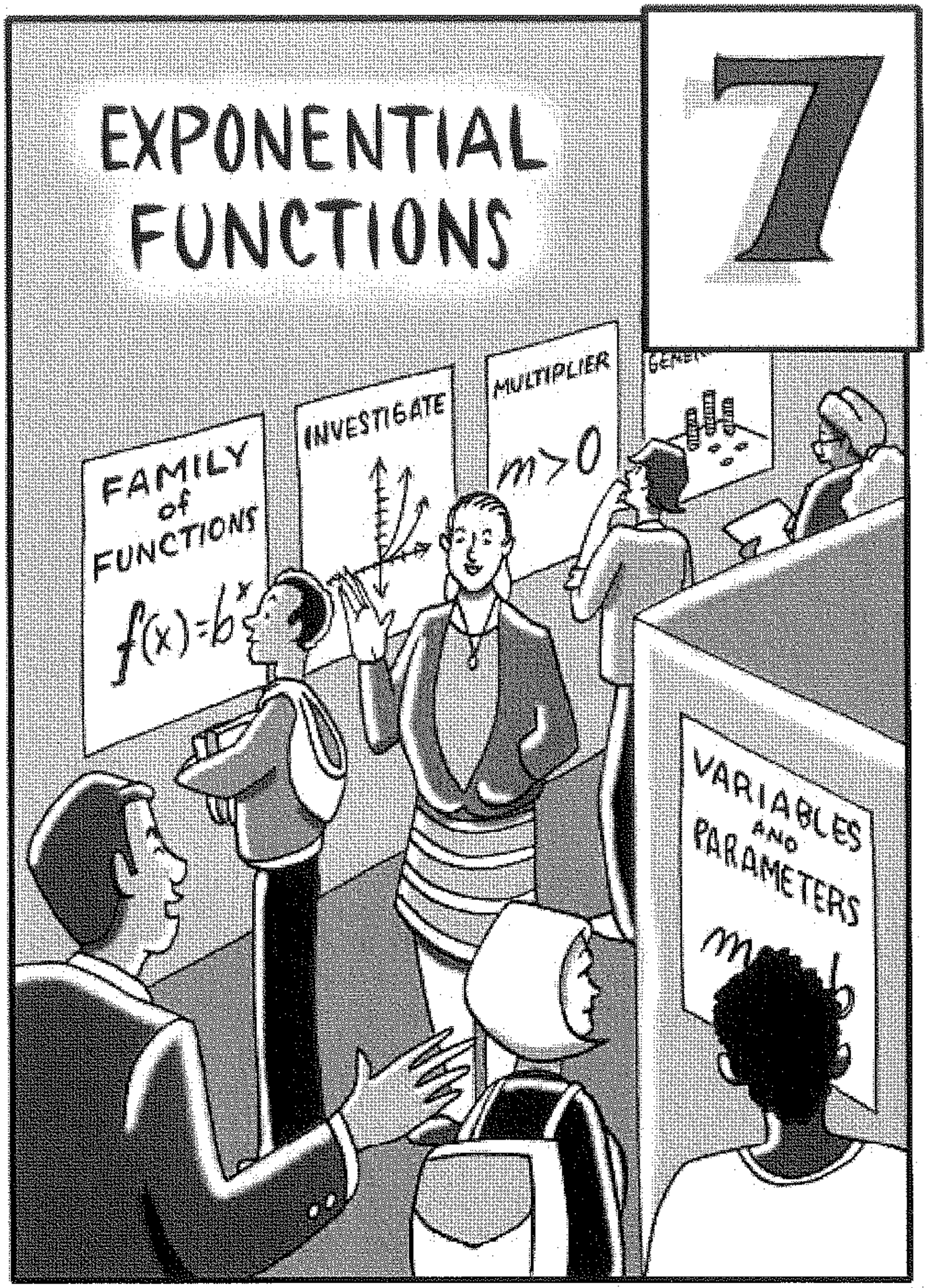
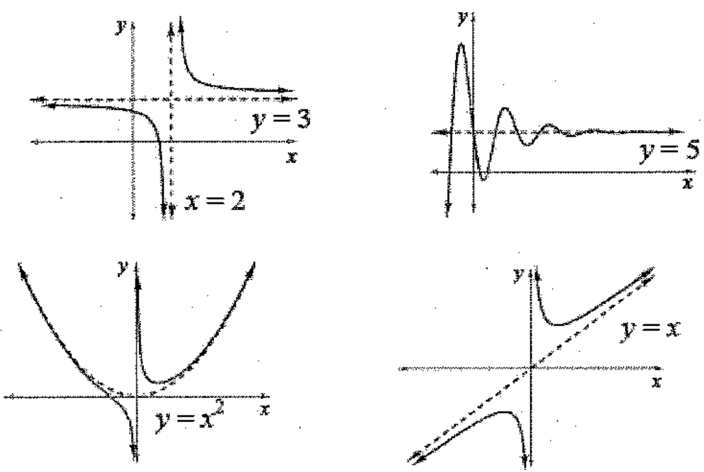

EXPONENTIAL FUNCTIONS
7
FAMILY of FUNCTIONS
f(x)=b^x
INVESTIGATE
MULTIPLIER
m>0
VARIABLES AND PARAMETERS

MATH NOTES

METHODS AND MEANINGS

Graphs with Asymptotes

A mathematically clear and complete definition of an asymptote requires some ideas from calculus, but some examples of **graphs with asymptotes** might help you recognize them when they occur. In the following examples, the dotted lines are the asymptotes, and their equations are given. In the two lower graphs, the y-axis, $x = 0$, is also an asymptote.



