?

6-68. AT YOUR SERVICE

Carina, a tennis player, wants to make her serve a truly powerful part of her game. She wants to hit the ball so hard that it appears to travel in a straight path. Unfortunately, the ball always lands beyond the service box. After a few practice serves, she realizes that the height at which you hit the ball determines where the ball lands. Before she gets tired from serving, she sits down to figure out how high the ball must be when she hits it so that her serve is legal.

In the game of tennis, every point begins with one player serving the ball. For a serve to be legal, the player must stand outside the court and hit the ball so that it crosses over the net and lands within the service box (shown shaded below). It can be difficult to make the ball land in the service box because the ball is often hit too low and touches the net or is hit too high and lands beyond the service box.




A tennis court is 78 feet long with the net stretched across the center. The distance from the net to the back of the service box is 21 feet, and the net is 3 feet tall.

**Your Task:** Assuming Carina can hit the ball so hard that its path is linear, from what height must she hit the ball to have the serve just clear the net and land in the service box? Decide whether or not it is reasonable for Carina to reach this height if she is 5'7" tall. Also, at what angle does the ball hit the ground?

Your solution should include:

- A labeled diagram that shows a birds' eye view of the path of the ball.
- A labeled diagram that shows the side view of Carina, the ideal height of the tennis racket, the ideal path of the tennis ball, and the measurements that are needed from the birds' eye view diagram.

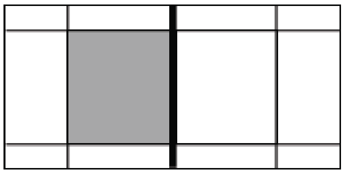
### *Discussion Points*

What would you see if you were a bird looking down on the court as Carina served?

Which distances do you know and which do you need to find?

## Further Guidance

6-69. To help solve this problem, copy the diagram at right, which shows the tennis court from above (called a "birds-eye" view). On your diagram, locate the position of Carina and the spot where the ball should land. Be sure to include the path of the ball.



a. Label all distances you already know. Do you see any triangles?

b. What distance(s) can you find? What geometric tool(s) can you use?

6-70. Next visualize the situation from the side and draw the path of the ball.

a. As you draw this diagram, be sure to include Carina, the net, and the spot where the ball should land.

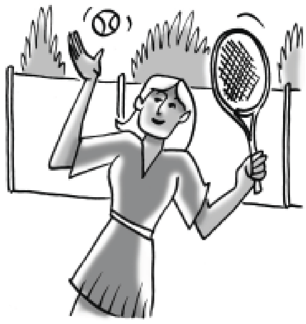
b. Are there any similar triangles? Be careful to include any measurements that might help you determine the height of the serve.

c. What triangle tool can you use to find the angle of depression of the path of the ball? Find the acute angle the path of the ball makes with the ground.

6-71. How high must the ball be hit so that it just clears the net and lands in the service box? If Carina is 5'7" tall, is it possible for her to accomplish this serve? Assume that a tennis racquet is 2 feet long.

*Further Guidance section ends here.*

6-72. You've been hired as a consultant for the National Tennis Association. They are considering raising the net to make the serve even more challenging. They want players to have to jump to make a successful serve (assuming that the powerful serves will be hit so hard that the ball will travel in a straight line). Determine how high the net should be so that the players must strike the ball from at least 10 feet.

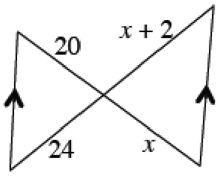


Review & Preview

6-73. Examine the triangles in the diagram at right.

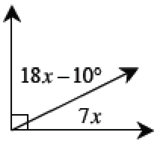
a. Are the triangles similar? If you decide that they are, then justify your conclusion using a flowchart.

b. Solve for $x$. Show all work.

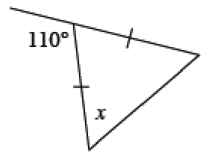



6-74. For each diagram below, use geometric relationships to solve for the given variable(s). Check your answer.

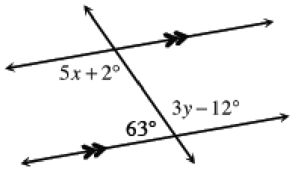
a.



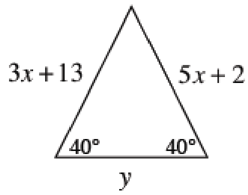

b.




c.

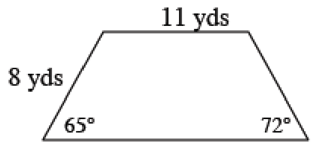



d.




6-75. Refrigerators that are produced on an assembly line sometimes contain defects. The probability a refrigerator has a paint blemish is 4%. The probability that it has a dent is $\frac{1}{2}$%. The probability it has both a paint blemish and a dent is also $\frac{1}{2}$%. What is the probability a refrigerator has a paint blemish or a dent? What can you conclude about defects on these refrigerators?

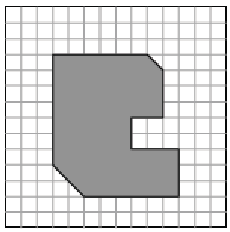
6-76. Find the area and perimeter of the trapezoid at right.




