



CHAPTER 7

Proof and Quadrilaterals

This chapter opens with a set of explorations designed to introduce you to new geometric topics that you will explore further in Chapters 8 through 12. You will learn about the special properties of a circle, explore three-dimensional shapes, and use a hinged mirror to learn more about a rhombus.

Section 7.2 then builds on your work from Chapters 3 through 6. Using congruent triangles, you will explore the relationships of the sides and diagonals of a parallelogram, kite, trapezoid, rectangle, and rhombus. As you explore new geometric properties, you will formalize your understanding of proof.

This chapter ends with an exploration of coordinate geometry.

Guiding Question

Mathematically proficient students will construct viable arguments and critique the reasoning of others.


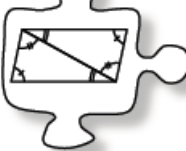

As you work through this chapter, ask yourself:

How can I use the given information to construct arguments, how can I justify my conclusions and how can I respond to the arguments of others.

In this chapter, you will learn:

- The relationships of the sides, angles, and diagonals of special quadrilaterals, such as parallelograms, rectangles, kites, and rhombi (plural of rhombus).
- How to write a convincing proof in a variety of formats, such as a flowchart or two-column proof.
- How to find the midpoint of a line segment.
- How to use algebraic tools to explore quadrilaterals on coordinate axes.

Chapter Outline

	Section 7.1	This section contains four large investigations introducing you to geometry topics that will be explored further in Chapters 8 through 12.
	Section 7.2	While investigating what congruent triangles can inform you about the sides, angles, and diagonals of a quadrilateral, you will develop an understanding of proof.
	Section 7.3	This section begins a focus on coordinate geometry, that is, the study of geometry on coordinate axes. During this section, you will use familiar algebraic tools (such as slope) to make and justify conclusions about shapes.

Chapter 7 Teacher Guide

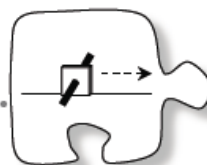
You may wish to omit some of the lessons in this chapter depending on the specific needs of your students. Before skipping any lesson, please refer to the Course Timeline in the "Preparing to Teach This Course" tab in the front matter of this Teacher Edition.

Section	Lesson	Days	Lesson Objective	Materials	Homework
7.1	7.1.1	1	Properties of a Circle	<ul style="list-style-type: none"> Cardboard Lesson 7.1.1A-B Res. Pgs. 	7-6 to 7-11
	7.1.2	1	Building a Tetrahedron	<ul style="list-style-type: none"> Lesson 7.1.2A Res. Pg. or paper plates Lesson 7.1.2B Res. Pg. (opt.) Scissors and Tape 	7-15 to 7-20
	7.1.3	2	Shortest Distance Problems	<ul style="list-style-type: none"> Lesson 7.1.3 Res. Pg. Soup cans with labels Cubes String or yarn Tracing paper 	7-28 to 7-33 and 7-34 to 7-39
	7.1.4	1	Using Symmetry to Study Polygons	<ul style="list-style-type: none"> Hinged mirrors Colored paper Protractors or Lesson 1.1.5 Res. Pg. 	7-43 to 7-48
7.2	7.2.1	1	Special Quadrilaterals and Proof	<ul style="list-style-type: none"> Lesson 7.2.1A-B Res. Pgs. Masking tape Tracing paper 	7-54 to 7-60
	7.2.2	1	Properties of Rhombi	<ul style="list-style-type: none"> Hinged mirror (opt.) Poster-making materials (opt.) 	7-65 to 7-71
	7.2.3	1	More Proofs with Congruent Triangles	None	7-75 to 7-81
	7.2.4	1	More Properties of Quadrilaterals	<ul style="list-style-type: none"> Poster-making materials 	7-83 to 7-89
	7.2.5	1	Two-Column Proofs	<ul style="list-style-type: none"> Posters from Lesson 7.2.4 	7-96 to 7-102
	7.2.6	1	Explore-Conjecture-Prove	None	7-108 to 7-114
7.3	7.3.1	1	Studying Quadrilaterals on a Coordinate Grid	<ul style="list-style-type: none"> Graph paper 	7-119 to 7-125
	7.3.2	1	Coordinate Geometry and Midpoints	<ul style="list-style-type: none"> Graph paper 	7-131 to 7-137
	7.3.3	1	Identifying Quadrilaterals on a Coordinate Grid	<ul style="list-style-type: none"> Graph paper 	7-140 to 7-146
Chapter Closure		Various Options			

Total: 14 days plus optional time for Chapter Closure and Assessment

7.1.1 Does it roll smoothly?

Properties of a Circle



In Chapters 1 through 6, you studied many different types of two-dimensional shapes, explored how they could be related, and developed tools to measure their lengths and areas. In Chapters 7 through 12, you will examine ways to extend these ideas to new shapes (such as polygons and circles) and will thoroughly investigate what you can learn about three-dimensional shapes.