As you can see in the examples above, asymptotes can be diagonal lines or even curves. However, in this course, asymptotes will almost always be horizontal or vertical lines. The graph of a function has a **horizontal asymptote** if, as you trace along the graph out to the left or right (that is, as you choose x-coordinates farther and farther away from zero, either toward infinity or toward negative infinity), the distance between the graph of the function and the asymptote gets closer to zero.

A graph has a **vertical asymptote** if, as you choose x-coordinates closer and closer to a certain value, from either the left or right (or both), the y-coordinate gets farther away from zero, either toward infinity or toward negative infinity.

Review & Preview

7-7. A grocery store is offering a sale on bread and soup. Khalil buys four cans of soup and three loaves of bread for $11.67. Ronda buys eight cans of soup and one loaf of bread for $12.89.

a. Write equations for both Khalil's and Ronda's purchases.

b. Solve the system to find the price of one can of soup and the price of one loaf of bread.

7-8. If two expressions are equivalent, they can form an equation that is considered to be **always true**. For example, since $3(x-5)$ is equivalent to $3x-15$, then the equation $3(x-5)=3x-15$ is always true, that is, true for any value of x.

If two expressions are equal only for certain values of the variable, they can form an equation that is considered to be **sometimes true**. For example, $x+2$ is equal to $3x-8$ only when $x=5$, so the equation $x+2=3x-8$ is said to be sometimes true.

If two expressions are not equal for any value of the variable, they can form an equation that is considered to be **never true**. For example, $x-5$ is not equal to $x+1$ for *any* value of x, so the equation $x-5=x+1$ is said to be never true.

Is the equation $(x+3)^2=x^2+9$ always, sometimes or never true? Justify your reasoning completely.

7-9. Consider the sequence that begins 40, 20, 10, 5, ...

a. Based on the information given, can this sequence be arithmetic? Can it be geometric? Why?

b. Assume this is a geometric sequence. On graph paper, plot the sequence on a graph up to $n=6$.

c. Will the values of the sequence ever become zero or negative? Explain.

7-10. If a ball is dropped from 160 cm and rebounds to 120 cm on the first bounce, how high will the ball be:

a. On the 2^{nd} bounce? b. On the 5^{th} bounce? c. On the n^{th} bounce?

7-11. Data for a study of a vitamin supplement that claims to shorten the length of the common cold is shown below:

Number of months taking supplement	0.5	2.5	1	2	0.5	1	2	1	1.5	2.5
Number of days cold lasted	4.5	1.6	3	1.8	5	4.2	2.4	3.6	3.3	1.4

a. You previously created a linear model for this data by "eyeballing" it. Now create a model that is consistent with your classmates by finding the LSRL. Sketch the graph and the LSRL.

b. Is a linear model appropriate? Provide evidence.

c. Find r and interpret *R-squared* in context.

d. Describe the association. Make sure you describe the *form* and provide evidence for the form. Provide numerical values for *direction* and *strength* and interpret them in context. Describe any *outliers*.

7-12. Simplify/Multiply each of the following expressions.

a. $(3x^2yz^4)^2$

b. $\left(\frac{r^2s}{rs^3t}\right)^3$

c. $(3m+7)(2m-1)$

d. $(x-3)^2$

7-13. Write and solve an equation for the problem below.

If 150 empty water bottles weigh 4.5 pounds, what would you expect 90 empty water bottles to weigh?

7-14. Sketch the shape of the graph of the function $y = b^x$ given each of the following values of b.

a. b is a number larger than 1.

b. b is a number between 0 and 1.

c. b is equal to 1.

7-15. For parts (a) and (b), find a recursive equation in a_n form for each sequence. (For a reminder about a_n form see the Math Notes box in Lesson 5.3.2.) For parts (c) and (d) find an explicit equation for each sequence.

a. 108, 120, 132, ...

b. $\frac{2}{5}, \frac{4}{5}, \frac{8}{5}, \ldots$

c. 3741, 3702, 3663, ...

d. 117, 23.4, 4.68, ...

7-16. Write the multiplier for each increase or decrease described below.

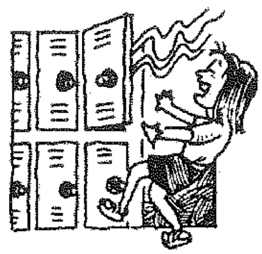
a. A 25% increase

b. A decrease of 18%

c. An increase of 39%

d. A decrease of 94%

7-17. Eeeeew! Hannah's volleyball team left their egg salad sandwiches sitting in their lockers over the weekend. When they got back on Monday they were moldy. "Perfect!" said Hannah. "I can use these sandwiches for my biology project. I'll study how quickly mold grows."