6-77. A map of an island is shown at right. Each unit of length on the grid represents 32 feet.

a. Find the actual dimensions of the island (the overall width and length).

b. Find the area of the shape at right and the actual area of the island.



6-78. **Multiple Choice:** The length of a rectangle is three units shorter than twice its width. Which expression below could represent the area of the rectangle?

a. $2x^2 - 3$   b. $2x^2 - 6x$   c. $2x^2 - 3x$   d. $(2x - 3)^2$

# 6.2.4 How can I use it? What's the connection?

## Analyzing a Game

The activities in this section review several big topics you have studied so far. Work with your team to decide which combination of tools you will need for each problem. As you work together, think about which skills and tools you are comfortable using and which ones you need more practice with.

As you work on this activity, keep in mind the following questions:

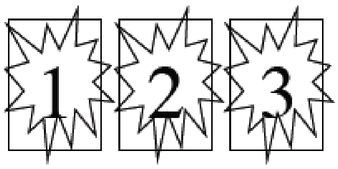
What mathematical concepts did you use to solve this problem?

What do you still want to know more about?

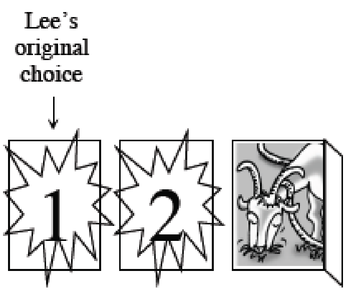
What connections did you find?

6-79. THE MONTY HALL PROBLEM

Wow! Your best friend, Lee, has been selected as a contestant in the popular "Pick-A-Door" game show. The game show host, Monty, has shown Lee three doors and, because he knows what is behind each door, has assured her that behind one of the doors lies a new car! However, behind each of the other two doors is a goat.




"*Which door do you pick?*" Monty asks.

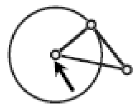



"*I pick Door #1,*" Lee replies confidently.

"*Okay. Now, before I show you what is behind Door #1, let me show you what is behind Door #3. It is a goat! Now, would you like to change your mind and choose Door #2 instead?*" Monty asks.

What should Lee do? Should she stay with Door #1 or should she switch to Door #2? Does she have a better chance of winning if she switches, or does it not matter? Discuss this situation with the class and make sure you provide reasons for your statements.

6-80. Now test your prediction from problem 6-79 by simulating this game with a partner using either a computer or a programmable calculator. If no technology is available, collect data by playing the game with a partner as described below.



Choose one person to be the contestant and one person to be the game show host. As you play, carefully record information about whether the contestant switches doors and whether the contestant wins. Play as many times as you can in the time allotted, but be sure to record at least 10 results from switching and 10 results from not switching. Be ready to report your findings with the class.

If playing this game without technology, the host should:

- Secretly choose the winning door. Make sure that the contestant has no way of knowing which door has been selected.
- Ask the contestant to choose a door.
- "Open" one of the remaining two doors that does not have the winning prize.
- Ask the contestant if he or she wants to change his or her door.
- Show if the contestant has won the car and record the results.

6-81. Examine the data the class collected in problem 6-80.

a. What does this data tell you? What should Lee do in problem 6-79 to maximize her chance of winning?

b. Your teammate, Kaye, is confused. *"Why does it matter? At the end, there are only two doors left. Isn't there a 50-50 chance that I will select the winning door?"* Explain to Kaye why switching is better.

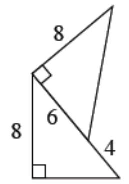
c. Gerald asks, *"What if there are 4 doors? If Monty now reveals two doors with a goat, is it still better to switch?"* What do you think? Analyze this problem and answer Gerald's question.

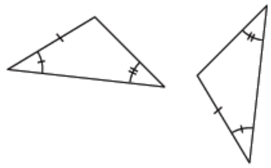
6-82. LEARNING LOG

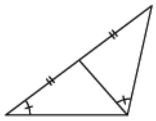
One of the topics you studied during Chapters 1 through 6 was probability. You investigated what made a game fair and how to predict if you would win or lose. Reflect on today's activity and write a Learning Log entry about the mathematics you used today to analyze the "Monty Hall" game. Title this entry "Game Analysis" and label it with today's date.

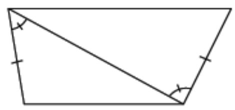
## Review & Preview

6-83. Determine whether or not the triangles in each pair below are congruent. Justify your conclusion with a triangle congruency condition. Then choose one pair of congruent triangles and show your reasoning with a flowchart.

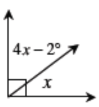
a.


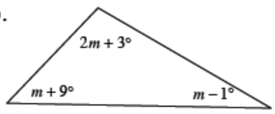

b.


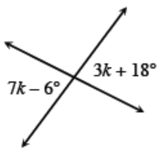
c.


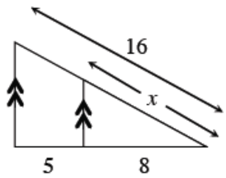
d.


6-84. Examine the geometric relationships in each of the diagrams below. For each one, write and solve an equation to find the value of the variable. Name all geometric relationships or theorems that you use.