Section 7.1 contains four key investigations that will touch upon the big ideas of the following chapters. As you explore these lessons, take note of what mathematical tools from Chapters 1 through 6 you are using and think about what new directions this course will take. Generate “What if...” questions that can be answered later once new tools are developed.

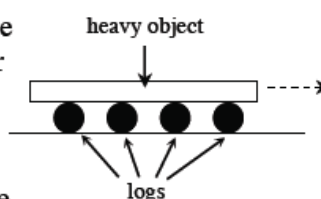
Since much of the focus of Chapters 7 through 12 is on the study of circles, this lesson will first explore the properties of a circle. What makes a circle special? Today you are going to answer that question and, at the same time, explore other shapes with surprisingly similar qualities.

7-1. THE INVENTION OF THE WHEEL

One of the most important human inventions was the wheel. Many archeologists estimate that the wheel was probably first invented about 10,000 years ago in Asia. It was an important tool that enabled humans to transport very heavy objects long distances. Most people agree that impressive structures, such as the Egyptian pyramids, could not have been built without the help of wheels.



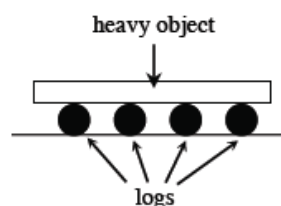
- One of the earliest types of “wheels” used were actually logs. Ancient civilizations laid multiple logs on the ground, parallel to each other, under a heavy item that needed to be moved. As long as the logs had the same diameter and the road was even, the heavy object had a smooth ride. What is special about a circle that allows it to be used in this way? In other words, why do circles enable this heavy object to roll smoothly?



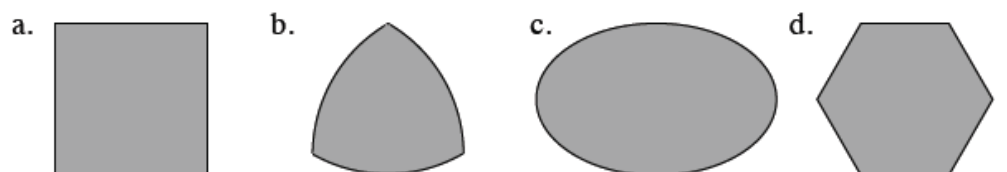
- What happens to a point on a wheel as it turns? For example, as the wheel at right rolls along the line, what is the path of point P ? Imagine a piece of gum stuck to a tire as it rolls. On your paper, draw the motion of point P . If you need help, find a coin or other round object and test this situation.
- Now turn your attention to the center of the wheel (labeled C in the diagram above). As the wheel rolls along the line, what is the path of point C ? Describe its motion. Why does that happen?

7-2. DO CIRCLES MAKE THE BEST WHEELS?

As you read in problem 7-1, ancient civilizations used circular logs to roll heavy objects. However, is a circle the only shape they could have chosen? Are there any other shapes that could rotate between a flat road and a heavy object in a similar fashion?



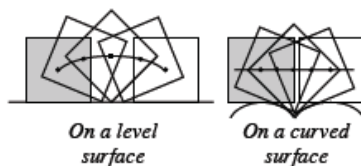
Examine the shapes below. Would logs of any of these shapes be able to roll heavy objects in a similar fashion? Be prepared to defend your conclusion!



- 7-3. Stan says that he has a tricycle with square wheels and claims that it can ride as smoothly as a tricycle with circular wheels! Rosita does not believe him. Analyze this possibility with your team as you answer the questions below.



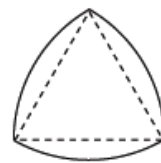
- Is Stan's claim possible? Describe what it would be like to ride a tricycle with square tires. What type of motion would the rider experience? Why does this happen?
- When Rosita challenged him, Stan confessed that he needed a special road so that the square wheels would be able to rotate smoothly and would keep Stan at a constant height. What would his road need to look like? Draw an example on your paper.
- How would Stanley need to change his road to be able to ride a tricycle with rectangular (but non-square) wheels? Draw an example on your paper.
- As the picture at right¹ shows, square wheels are possible if the road is specially curved to accommodate the change in the length of the radius of the wheel as it rotates. An example is shown below.