Using a transparent grid, Hannah estimated that about 12% of the surface of one sandwich had mold on it. She threw the sandwich out. For the rest of the week, Hannah came back when she had time. Each time she measured somebody else's sandwich and threw it out. She collected the following data:

Day 1 (Monday)	Day 2 (Tuesday)	Day 2 (Tuesday)	Day 4 (Thursday)	Day 4 (Thursday)	Day 4 (Thursday)	Day 5 (Friday)
12%	15%	13%	26%	27%	24%	38%

a. Create a scatterplot and sketch it. Is a linear model reasonable?

b. Based on the story, what kind of equation do you think will best fit the situation?

c. Fit the data with an exponential model and write the equation. What percentage of a sandwich did Hannah predict was covered on Wednesday? Consider the precision of Hannah's measurements when deciding how many decimal places to use in your answer.

7-18. In 1999, Charlie received the family heirloom marble collection consisting of 1239 marbles. Charlie's great-grandfather had started the original marble collection in 1905. Each year, Charlie's great-grandfather had added the same number of marbles to his collection. When he passed them on to his son, he insisted that each future generation add the same number of marbles per year to the collection. When Charlie's father received the collection in 1966, there were 810 marbles.

a. By the time Charlie inherited the collection, for how many years had it been in existence?

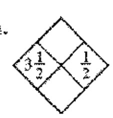
b. How many marbles are added to the collection each year?

c. Use the information you found in part (b) to figure out how many marbles were in the original collection when Charlie's great-grandfather started it.

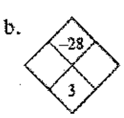
d. Generalize this situation by writing a function describing the growth of the marble collection for each year (n) since Charlie's great-grandfather started it.

7-19. Copy and complete each of the Diamond Problems below. The pattern used in the Diamond Problems is shown at right.

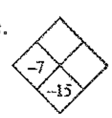
a.



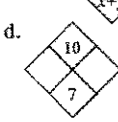
b.



c.



d.



Review & Preview

7-24. Each table below represents an exponential function of the form $y = ab^x$. Copy and complete each table on your paper and find the corresponding equation.

a.

x	y
0	1.2
1	3.96
2	13.068
3	
4	

b.

x	y
0	5
1	
2	180
3	
4	

7-25. Brianna is working on her homework. Her assignment is to come up with four representations for an exponential function of her choosing. She decides it is easiest to start by writing an equation, so she chooses $y = 1200\left(\frac{1}{2}\right)^x$. Help Brianna create the other three components of the web.

7-26. Sketch the graphs of $y = x^2$, $y = 2x^2$, and $y = \frac{1}{2}x^2$ on the same set of axes. Describe the similarities and differences among the graphs.

7-27. Write an equation or system of equations to solve this problem.

Morgan started the year with $615 in the bank and is saving $25 per week. Kendall started with $975 and is spending $15 per week. When will they both have the same amount of money in the bank?

7-28. Examine each sequence below. State whether it is arithmetic, geometric, or neither. For the sequences that are arithmetic, find the formula for $t(n)$. For the sequences that are geometric, find the sequence generator for $t(n)$.

a. $1, 4, 7, 10, 13, \ldots$

b. $0, 5, 12, 21, 32, \ldots$

c. $2, 4, 8, 16, 32, \ldots$

d. $5, 12, 19, 26, \ldots$

e. $x, x+1, x+2, x+3, \ldots$

f. $3, 12, 48, 192, \ldots$

7-29. Scientists hypothesized that dietary fiber would impact the blood cholesterol level of college students. They collected data and found $r = -0.45$ with a scattered residual plot. Interpret the scientists' findings in context.

MATH NOTES

METHODS AND MEANINGS

Compounding Interest

A bank can pay **simple interest** in which case the amount in the bank grows linearly. For example, 3% simple interest compounded annually on an initial investment of $2500 would grow in a sequence with a common difference: $0.03(2500) = \$75$. The equation and table follow:

$$t(n) = 2500 + 75n$$

Number of Years, n	0	1	2	3	...	10
Amount in Bank, $t(n)$	2500.00	2575.00	2650.00	2725.00		3250.00

If the bank **compounds interest**, the relationship is exponential. For example, 3% annual interest, *compounded annually*, would have a multiplier of 1.03 every year. The equation and table using the example above are:

$$t(n) = 2500 \cdot 1.03^{n}$$

Number of Years, n	0	1	2	3	...	10
Amount in Bank, $t(n)$	2500	2575.00	2652.25	2731.82		3359.79

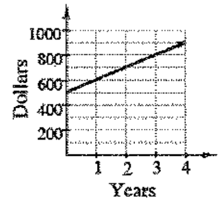
If the bank *compounds monthly*, the 3% annual interest becomes $\frac{3\%/\text{year}}{12\ \text{months/year}} = 0.25\%$ per month, and the multiplier becomes 1.0025. The equation and table for the first ten years follows:

$$t(m) = 2500 \cdot 1.0025^{m}$$

Number of Months, m	0	12	24	36	...	120
Amount in Bank, $t(m)$	2500	2576.00	2654.39	2735.13		3373.38

Review & Preview

7-35. Your banker shows you the graph at right to explain what you can earn if you invest with him. Does this graph represent simple or compound interest? How can you tell? Write an equation to represent how much money you would have as time passes. Make sure you write a "let" statement.