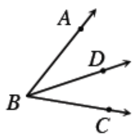
a.



b.



c.



d.



6-85. In the diagram at right, $\overrightarrow{BD}$ **bisects** $\angle ABC$. This means that $\overrightarrow{BD}$ divides the angle into two equal parts. If $m\angle ABD = 3x + 24°$ and if $m\angle CBD = 5x + 2°$, solve for $x$. Then find $m\angle ABC$.




6-86. Write a converse for each conditional statement below. Then, assuming the original statement is true, decide if the converse must be true or not.

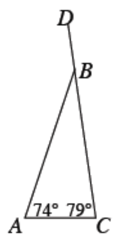
a. If the base angles of a triangle are congruent, the it is isosceles.

b. If a figure is a triangle, then the sum of the angles in the figure is 180°.

c. If I clean my room, then my mom will be happy.

6-87. A particular spinner only has two regions: green and purple. If the spinner is randomly spun twice, the probability of it landing on green twice is 16%. What is the probability of the spinner landing on purple twice?

6-88. **Multiple Choice:** The measure of $\angle ABD$ at right is:




a. 27° b. 161°

c. 118° d. 153°

e. None of these

Lesson 6.2.4 Resource Page

## The Monty Hall Problem

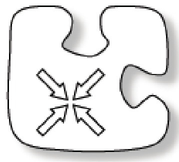
6-80. **Directions:** Simulate the "Pick-A-Door" game with a partner. Carefully record your data below. Record at least 10 results that occurred when the contestant switched his or her door and at least 10 results that occurred when the contestant did not switch. Space is provided here for extra data, however. Gather as much as you can.

| Contestant Switched Doors | | | Contestant Did Not Switch Doors | |
|---|---|---|---|---|
| Game Number | Win or Lose? | | Game Number | Win or Lose? |
| 1 | | | 1 | |
| 2 | | | 2 | |
| 3 | | | 3 | |
| 4 | | | 4 | |
| 5 | | | 5 | |
| 6 | | | 6 | |
| 7 | | | 7 | |
| 8 | | | 8 | |
| 9 | | | 9 | |
| 10 | | | 10 | |
| 11 | | | 11 | |
| 12 | | | 12 | |
| 13 | | | 13 | |
| 14 | | | 14 | |
| 15 | | | 15 | |
| 16 | | | 16 | |
| 17 | | | 17 | |
| 18 | | | 18 | |
| 19 | | | 19 | |
| 20 | | | 20 | |

Percentage wins: _______ Percentage wins: _________

Percentage losses: _______ Percentage losses: _________

# 6.2.5 How can I use it? What is the connection?



Using Transformations and Symmetry to Design Snowflakes

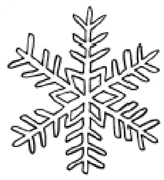
The activities in this section review several big topics you have studied so far. Work with your team to decide which combination of tools you will need for each problem. As you work together, think about which skills and tools you are comfortable using and which ones you need more practice with. As you work on this activity, keep in mind the following questions:

What mathematical concepts did you use to solve this problem?

What do you still want to know more about?

What connections did you find?

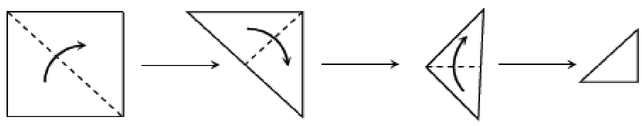
6-89. THE PAPER SNOWFLAKE



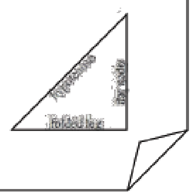
You have volunteered to help the decorating committee make paper snowflakes for the upcoming winter school dance. A paper snowflake is made by folding and cutting a square piece of paper in such a way that when the paper is unfolded, the result is a beautiful design with symmetric patterns similar to those of a real snowflake.

Looking through your drawer of craft projects, you find the directions for how to fold the paper snowflake (see below). However you cannot find any directions for how to cut the folded paper to make specific designs in the final snowflake.

DESIGNING A PAPER SNOWFLAKE: Cut out a 20 unit by 20 unit square from a piece of graph paper. Making sure that the gridlines are visible on the outside of the shape, fold the square three times as shown in the diagram below. Your final shape should be a 45°- 45°- 90° triangle that has folds along the hypotenuse.



Once you have folded your paper, correctly label the sides of your folded triangle "hypotenuse," "folded leg," and "open leg" (the leg comprised of the edges of your original square). Orient your triangle as shown in the diagram at right.



**Your Task:** You want to be fully prepared to help the decorating committee for the school dance. Explore and be ready to explain the relationships between the shapes that are cut out and the design that appears after unfolding the paper. For each possible location of a cutout, use what you know about symmetry and transformations to describe the shapes that result when you unfold the paper.

### *Discussion Points*

What are your goals for this task?

What tools would be useful to complete this task?

Visualize the result when a shape is cut along the hypotenuse. What qualities will the result have?