Using mathematical terms, explain why the square wheel needed a modified road to ride smoothly, while the circular wheel did not. What is different between the two shapes?

7-4. REULEAUX CURVES

Reuleaux (pronounced “roo LOW”) curves are special because they have a constant diameter. That means that as a Reuleaux curve rotates, its height remains constant. Although the diagram at right is an example of a Reuleaux curve based on an equilateral triangle, these special curves can be based on any regular polygon with an odd number of sides.



- a. What happens to the center (point C) as the Reuleaux wheel at right rolls?
- b. Since logs with a Reuleaux curve shape can also smoothly roll heavy objects, why are these shapes not used for bicycle wheels? In other words, what is the difference between a circle and a Reuleaux curve?



7-5. LEARNING LOG

A big focus of Chapters 7 through 12 is on circles. What did you learn about circles today? Did you learn anything about other shapes that was new or that surprised you?

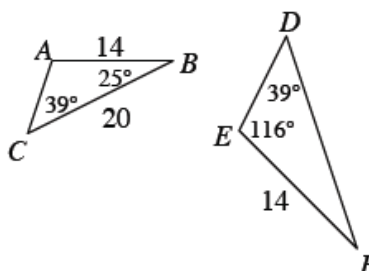
Write a Learning Log entry explaining what you learned about the shapes of wheels. Title this entry “Shapes of Wheels” and label it with today’s date.





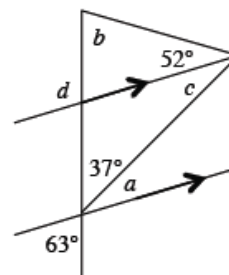
7-6. Examine $\triangle ABC$ and $\triangle DEF$ at right.

- a. Assume the triangles at right are not drawn to scale. Complete a flowchart to justify the relationship between the two triangles.



- b. Find AC and DF .

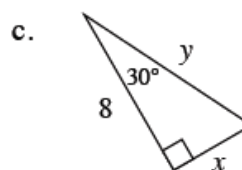
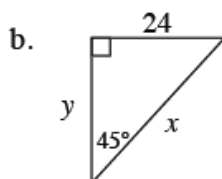
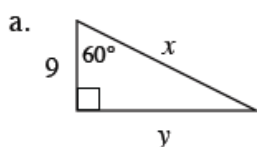
7-7. Use the relationships in the diagram at right to find the values of each variable. Name which geometric relationships you used.



7-8. A rectangle has one side of length 11 mm and a diagonal of 61 mm. Draw a diagram of this rectangle and find its width and area.

7-9. Troy is thinking of a shape. He says that it has four sides and that no sides have equal length. He also says that no sides are parallel. What is the best name for his shape?

7-10. Without using your calculator, find the exact values of x and y in each diagram below.



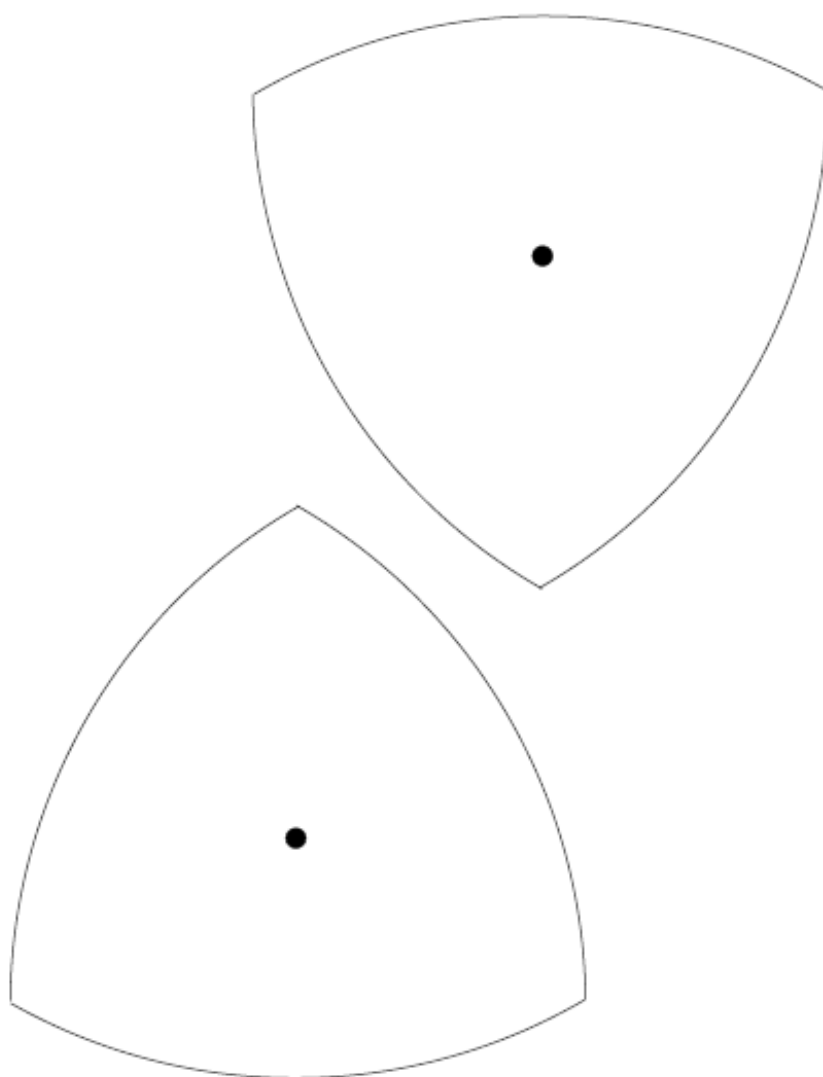
7-11. Solve each system of equations below, if possible. If it is not possible, explain what the lack of an algebraic solution tells you about the graphs of the equations. Write each solution in the form (x, y) . Show all work.