7-36. Each table below represents an exponential function in $y = ab^x$ form. Copy and complete each table on your paper and find a corresponding equation.

a.

x	y
−1	3
0	
1	75
2	
3	

b.

x	y
0	
1	
2	96.64
3	77.312
4	

7-37. Tickets for a concert have been in incredibly high demand, and as the date for the concert draws closer, the price of tickets increases exponentially. The cost of a pair of concert tickets was $150 yesterday, and today it is $162. As you complete parts (a) through (c) below, assume that each day's percent increase from the day before is the same.

a. What is the daily percent rate of increase? What is the multiplier?

b. What will be the cost of a pair of concert tickets one week from now?

c. What was the cost of a pair of tickets two weeks ago?

7-38. Dusty won $125,000 on the *Who Wants to be a Zillionaire?* game show. He decides to place the money into an account that earns 6.25% interest compounded annually and plans not to use any of it until he retires.

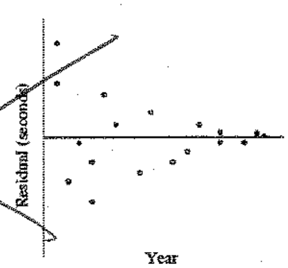
a. Write an expression that represents how much money Dusty will have in t years.

b. How much money will be in the account when he retires in 23 years?

7-39. The winning times for various swim meets at Smallville High School were compared to the height of the swimmer. The residual is shown at right.

a. Sketch what the original scatterplot may have looked like.

b. What does the residual plot tell you about predictions made with the LSRL in more recent years?



7-40. Solve the following systems of equations.

a. $3x - 2y = 14$
$-2x + 2y = -10$

b. $y = 5x + 3$
$-2x - 4y = 10$

c. Which system above is most efficiently solved by using the Substitution Method? Explain.

d. Which system above is most efficiently solved by using the Elimination Method? Explain.

7-41. If you flip a fair coin, what is the probability that it comes up "heads"? "Tails"?

Review & Preview

7-47. The leadership class at Mt. Heron High School is organizing a shoe drive. A local business has agreed to donate boxes to collect the shoes in. If each box can hold 20 pairs of shoes, draw a step graph relating the number of shoes collected to the number of boxes needed for up to 200 pair of shoes.

7-48. Assume that a DVD loses 60% of its value every year it is in a video store. Suppose the initial value of the DVD was $80.

a. What multiplier would you use to calculate the video's new values?

b. What is the value of the DVD after one year? After four years?

c. Write a continuous function, $V(t)$, to model the value of a DVD after t years.

d. When does the video have no value?

e. Sketch a graph of this function. Be sure to scale and label the axes.

7-49. This problem is a checkpoint for solving problems by writing equations. It will be referred to as Checkpoint 7A.

For each problem, write one or two equations to represent the situation and then solve. Be sure to define your variable(s) and clearly answer the question.

a. The Lee's have three children. The oldest is twice as old as the youngest. The middle child is five years older than the youngest. If the sum of the ages is 57, how old is each child?

b. In Katy's garden there are 105 ladybugs. They are increasing at two ladybugs per month. There are currently 175 aphids and the number of aphids is decreasing at three aphids per month. When will the number of ladybugs and aphids in Katy's garden be the same?

c. At the farmer's market Laura bought three pounds of heirloom tomatoes. If the tomatoes are priced at $8 for five pounds, what did Laura pay for her tomatoes?

d. Adult tickets for the school play cost $5 and student tickets cost $3. Thirty more student tickets were sold than adult tickets. If $1770 was collected, how many of each type of ticket was sold?

Check your answers by referring to the Checkpoint 7A materials located at the back of your book.

If you needed help solving these problems correctly, then you need more practice. Review the Checkpoint 7A materials and try the practice problems. Also, consider getting help outside of class time. From this point on, you will be expected to do problems like these quickly and easily.

7-50. Multiply and simplify each expression below.

a. $(x-3)^2$ b. $(2m+1)^2$

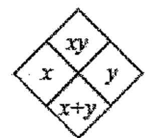
c. $x(x-3)(x+1)$ d. $(2y-1)(y^2+7)$

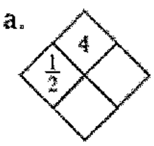
7-51. Consider the sequence $2, 8, 3y+5, \ldots$

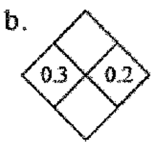
a. Find the value of y if the sequence is arithmetic.

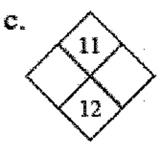
b. Find the value of y if the sequence is geometric.

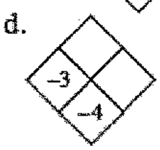
7-52. Copy and complete each of the Diamond Problems below. The pattern used in the Diamond Problems is shown at right.



a.


b.


c.


d.


7-53. Solve each equation below for x. Check your solution.

a. $3x-7(4+2x)=-x+2$ b. $-5x+2-x+1=0$

7-54. Jerry says, "I've got my money in a great account that compounds interest monthly. The equation $y=388(1.008)^m$ represents how much money I have at the end of any month." What is Jerry's monthly interest rate? What is his annual interest rate? Write an equation to represent your total money if you invest \$500 in an account with the same rate of return. Let m represent the number of months the money has been invested.