## *Further Guidance*

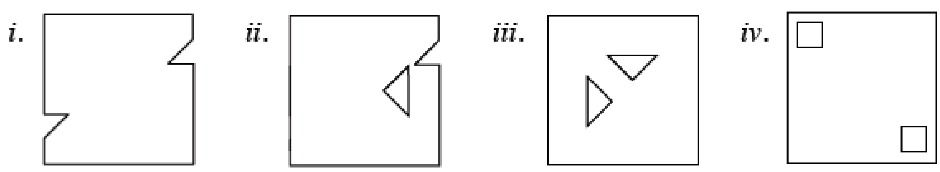
6-90. Since there are so many cuts that are possible, it is helpful to begin by considering a simple cutout along one side of the triangle.

a. Sketch triangles onto your folded paper according to these directions, making sure that none of the shapes share a side or overlap:

- A 45°-45°-90° triangle with its hypotenuse along the paper triangle's hypotenuse. Each leg should be 3 units long.
- A 45°-45°-90° triangle with a 3-unit leg along the paper triangle's folded leg.
- A 45°-45°-90° triangle with a 3-unit leg along the paper triangle's open leg.

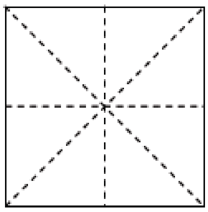
b. Cut out the shapes you sketched from part (a). Then unfold the paper to view your snowflake. Identify the three different kinds of shapes that resulted from the triangles you cut. Describe them with as much specific vocabulary as you can.

c. Find each pair of shapes shown below on your own snowflake. For each pair, describe *two different ways* to transform one shape into the other.

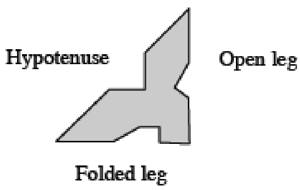



*Further Guidance section ends here.*

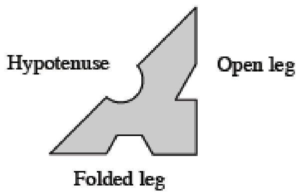
6-91. Some possible folded triangles with cutouts are shown below. What would each of these snowflakes look like when unfolded? Draw the resulting designs on the squares provided on the Lesson 6.2.5 Resource Page (or draw the result on your paper.)



a.




b.



6-92. What shape must you cut out along your folded triangle hypotenuse to get the following shapes on your snowflake when your paper is unfolded? Sketch and label diagrams to show that you can accomplish each result.

a. Rectangle with a length that is twice its width

b. Kite

c. Rhombus

d. Pentagon

6-93. EXTENSION

Get a final piece of grid paper from your teacher. Make one more snowflake that includes at least four of the following shapes. Make sure you sketch all of your planned cuts before cutting the paper with scissors. After you cut out your shapes and unfold your snowflake, answer questions (a) through (d) below.

- Draw a shape at the vertex where the hypotenuse and folded leg intersect that results in a shape with 8 lines of symmetry. Note that you will actually have to cut the vertex off to do this.
- Many letters, such as E, H, and I have reflection symmetry. Pick a letter (maybe one of your initials) and draw a shape along the folded triangle's hypotenuse or folded leg that results in a box letter when cut out and the snowflake is unfolded.
- Draw a shape along the folded triangle's hypotenuse or folded leg that results in a regular hexagon when cut out and the snowflake is unfolded.
- Draw a shape along the folded triangle's hypotenuse or folded leg that results in a star when cut out and the snowflake is unfolded.
- Draw curved shapes along the open leg so that there is at least one line of symmetry that is perpendicular to the open leg.

a. What has to be true about the shape you cut out in order for your unfolded shape to have 8 lines of symmetry?

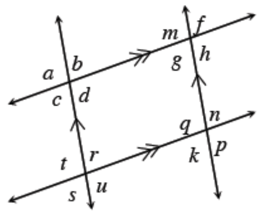
b. There are two different shapes you could have drawn that would result in a regular hexagon. Sketch both shapes. Why do both shapes work?

c. Some shapes are impossible to create by cutting along the hypotenuse of the folded paper triangle. Sketch several different examples of these shapes. What is the common characteristic of these impossible shapes?

d. How is cutting along the open leg different from cutting along the folded leg or hypotenuse? Try to describe the difference in terms of symmetry. Use a diagram to help make your description clear.

## Review & Preview

6-94. Examine the angles formed by parallel lines at right.




a. If $r = 5x + 3°$ and $k = 4x + 9°$, solve for $x$. Justify your answer.

b. If $c = 114°$, what is $q$? Justify your answer.

c. If $g = 88°$, then what is $q$? Justify your answer.

6-95. Write the equation of each line described below in slope-intercept form ($y = mx + b$).

a. $m = \frac{6}{5}$ and $b = -3$

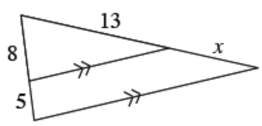
b. $m = -\frac{1}{4}$ and $b = 4.5$

c. $m = \frac{1}{3}$ and the line passes through the origin (0, 0)

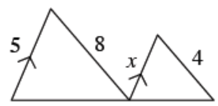
d. $m = 0$ and $b = 2$

6-96. For each part below, decide if the triangles are similar. If they are similar, use their similarity to solve for $x$. If they are not similar, explain why not.

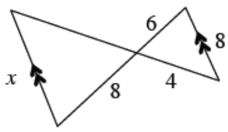
a.




b.