a. $y = -2x - 1$
 $y = \frac{1}{2}x - 16$

b. $y = x^2 + 1$
 $y = -x^2$

Lesson 7.1.1A Resource Page

Reuleaux Wheels



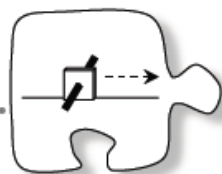
Lesson 7.1.1B Resource Page

Team Roles

Resource Manager:	<ul style="list-style-type: none"> Get all materials for your team. Make sure all your questions are team questions. Do not let your team stay stuck! <i>"Does anyone have any ideas?"</i> <i>"What team question can we ask the teacher?"</i> <i>"Are we sure no one here can answer the question?"</i>
Facilitator:	<ul style="list-style-type: none"> Help your team get started by having someone read the task. Then make sure everyone understands what to do. <i>"Who wants to read?"</i> <i>"I'm not sure how to start – what are we being asked to do?"</i> <i>"What does the first question mean?"</i> Make sure everyone's ideas are heard. <i>"Does anyone see it a different way?"</i> <i>"Does anyone have a different idea?"</i> Keep your team together. Make sure everyone understands your team's conjectures and conclusions before moving on. <i>"Can you show us what you're doing?"</i> <i>"Do we all agree?"</i> <i>"Are we ready to move on?"</i> <i>"I'm not sure I get it yet – can someone explain?"</i>
Recorder/Reporter:	<ul style="list-style-type: none"> Your team needs to have a comprehensive list of brainstormed conjectures, tests performed, and counterexamples found with reasons. You will report on your team's process and results at the end of class. As you work, it is your job to ensure your teammates can see each other's work, reasons, and connections. You might want to sketch information on scratch paper to put in the middle of your table so your team can talk about it. <i>"How can we show that idea?"</i> <i>"Should I make a picture for the middle of the table?"</i> <i>"How can we record that idea?"</i> <i>"Can you explain that idea again?"</i>
Task Manager:	<ul style="list-style-type: none"> You need to make sure your team is accomplishing the task effectively and efficiently. Make sure that all talking is <u>within</u> your team and is helping you to accomplish the task. Eliminate side conversations. <i>"Okay, let's get back to work."</i> <i>"What does the next question say?"</i> <i>"Listen for statements and reasons."</i>

7.1.2 What can I build with a circle?

Building a Tetrahedron



In later chapters, you will learn more about polygons, circles, and three-dimensional shapes. Later investigations will require that you remember key concepts you have already learned about triangles, parallel lines, and other angle relationships. Today you will have the opportunity to review some of the geometry you have learned while also beginning to think about what you will be studying in the future.

As you work with your team, consider the following focus questions:

Is there more than one way?

How can you be sure that is true?

What else can we try?

7-12. IS THERE MORE TO THIS CIRCLE?

Circles can be folded to create many different shapes. Today, you will work with a circle and use properties of other shapes to develop a three-dimensional shape. Be sure to have reasons for each conclusion you make as you work. Each person in your team should start by obtaining a copy of a circle from your teacher and cutting it out.