7-55. Solve each system of equations below.

a. $2x+y=-7y$
$y=x+10$

b. $3x=-5y$
$6x-7y=17$

7-56. Find the equation of the line with x-intercept $(-4, 0)$ and y-intercept $(0, 9)$.

7-57. In problem 6-10, Battle Creek Cereal was considering a variety of packaging options for Toasted Oats cereal. They wish to predict the net weight of cereal based on the amount of cardboard used for the package. Below is a list of six current packages.

Packaging cardboard (in^2)	Net weight of cereal (g)
34	21
150	198
218	283
325	567
357	680
471	1020
checksum 1555	*checksum 2769*

a. In a previous lesson, you may have hand-drawn a line of best fit for this data. Now use your calculator to find the equation of the LSRL. Sketch the scatterplot.

b. Sketch the residual plot and interpret it.

c. Since this equation involves area (a function with an exponent of 2) and weight (a function with an exponent of 3), try fitting a power model to your data. Recall that a power model has an exponent in it, and has the form $y = ax^b$. Make a residual plot and interpret it.

d. What is the equation of the model that fits your data best?

7-58. Determine which of the following equations are true for all values (always true). For those that are not, decide whether they are true for certain values (sometimes true) or not true for any values (never true). Justify your decisions clearly.

a. $(x-5)^2 = x^2 + 25$

b. $(2x-1)(x+4) = 2x^2 + 7x - 4$

c. $\frac{2x^2y^3}{y^2} = 2x^2y$

d. $(3x-2)(2x+1) = 6x^2 - x - 5$

MATH NOTES

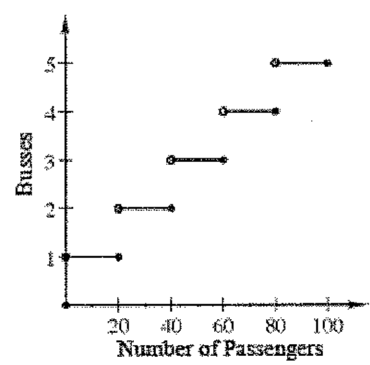
METHODS AND MEANINGS

Step Functions

A **step function** is a special kind of piecewise function (a function composed of parts of two or more functions). A step function has a graph that is a series of line segments that often looks like a set of steps. Step functions are used to model real-world situations where there are abrupt changes in the output of the function.

The endpoints of the segments on step functions are either open circles (indicating this point is not part of the segment) or filled-in circles (indicating this point is part of the segment).

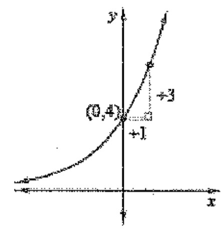
The graph at right models a situation in which a tour bus company has busses that can each hold up to 20 passengers with 5 available busses.



Review & Preview

7-61. The drama club found that the best price for renting a fog machine was $38 for every three days, plus a one-time $60 delivery fee. Make a step graph that shows the cost of renting the fog machine for up to three weeks.

7-62. Use the clues in the graph at right to find a possible corresponding equation in $y = ab^x$ form. Assume the graph has an asymptote at the x-axis.



7-63. Kristin's grandparents started a savings account for her when she was born. They invested $500 in an account that pays 8% interest compounded annually.

a. Write an equation to model the amount of money in the account on Kristin's x^{th} birthday.

b. How much money is in the account on Kristin's 16^{th} birthday?

c. What are the domain and range of the equation that you wrote in part (a)?

7-64. Graph $y = x^2 + 3$ and $y = (x+3)^2$. What are the similarities and differences between the graphs? How do these graphs compare to the graph of $y = x^2$?

7-65. Simplify.

a. $\sqrt[3]{-1000}$ b. $\sqrt[3]{\frac{1}{8}}$ c. $\sqrt[3]{-125}$ d. $\sqrt[4]{81}$

7-66. Solve each equation for the variable. Check your solutions, if possible.

a. $8a + a - 3 = 6a - 2a - 3$ b. $8(3m-2) - 7m = 0$

c. $\frac{x}{2} + 1 = 6$ d. $|x-3| + 5 = 11$

7-73. The U.S. Census Bureau takes a census every 10 years. The population in 2000 was estimated at 281.4 million people. A model created at the time predicted that the population grew at about 2% per year.

a. If the Census Bureau had conducted a count in 2005, how many people would it have expected to count?

b. Using the model, how many people would the Census Bureau have expected to count in the 2010 census?

c. The actual 2010 census put the population at 309 million people. What is the residual? What does it mean?