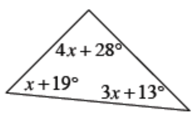



c.

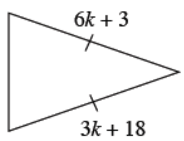



6-97. For each shape below, use the geometric relationships to solve for the given variable(s). Show all work. Name the geometric relationships you used.

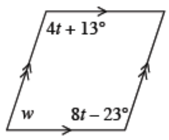
a.



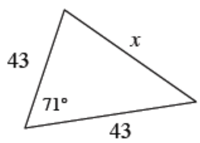

b.




c.




d.




6-98. Jinning is going to flip a coin. If the result is "heads," he wins \$4. If the result is "tails," he loses \$7.

a. What is his expected value per flip?

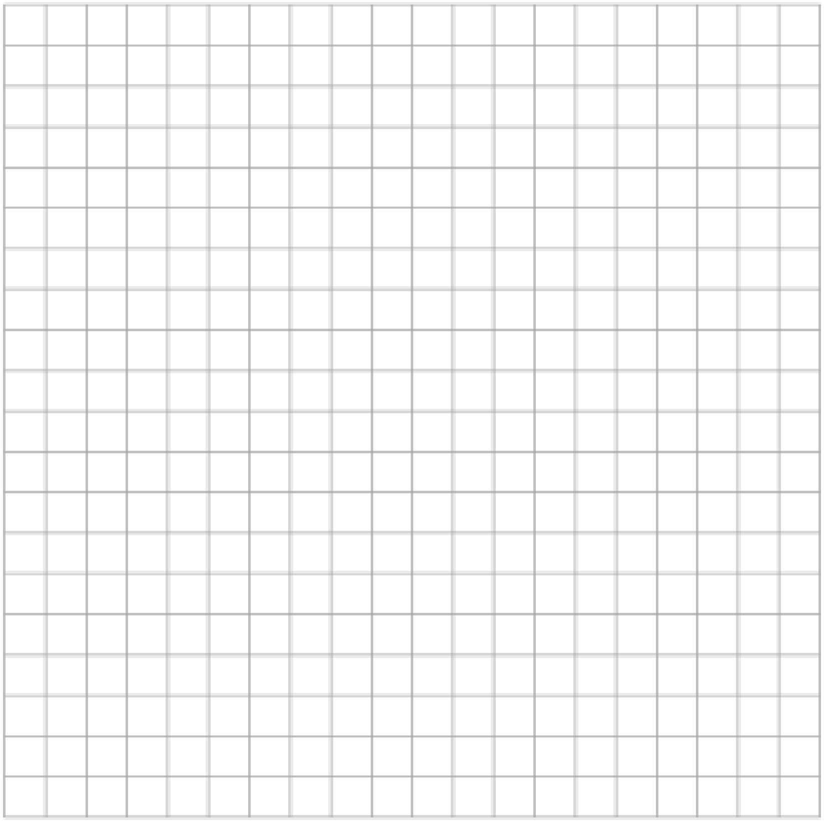
b. If he flips the coin 8 times, how much should he win or lose?

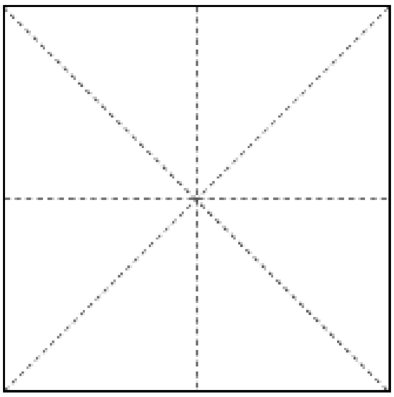
6-99. **Multiple Choice:** What is the distance between the points (–2, –5) and (6, 3)?

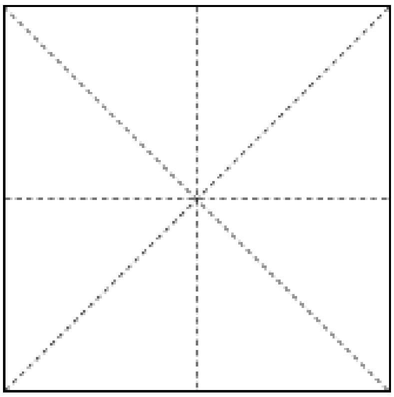
a. 8 b. $8\sqrt{2}$ c. 16 d. 64

Lesson 6.2.5 Resource Page

## Designing a Snowflake







## Chapter 6 Closure What have I learned?

Reflection and Synthesis

The activities below offer you a chance to reflect about what you have learned during this chapter. As you work, look for concepts that you feel very comfortable with, ideas that you would like to learn more about, and topics you need more help with. Look for connections between ideas as well as connections with material you learned previously.

① TEAM BRAINSTORM

What have you studied in this chapter? What ideas were important in what you learned? With your team, brainstorm a list. Be as detailed as you can. To help get you started, lists of Learning Log entries and Math Notes boxes are below.

What topics, ideas, and words that you learned *before* this chapter are connected to the new ideas in this chapter? Again, be as detailed as you can.

How long can you make your list? Challenge yourselves. Be prepared to share your team's ideas with the class.