- Fold the circle in half to create a crease that lies on a line of symmetry of the circle. Unfold the circle and then fold it in half again to create a new crease that is perpendicular to the first crease. Unfold your paper back to the full circle. How could you convince someone else that your creases are perpendicular? What is another name for the line segment represented by each crease?
- On the circle, label the endpoints of one diameter A and B . Fold the circle so that point A touches the center of the circle and create a new crease. Then label the endpoints of this crease C and D . What appears to be the relationship between \overline{AB} and \overline{CD} ? Discuss and justify with your team. Be ready to share your reasons with the class.
- Now fold the circle twice to form creases \overline{BC} and \overline{BD} and use scissors to cut out $\triangle BCD$. What type of triangle is $\triangle BCD$? How can you be sure? Be ready to convince the class.

7-13. ADDING DEPTH

Your equilateral triangle should now be flat (also called two-dimensional).

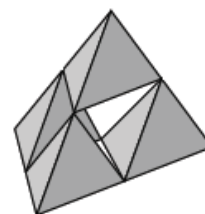
Two-dimensional shapes have length and width, but not depth (or “thickness”).

- a. If the labels were cut off, then label the vertices of $\triangle BCD$ again now. Then, with the unmarked side of the triangle facedown, fold and crease the triangle so that B touches the midpoint of \overline{CD} . Keep it in the folded position.

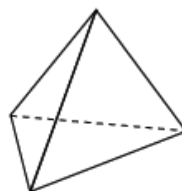
What does the resulting shape appear to be? What smaller shapes do you see inside the larger shape? Justify that your ideas are correct. For example, if you think that lines are parallel, you must provide evidence.

- b. Open your shape again so that you have the large equilateral triangle in front of you. How does the length of a side of the large triangle compare to the length of the side of the small triangle formed by the crease? How many of the small triangles would fit inside the large triangle? In what ways are the small and large triangles related?
- c. Repeat the fold in part (a) so that C touches the midpoint of \overline{BD} . Unfold the triangle and fold again so that D touches the midpoint of \overline{BC} . Create a three-dimensional shape by bringing points B , C , and D together. A **three-dimensional** shape has length, width, and depth. Use tape to hold your shape together.
- d. Three-dimensional shapes formed with polygons have **faces** and **edges**, as well as **vertices** (plural of vertex). Faces are the flat surfaces of the shape, while edges are the line segments formed when two faces meet. Vertices are the points where edges intersect. Discuss with your team how to use these words to describe your new shape. Then write a complete description. If you think you know the name of this shape, include it in your description.

- 7-14. Your team should now have 4 three-dimensional shapes (called tetrahedra). If you are working in a smaller team, you should quickly fold more shapes so that you have a total of four.



- Put four tetrahedra together to make an enlarged tetrahedron like the one pictured at right. Is the larger tetrahedron similar to the small tetrahedron? How can you tell?
- To determine the edges and faces of the new shape, pretend that it is solid. How many edges does a tetrahedron have? Are all of the edges the same length? How does the length of an edge of the new shape compare with the length of an edge of one of the small shapes?
- How many faces of the small tetrahedron would it take to cover the face of the large tetrahedron? Remember to count gaps as part of a face. Does the area of the tetrahedron change in the same way as the length?



Enlarged tetrahedron



Original



MATH NOTES

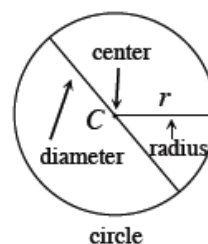
METHODS AND MEANINGS

Parts of a Circle

A **circle** is the set of all points on a flat surface that are the same distance from a fixed central point, C , referred to as its **center**. This text will use the notation $\odot C$ to name a circle with center at point C .

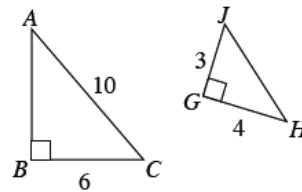
The **radius** is a line segment from the center to a point on the circle. Its length is usually denoted r . However, a line segment drawn through the center of the circle with both endpoints on the circle is called a **diameter** and its length is usually denoted d .

Notice that a diameter of a circle is always twice as long as the radius.

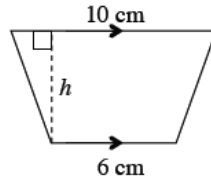




- 7-15. What is the relationship between $\triangle ABC$ and $\triangle GHJ$ at right? Create a flowchart to justify your conclusion.



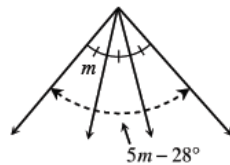
- 7-16. The area of the trapezoid at right is 56 cm^2 . What is h ? Show all work.