7-74. Find values of a and b that make each system of equations true (i.e., solve each system). Be sure to show your work or explain your thinking clearly.

a. $6 = a \cdot b^0$
$24 = a \cdot b^2$

b. $32 = a \cdot b^2$
$128 = a \cdot b^3$

7-75. Jack and Jill were working on simplifying the expression at right, but they were having some trouble. Then Jill had an idea. $\frac{3x^2y^{-3}}{x^{-1}y^2}$

"Can't we separate the parts?" she said. *"That way, it might be easier to tell what we can simplify."* She rewrote the expression as shown at right. $3 \cdot x^2 \cdot \frac{1}{x^{-1}} \cdot y^{-3} \cdot \frac{1}{y^2}$

"Okay," said Jack. *"Now we can rewrite each of the parts with negative exponents and simplify."*

a. Help Jack and Jill finish simplifying their expression.

b. Use their idea to rewrite and simplify $\frac{m^2pq^{-1}}{4m^{-2}pq^3}$.

7-76. A particular sequence can be represented by $t(n) = 2(3)^n$.

a. What are $t(0)$, $t(1)$, $t(2)$, and $t(3)$?

b. Graph this sequence. What is the domain?

c. On the same set of axes, graph the function $f(x) = 2(3)^x$.

d. How are the two graphs similar? How are they different?

7-77. Solve the system of equations at right algebraically.

$2x - 3y = 12$
$y + x = -9$

7-78. Each teams in Ms. Zaleski's class cut a circular disks from cardboard file folders. They could cut the disks any size they chose. The weight and radius were recorded. The information is shown in the table below. Since students chose the radius, consider radius the independent variable.

Radius (cm)	9.6	9	7.7	6.3	5.3	4.7	3.7	2.4	1.3
Weight (g)	5.4	4.6	3.4	2.3	1.6	1.2	0.8	0.3	0.1

a. Make a scatterplot for the data and sketch it onto your paper. Describe the association between weight and radius.

b. The weight of the cardboard disk depends on its area. What kind of equation do you suggest to model this data?

c. Model the data with a regression equation. Does the y-intercept of your model make sense in this context?.

d. What do you predict a 7-cm disk will weigh? Use appropriate precision in your answer.

Review & Preview

7-87. Find a possible exponential function in $y = a \cdot b^x$ form that represents each situation described below.

a. Has an initial value of 2 and passes through the point (3, 128).

b. Passes through the points (0, 4) and (2, 1).

7-88. Solve the following systems of equations. In other words, find values of a and b that make each system true. Be sure to show your work or explain your thinking clearly.

a. $3 = a \cdot b^0$
$75 = a \cdot b^2$

b. $18 = a \cdot b^2$
$54 = a \cdot b^3$

7-89. Evaluate each expression below.

a. $\sqrt[3]{-64}$ b. $\sqrt[5]{32}$ c. $\sqrt[3]{-8}$ d. $\sqrt[4]{10000}$

7-90. Rewrite $16^{3/4}$ in as many different ways as you can.

7-91. Find the equation of the line passing through the points (7, 16) and (2, −4). Then state the slope and x- and y-intercepts. Explain how you found them.

MATH NOTES

METHODS AND MEANINGS

Negative and Fractional Exponents

For all x not equal to zero:

$x^0 = 1$ Examples: $2^0 = 1$, $(-3)^0 = 1$, $\left(\frac{1}{4}\right)^0 = 1$

For positive values of x:

$x^{-n} = \frac{1}{x^n}$ Examples: $x^{-3} = \frac{1}{x^3}$, $y^{-4} = \frac{1}{y^4}$, $4^{-2} = \frac{1}{4^2} = \frac{1}{16}$

$\frac{1}{x^{-n}} = x^n$ Examples: $\frac{1}{x^{-5}} = x^5$, $\frac{1}{x^{-2}} = x^2$, $\frac{1}{3^{-2}} = 3^2 = 9$

$x^{a/b} = (x^a)^{1/b} = \sqrt[b]{x^a}$ or $x^{a/b} = (x^{1/b})^a = (\sqrt[b]{x})^a$

Examples: $5^{1/2} = \sqrt{5}$, $3^{2/3} = \sqrt[3]{3^2} = \sqrt[3]{9}$,

$16^{3/4} = (16^{1/4})^3 = (\sqrt[4]{16})^3 = 2^3 = 8$

Review & Preview

7-96. Find an exponential function that passes through each pair of points.

a. (1, 7.5) and (3, 16.875) b. (–1, 1.25) and (3, 0.032)

7-97. Consider the pattern at right.

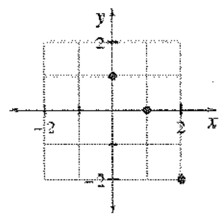
$\frac{1}{2^3}=\frac{1}{8}$
$\frac{1}{2^2}=\frac{1}{4}$
$\frac{1}{2^1}=\frac{1}{2}$
$\frac{1}{2^0}=1$

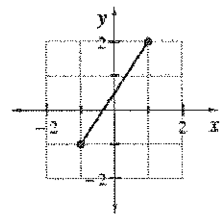
a. Continue the pattern to find $\frac{1}{2^{-1}}$, $\frac{1}{2^{-2}}$, $\frac{1}{2^{-3}}$, and $\frac{1}{2^{-4}}$.

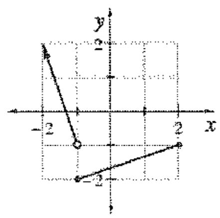
b. What is the value of $\frac{1}{2^{-n}}$?