**Learning Log Entries**

- Lesson 6.1.5 – Converses
- Lesson 6.2.4 – Game Analysis

**Math Notes**

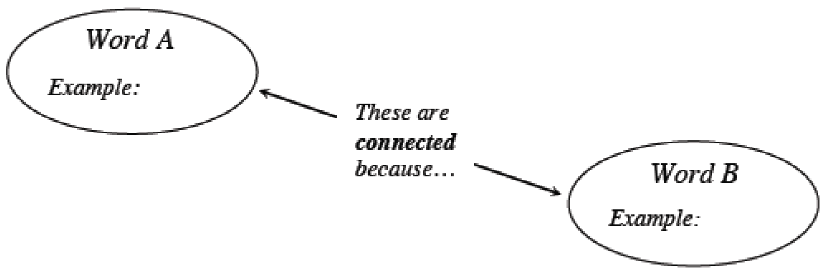
- Lesson 6.1.1 – Congruent Shapes
- Lesson 6.1.4 – Triangle Congruence Conditions
- Lesson 6.1.5 – Converses

② MAKING CONNECTIONS

Below is a list of the vocabulary used in this chapter. Make sure that you are familiar with all of these words and know what they mean. Refer to the glossary or index for any words that you do not yet understand.

| | | |
|---|---|---|
| AAS ≅ | arrow diagrams | ASA ≅ |
| bisect | conditional statement | congruent |
| conjecture | converse | corresponding parts |
| flowchart | HL ≅ | SAS ≅ |
| similar | SSS ≅ | |
| triangle congruence conditions | | |

Make a concept map showing all of the connections you can find among the key words and ideas listed above. To show a connection between two words, draw a line between them and explain the connection, as shown in the model below. A word can be connected to any other word as long as you can justify the connection. For each key word or idea, provide an example or sketch that shows the idea.




Your teacher may provide you with vocabulary cards to help you get started. If you use the cards to plan your concept map, be sure either to re-draw your concept map on your paper or to glue the vocabulary cards to a poster with all of the connections explained for others to see and understand.

While you are making your map, your team may think of related words or ideas that are not listed here. Be sure to include these ideas on your concept map.

③ PORTFOLIO: EVIDENCE OF MATHEMATICAL PROFICIENCY

This portfolio entry gives you the opportunity to showcase your understanding of the geometric concepts studied so far in the course.

Your teacher will indicate which of the problems from Lessons 6.2.1 through 6.2.5 you are to use as your portfolio entry. Revise your initial work if you need to so that you are showing off the very best work you know how to do. Record your work neatly and justify each step as evidence of the mathematics you are now able to do. Include as much detail as you can.

④ WHAT HAVE I LEARNED?


Most of the problems in this section represent typical problems found in this chapter. They serve as a gauge for you. You can use them to determine which types of problems you can do well and which types of problems require further study and practice. Even if your teacher does not assign this section, it is a good idea to try these problems and find out for yourself what you know and what you still need to work on.

Solve each problem as completely as you can. The table at the end of the closure section has answers to these problems. It also tells you where you can find additional help and practice with problems like these.

CL 6-100. Write the converse of each statement and then determine whether or not the converse is true.

a. If two lines are parallel, then pairs of corresponding angles are equal.

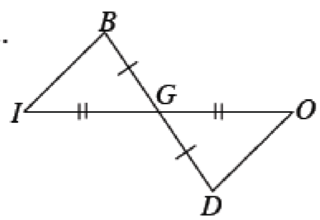
b. In $\Delta ABC$, if the sum of $m\angle A$ and $m\angle B$ is $110°$, then $m\angle C = 70°$.

c. If alternate interior angles $k$ and $s$ are not equal, then the two lines cut by the transversal are not parallel.

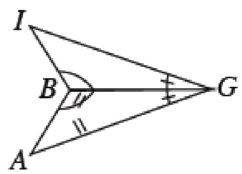
d. If Johan throws coins in the fountain, then he loses his money.

CL 6-101. Determine whether or not the two triangles in each part below are congruent. If they are congruent, show your reasoning in a flowchart. If the triangles are not congruent or you cannot determine that they are, justify your conclusion.

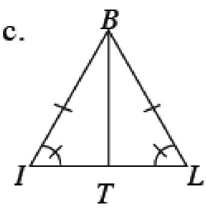
a.



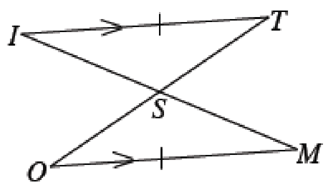

b.



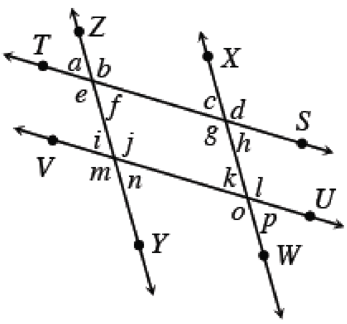

c.




d.