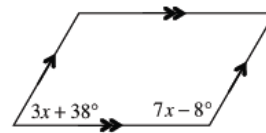
- 7-17. Line L is perpendicular to the line $6x - y = 7$ and passes through the point $(0, 6)$. Line M is parallel to the line $y = \frac{2}{3}x - 4$ and passes through the point $(-3, -1)$. Where do these lines intersect? Explain how you found your solution.

- 7-18. Examine the geometric relationships in each of the diagrams below. For each one, write and solve an equation to find the value of the variable. Name any geometric property or conjecture that you used.

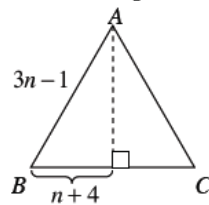
a.



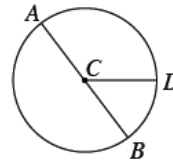
b.



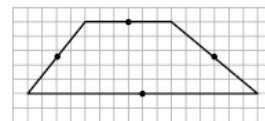
c. $\triangle ABC$ is equilateral.



d. Point C is the center of the circle.
 $AB = 11x - 1$ and $CD = 3x + 12$



- 7-19. In the Shape Factory, you created many shapes by rotating triangles about the midpoint of its sides. (Remember that the **midpoint** is the point exactly halfway between the endpoints of the line segment.) However, what if you rotate a trapezoid instead?

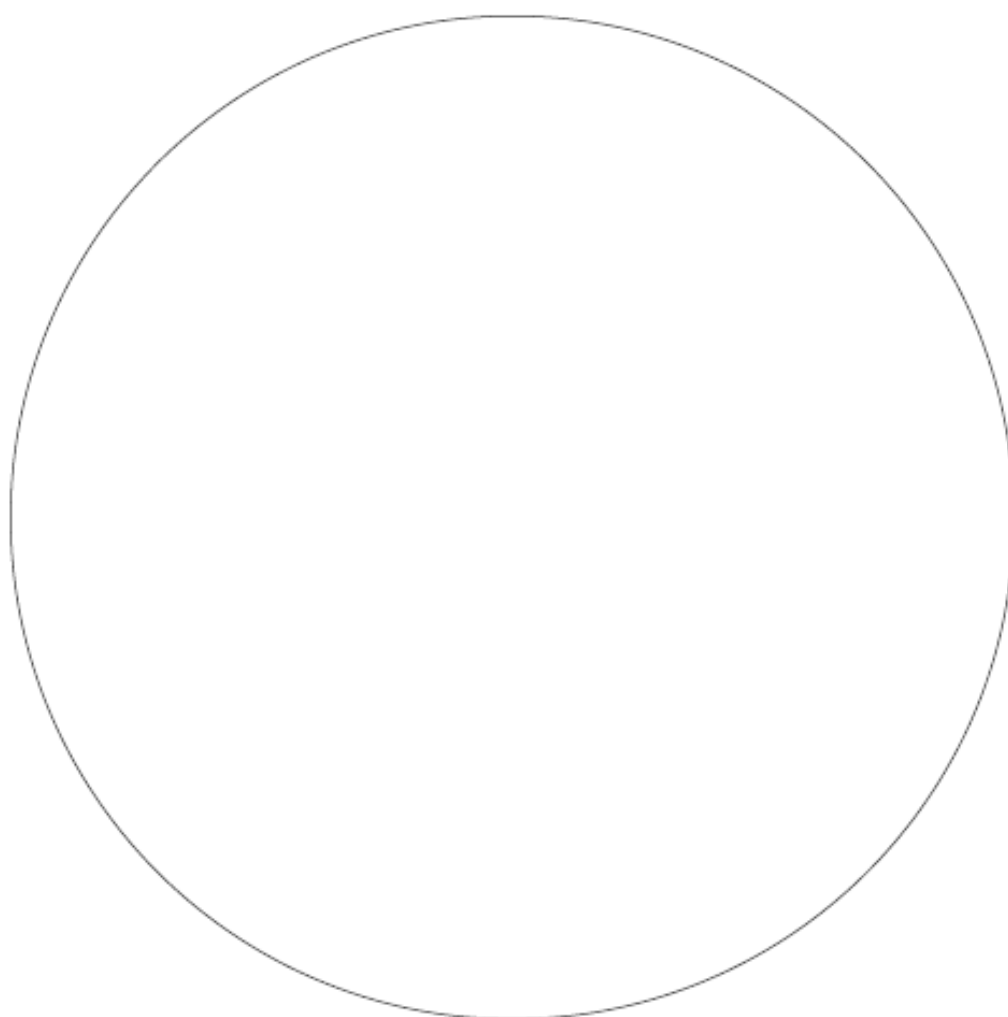


Carefully draw the trapezoid above on graph paper, along with the given midpoints. Then rotate the trapezoid 180° about one of the midpoints and examine the resulting shape formed by both trapezoids (the original and its image). Continue this process with each of the other midpoints, until you discover all the shapes that can be formed by a trapezoid and its image when rotated 180° about the midpoint of one of its sides.