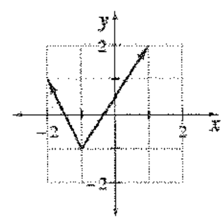
c. Write a conjecture about how to rewrite $\frac{1}{a^{-n}}$ without a negative exponent.

7-98. Find the domain and range for each of the relations graphed below.

a.


b.


c.


d.


7-99. If $f(x)=3(2)^x$, find the value of the expressions in parts (a) through (c) below. Then complete parts (d) through (f).

a. $f(-1)$ b. $f(0)$ c. $f(1)$

d. What value of x gives $f(x)=12$?

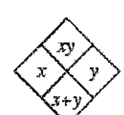
e. Where does the graph of this function cross the x-axis? The y-axis?

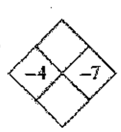
f. If $g(x)=\frac{1}{3x}$, find $f(x)\cdot g(x)$.

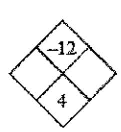
7-100. Show two steps to simplify each of the following expressions, and then calculate the value of each expression.

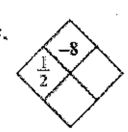
a. $64^{2/3}$ b. $25^{5/2}$ c. $81^{7/4}$

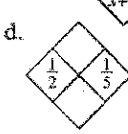
7-101. Copy and complete each of the Diamond Problems below. The pattern used in the Diamond Problems is shown at right.



a.


b.


c.


d.


MATH NOTES

METHODS AND MEANINGS

Equations for Sequences

Arithmetic Sequences

The equation for an arithmetic sequence is: $t(n) = mn + b$ or $a_n = mn + a_0$ where n is the term number, m is the sequence generator (the common difference), and b or a_0 is the zeroth term. Compare these equations to a continuous linear function $f(x) = mx + b$ where m is the growth (slope) and b is the starting value (y-intercept).

For example, the arithmetic sequence 4, 7, 10, 13, ... could be represented by $t(n) = 3n + 1$ or by $a_n = 3n + 1$. (Note that "4" is the first term of this sequence, so "1" is the zeroth term.)

Another way to write the equation of an arithmetic sequence is by using the first term in the equation, as in $a_n = m(n-1) + a_1$, where a_1 is the first term. The sequence in the example could be represented by $a_n = 3(n-1) + 4$.

You could even write an equation using any other term in the sequence. The equation using the fourth term in the example would be $a_n = 3(n-4) + 13$.

Geometric Sequences

The equation for a geometric sequence is: $t(n) = ab^n$ or $a_n = a_0 \cdot b^n$ where n is the term number, b is the sequence generator (the multiplier or common ratio), and a or a_0 is the zeroth term. Compare these equations to a continuous exponential function $f(x) = ab^x$ where b is the growth (multiplier) and a is the starting value (y-intercept).

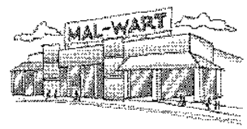
For example, the geometric sequence 6, 18, 54, ... could be represented by $t(n) = 2 \cdot 3^n$ or by $a_n = 2 \cdot 3^n$.

You can write a first term form of the equation for a geometric sequence as well: $a_n = a_1 \cdot b^{n-1}$. For the example, first term form would be $a_n = 6 \cdot 3^{n-1}$.

Review & Preview

7-106. Find the equation of an exponential function that passes through the points (2, 48) and (5, 750).

7-107. After noon, the number of people in Mal-Wart grows steadily until 6:00 p.m. If the equation $y = 228 + 58x$ represents the number of people in the store x hours after noon:



a. How many people were in the store at noon?

b. At what rate is the number of shoppers growing?

c. When were there 402 shoppers in the store?

7-108. Wade and Dwayne were working together writing an equation for the sequence 12, 36, 108, 324, ... Wade wrote $t(n) = 4 \cdot 3^n$ and Dwayne wrote $t(n) = 12 \cdot 3^{n-1}$.

a. Make a table for the first four terms of each of their sequences. What do you notice?

b. How do you think Dwayne explained his method of writing the equation to Wade?

c. For the sequence 10.3, 11.5, 12.7, ..., Wade wrote $t(n) = 9.1 + 1.2n$ while Dwayne wrote $t(n) = 10.3 + 1.2(n-1)$. Make a table for the first four terms of each of their sequences. Are both forms of the equation correct?

d. Dwayne calls his equations the **"first term" form**. Why do you think he calls them "first term" form? Why does Dwayne subtract one in both situations?

7-109. Find the dimensions of the generic rectangle shown at right and write its area as a sum and a product.

$-6x$	4
$9x^2$	$-6x$

7-110. Chris got a summer job at an environmental sciences lab. He was using a microscope to measure the length of a particular organelle in tree sap cells and the diameter of the cell itself. Chris collected the following data. A "μm" is a micrometer or one-millionth of a meter.

Length of Organelle (μm)	2	9	7	4	4	9	6	2
Diameter of Cell (μm)	46	34	36	42	46	36	42	45

a. What would you include in a mini-report that fully describes all aspects of a linear association? Make as detailed a list as you can.

b. Fully describe all aspects of the linear association in context. Include appropriate graphs.

7-111. This problem is a checkpoint for solving linear systems of equations. It will be referred to as Checkpoint 7B.