CL 6-102. For each part, determine which lines, if any, are parallel. Be sure to justify your decisions.




a. $\angle e \cong \angle m$

b. $\angle c \cong \angle o$

c. $\angle d \cong \angle o$

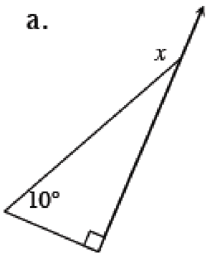
d. $\angle a \cong \angle m \cong \angle o$

e. $\angle a \cong \angle k$

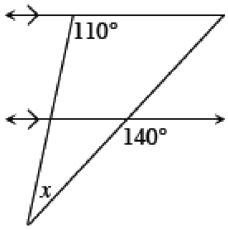
f. $\angle k \cong \angle c \cong \angle f$

CL 6-103. For each diagram, solve for the variable. Be sure to include the names of any relationships you used to get your solution.

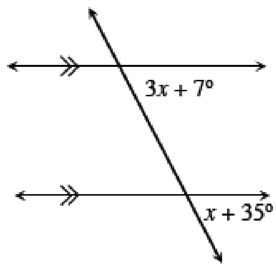
a.



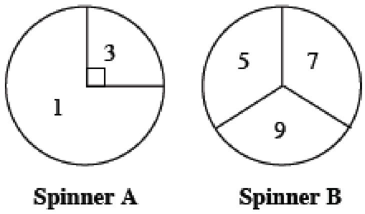

b.

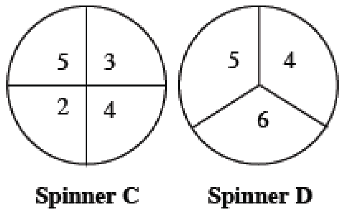



c.

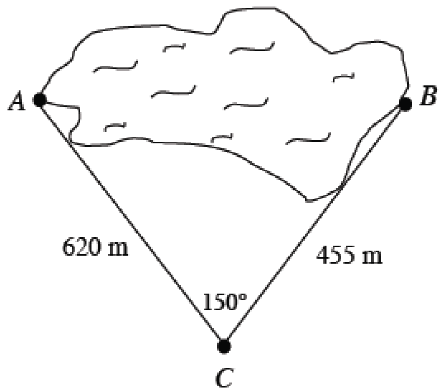


CL 6-104. Cynthia is planning a party. For entertainment, she has designed a game that involves spinning two spinners. If the sum of the numbers on the spinners is 10 or greater, the guests can choose a prize from a basket of candy bars. If the sum is less than 10, then the guest will be thrown in the pool. She has two possible pairs of spinners, shown below. For each pair of spinners, determine the probability of getting tossed in the pool. Assume that Spinners B, C, and D are equally subdivided.

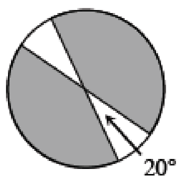
a. 


b. 

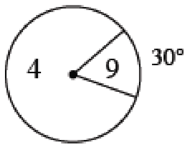

CL 6-105. Yee Ping thought about swimming across Redleaf Lake. She knows that she can swim about 1000 meters. She decided that she would feel more confident if she knew how far she would have to swim. To determine the length of the lake, she put posts at both ends of the lake (points $A$ and $B$) and a third post on one side of the lake (point $C$). The distances between the posts are shown in the diagram at right. She measured the angle between the two posts and found that it was 150°. Use this information to determine the length of the lake. Do you think that Yee Ping will be able to swim between points $A$ and $B$?

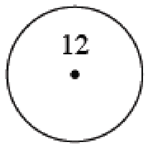



CL 6-106. Your teacher has constructed a spinner like the one at right. He has informed you that the class gets one spin. If the spinner lands on the shaded region, you will have a quiz tomorrow. What is the probability that you will have a quiz tomorrow? Explain how you know.



CL 6-107. For each spinner below, find the expected value of one spin.

a. 


b. 


CL 6-108. Margarite has 9 pieces of copper pipe with which she plans to make 3 triangular frames. She has organized them into groups of three based on their coloring. The lengths of the pipes in each group are listed below.

*i.* 23, 21, 4 *ii.* 2, 11, 10 *iii.* 31, 34, 3

a. Which groups, if any, will she actually be able to use to make a triangular frame if she is unable to cut any of the pipes? How do you know?

b. If possible, arrange the 9 pieces she has so that she can make 3 triangular frames. If so, how? If not, why not?

CL 6-109. At a story-telling class, Barbara took notes on the following story. However, all the parts of the story got mixed up. Help her make sense of the story by organizing the following details in a flowchart.

a. Maggie was happy she could play on the same team as Julie and Cheryl.

b. Julie was hoping to make the A team again this year as she grabbed her basketball and got on a bus in Bellingham.

c. Cheryl, having been named most valuable player in Port Townsend, started the drive to the statewide basketball camp.

d. Because of her skill in the first game, Maggie moved up to the A team.

e. At camp, Julie and Cheryl were placed on the A team.

f. Julie, Maggie and Cheryl met at a statewide basketball camp. Shortly after they met, they were placed on teams.

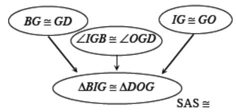
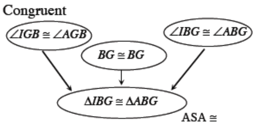
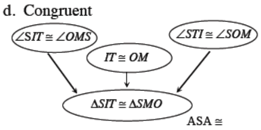
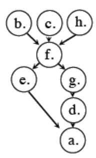
g. This year was Maggie's first year at camp, and was placed on the B team.

h. On the train to camp, Maggie thought about how surprising it was that her basketball coach chose her to attend camp.