- 7-20. On graph paper, plot the points $A(-5, 7)$ and $B(3, 1)$.
- Find AB (the length of \overline{AB}).
 - Locate the midpoint of \overline{AB} and label it C. What are the coordinates of C?
 - Find AC. Can you do this without using the Pythagorean Theorem?

Lesson 7.1.2A Resource Page

Circle



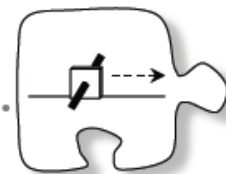
Lesson 7.1.2B Resource Page

General Team Roles

	Resource Manager: <ul style="list-style-type: none">• Get supplies for your team, and make sure your team cleans up.• Call the teacher over for team questions: <i>"No one has an idea? Should I call the teacher?"</i>
	Facilitator: <ul style="list-style-type: none">• Help your team get started by having someone read the task: <i>"Who wants to read?"</i>• Make sure everyone understands what to do: <i>"Does anyone know how to get started?"</i> <i>"What does the first question mean?"</i> <i>"I'm not sure – what are we supposed to do?"</i>• Make sure everyone understands your team's answer before you move on: <i>"Do we all agree?"</i> <i>"I'm not sure I get it yet – can someone explain?"</i>
	Recorder/Reporter: <ul style="list-style-type: none">• Share your team data with the class.• Be sure all team members have access to any team diagrams by placing them at the center of the table or desks.• Make sure your team agrees about how to show your work: <i>"How can we write this?"</i> <i>"How can we show it on the diagram?"</i>
	Task Manager: <ul style="list-style-type: none">• Make sure no one talks outside your team.• Help keep your team on-task and talking about math: <i>"Okay, let's get back to work!"</i> <i>"Let's keep working."</i>• Listen for statements and reasons: <i>"Explain how you know that."</i> <i>"Can you prove that?"</i>

7.1.3 What is the shortest distance?

Shortest Distance Problems



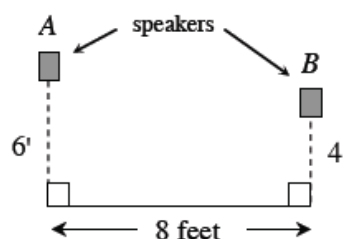
Questions such as, “What length will result in the largest area?” or “When was the car traveling the slowest?” concern *optimization*. To optimize a quantity is to find the “best” possibility. Calculus is often used to solve optimization problems, but geometric tools can sometimes offer surprisingly simple and elegant solutions.

7-21. INTERIOR DESIGN

Laura needs your help. She needs to order expensive wire to connect her sound system to her built-in speakers and would like your help to save her money.

She plans to place her sound system somewhere on a cabinet that is 8 feet wide. Speaker A is located 6 feet above one end of the cabinet, while speaker B is located 4 feet above the other end. She will need wire to connect the sound system to speaker A, and additional wire to connect it to speaker B.

Where should she place her sound system that she needs the least amount of wire?



Your Task: Before you discuss this with your team, make your own guess. What does your intuition tell you? Then, using the Lesson 7.1.3 Resource Page, work with your team to determine where on the cabinet the sound system should be placed. How can you be sure that you found the best answer? In other words, how do you know that the amount of wire you found is the least amount possible?

Discussion Points

What is this problem about? What are you supposed to find?

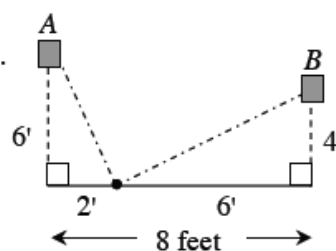
What is a reasonable estimate of the total length of speaker wire?

What mathematical tools could be helpful to solve this problem?

Further Guidance

7-22. To help solve problem 7-21, first collect some data.

- Calculate the total length of wire needed if the sound system is placed 2 feet from the left edge of the cabinet (the edge below Speaker A), as shown in the diagram at right.
- Now calculate the total length of wire needed if the sound system is placed 3 feet from the same edge. Does this placement require more or less wire than that from part (a)?
- Continue testing placements for the sound system and create a table with your results. Where should the sound system be placed to minimize the amount of wire?