Solve each system of equations.

a. $y = 3x + 11$
$x + y = 3$

b. $y = 2x + 3$
$x - y = -4$

c. $x + 2y = 16$
$x + y = 2$

d. $2x + 3y = 10$
$3x - 4y = -2$

Check your answers by referring to the Checkpoint 7B materials located at the back of your book.

If you needed help solving these problems correctly, then you need more practice. Review the Checkpoint 7B materials and try the practice problems. Also, consider getting help outside of class time. From this point on, you will be expected to do problems like these quickly and easily.

CL 7-112. Find an exponential function in $y = ab^x$ form that satisfies each of the following sets of conditions.

a. Has a y-intercept of $(0, 2)$ and a multiplier of 0.8.

b. Passes through the points $(0, 3.5)$ and $(2, 31.5)$.

CL 7-113. Sam wants to create an arithmetic sequence and a geometric sequence, both of which have $t(1) = 8$ and $t(7) = 512$. Is this possible? If it is, help Sam create his sequences. If not, justify why not.

CL 7-114. Write each expression below as an equivalent expression without negative exponents.

a. 3^{-2} b. m^{-4} c. $(\frac{1}{2})^{-3}$ d. $(\frac{3}{5x})^{-1}$

CL 7-115. Write each expression below in radical form and compute the value without using a calculator.

a. $8^{1/3}$ b. $16^{3/4}$ c. $125^{4/3}$

CL 7-116. Best Price Parking charges \$2 for the first hour of parking and \$0.50 for each additional hour. Create a step function graph that represents this information.

CL 7-117. A share of ABC stock was worth \$60 in 2005 and only worth \$45 in 2010.

a. Find the multiplier and the percent decrease.

b. Write an exponential function that models the value of the stock starting from 2005.

c. Assuming that the decline in value continues at the same rate, use your answer to (b) to predict the value in 2020.

CL 7-118. Write an equation or system of equations to solve this problem.

An adult ticket to the amusement park costs \$24.95 and a child's ticket costs \$15.95. A group of 10 people paid \$186.50 to enter the park. How many were adults?

CL 7-119. Solve each system of equations.

a. $2x - y = 9$
$y = x - 7$

b. $-4x + y = 5$
$2x = -y - 13$

CL 7-120. Below are several situations that can be described using exponential functions. They represent a small sampling of the situations where quantities grow or decay by a constant percentage over equal periods of time. For each situation (a) through (d):

- Find an appropriate unit of time (such as days, weeks, years).
- Find the multiplier that should be used.
- Identify the initial value.
- Write an exponential equation in the form $f(x) = ab^x$ that represents the growth or decay.

a. A house purchased for $120,000 has an annual appreciation of 6%.

b. The number of bacteria present in a colony is 180 at noon, and it increases at a rate of 22% per hour.

c. The value of a car with an initial purchase price of $12,250 depreciates by 11% per year.

d. An investment of $1000 earns 6% annual interest, compounded monthly.

CL 7-121. Write an equation for the line that passes through the points $(-5, 4)$ and $(3, -2)$.

CL 7-122. Mary helps prepare food in the Tiger Café. Mary notices that sales of fresh fruit cups seem to vary widely from day to day. This is a problem because preparing too many cups results in wasted fruit and making too few results in lost sales. She decides that the daily weather may have a strong association with demand. Mary chooses 12 days at random from last semester and pairs each day's high temperature with the number of fresh fruit cups sold each day.

Temp (°F)	# of Cups
89	150
52	85
72	136
65	101
72	122
45	66
31	63
89	137
57	86
38	80
37	58
71	118
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a. Create a linear model for this data by finding the LSRL. Sketch the graph and the LSRL.

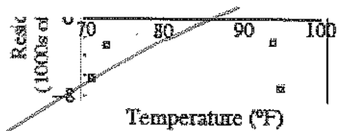
b. Is a linear model appropriate? Create a residual plot to provide evidence.

c. Find the correlation coefficient and interpret *R-squared* in context.

d. Describe the association. Make sure you describe the *form* and provide evidence for the form. Provide numerical values for *direction* and *strength* and interpret them in context. Describe any *outliers*.

e. If Mary wanted to be reasonably certain of not running out of fresh fruit cups on a day forecasted to be 90 degrees, how many should she prepare? (Hint: Consider the upper and lower bounds of sales.)

association between the attendance at the amusement park and the temperature. Marissa made the residual plot at right.



a. Was a linear model appropriate? Why or why not?

b. Marissa's data follows. She rounded attendance to the nearest hundred. Make a scatterplot of the data. What kind of model might better represent Marissa's data? Why?

Temperature (°F)	71	73	78	83	91	92	73	88	95	94	*checksum 838*
Attendance (thousands)	8.6	13	21.6	25.9	23.8	25.9	17.3	25.9	17.3	21.6	*checksum 200.9*

c. Fit a quadratic model to the data. What attendance does your model predict for a 95°F day? Use appropriate precision.

CL 7-124. Check your answers using the table at the end of this section. Which problems do you feel confident about? Which problems were hard? Have you worked on problems like these in previous math classes? Use the table to make a list of topics you need help on, and a list of topics you need to practice more.