CL 6-110. Check your answers using the table at the end of this section. Which problems do you feel confident about? Which problems were hard? Have you worked on problems like these in math classes you have taken before? Use the table to make a list of topics you need help on and a list of topics you need to practice more.

## Answers and Support for Closure Activity #4
### *What Have I Learned?*

MN = Math Notes, LL = Learning Log

| Problem | Solutions | Need Help? | More Practice |
|---|---|---|---|
| CL 6-100. | a. True; If a pair of corresponding angles formed by two lines and a transversal are equal, then the two lines are parallel.<br>b. True; In $\Delta ABC$, if angle $C$ is 70°, then the sum of angles $A$ and $B$ is 110°.<br>c. True; If lines cut by a transversal that form alternate interior angles are not parallel, then those angles are not equal.<br>d. False; If Johan loses money, then he has thrown coins in the fountain. | Lesson 6.1.5<br>MN: 6.1.5<br>LL: 6.1.5 | Problems 6-48, 6-64, and 6-86 |
| CL 6-101. | a. Congruent<br><br>b. Congruent<br><br>c. Not enough information.<br>d. Congruent<br> | Lessons 6.1.1, 6.1.2, 6.1.3, and 6.1.4<br>MN: 3.2.2, 3.2.4, 3.2.5, 6.1.1, and 6.1.4<br>LL: 3.2.2 | Problems 6-26, 6-47, 6-58, 6-63, and 6-83 |
| CL 6-102. | a. $\overline{ST} \parallel \overline{UV}$ ; corres. angles are equal.<br>b. Not enough information.<br>c. $\overline{ST} \parallel \overline{UV}$ ; alt. interior angles are equal.<br>d. $\overline{ZY} \parallel \overline{XW}$ ; corres. angles are equal.<br>e. Not enough information.<br>f. $\overline{ZY} \parallel \overline{XW}$; $\overline{ST} \parallel \overline{UV}$ ; corres. angles and alternate interior angles are equal. | Section 2.1<br>MN: 2.1.1, 2.1.4, and 2.2.1<br>LL: 2.1.1<br>Angle Relationship Toolkit | Problems CL 2-122, CL 4-125, 6-14, 6-34, 6-40, 6-46, 6-62, and 6-94 |
| CL 6-103. | Reasons may vary. One possible set of reasons is listed for each:<br>a. $x = 100°$ Triangle Angle Sum Theorem and supplementary angles<br>b. $x = 30°$ Triangle Angle Sum Theorem, corresponding and supplementary angles<br>c. $x = 14°$ corresponding angles | Section 2.1<br>MN: 2.1.1, 2.1.4, and 2.2.1<br>LL: 2.1.1<br>Angle Relationship Toolkit | Problems CL 2-122, CL 4-125, 6-14, 6-34, 6-40, 6-46, 6-62, 6-88, and 6-94 |
| CL 6-104. | a. $\frac{3}{12}+\frac{3}{12}+\frac{1}{12}=\frac{7}{12}$<br>b. $\frac{9}{12}$ | Section 4.2<br>MN: 1.2.1, 4.1.5, 4.2.3, and 4.2.4<br>LL: 4.2.3 | Problems CL 4-127, CL 5-145, 6-18, 6-28, 6-39, 6-75, and 6-87 |
| CL 6-105. | The lake is about 1039 meters wide. This means Yee might have a problem. | Lesson 5.3.3<br>MN: 5.3.3 | Problems CL 5-146, and 6-15(c) |
| CL 6-106. | $360° - 40° = 320°$, so $\frac{320}{360} = \frac{8}{9} \approx 89\%$ | Section 4.2<br>MN: 1.2.1, 4.1.5, 4.2.3, and 4.2.4<br>LL: 4.2.3 | Problems CL 4-127 and 6-51 |
| CL 6-107. | a. $\frac{53}{12}$ , or $\approx 4.42$<br>b. 12 | Lesson 4.2.5<br>MN: 5.2.2 | Problems CL 5-141, 6-65, and 6-98 |
| CL 6-108. | a. Sets *i* and *ii* work.<br>b. It is possible.<br>*i.* 4, 31, 34<br>*ii.* 3, 21, 23<br>*iii.* 2, 10, 11 | Lesson 2.3.1<br>LL: 2.3.1 | Problems CL 3-117, CL 5-144, and 6-19 |
| CL 6-109. | One possible solution:<br> | Lesson 3.2.2<br>MN: 3.2.4<br>LL: 3.2.2 | Problems CL 3-121, 4-72, 5-34, 5-56, and 6-8 |

| | |
|---|---|
| AAS ≅ | Arrow diagrams |
| ASA ≅ | Bisect |
| Conditional statement | Congruent |
| Conjecture | Converse |
| Corresponding parts | Flowchart |

| | |
|---|---|
| HL $\cong$ | SAS $\cong$ |
| Similar | SSS $\cong$ |
| Triangle congruence conditions | |
| | |