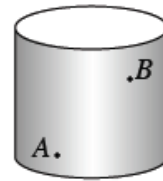


*Further Guidance
section ends here.*

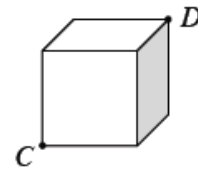
- 7-23. This problem reminds Bradley of problem 3-105, *You Are Getting Sleepy...*, in which you and a partner created two triangles by standing and gazing into a mirror. He remembered that the only way two people could see each other's eyes in the mirror was when the triangles were similar. Examine your solution to problem 7-21. Are the two triangles created by the speaker wires similar? Justify your conclusion.

- 7-24. Bradley enjoyed solving problem 7-21 so much that he decided to create other “shortest distance” problems. For each situation below, first predict where the shortest path would be using visualization and intuition. Then find a way to determine whether the path you chose is, in fact, the shortest.

- a. In this first puzzle, Bradley decided to test what would happen on the side of a cylinder, such as a soup can. On a can provided by your teacher, find points *A* and *B* labeled on the outside of the can. With your team, determine the shortest path from point *A* to point *B* along the surface of the can. (In other words, no part of your path can go inside the can.) Describe how you found your solution.



- b. What if the shape is a cube? Using a cube provided by your teacher, predict which path would be the shortest path from opposite corners of the cube (labeled points *C* and *D* in the diagram at right). Then test your prediction. Describe how you found the shortest path.



7-25. MAKING CONNECTIONS

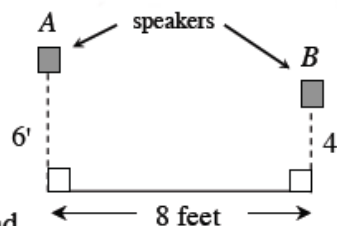
As Bradley looked over his answer from problem 7-21, he couldn't help but wonder if there is a way to change this problem into a straight-line problem like those in problem 7-24.



- a. On the Lesson 7.1.3 Resource Page, reflect one of the speakers so that when the two speakers are connected with a straight line, the line passes through the horizontal cabinet.

- b. When the speakers from part (a) are connected with a straight line, two triangles are formed. How are the two triangles related? Justify your conclusion.

- c. Use the fact that the triangles are similar to find where the sound system should be placed. Did your answer match that from problem 7-21?

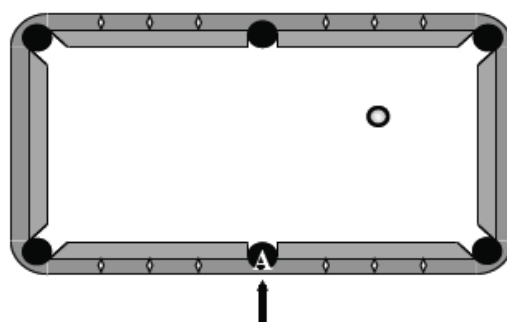


7-26. TAKE THE SHOT

While playing a game of pool, Montana Mike needed to hit the last remaining ball into pocket A, as shown in the diagram below. However, to show off, he decided to make the ball first hit at least one of the rails of the table.



Your Task: On the Lesson 7.1.3 Resource Page provided by your teacher, determine where Mike could bounce the ball off a rail so that it will land in pocket A. Work with your team to find as many possible locations as you can. Can you find a way he could hit the ball so that it would rebound twice before entering pocket A?

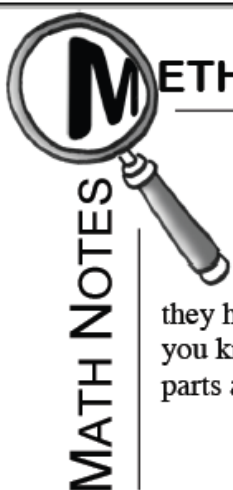


Be ready to share your solutions with the class.

7-27. LEARNING LOG

Look over your work from this lesson. What mathematical ideas did you use? What connections, if any, did you find? Can any other problems you have seen so far be solved using a straight line? Describe the mathematical ideas you developed during this lesson in your Learning Log. Title this entry "Shortest Distance" and label it with today's date.





METHODS AND MEANINGS

Congruent Triangles → Congruent Corresponding Parts

As you learned in Chapter 3, if two shapes are congruent, then they have exactly the same shape and the same size. This means that if you know two triangles are congruent, you can state that corresponding parts are congruent. This can be also stated with the arrow diagram:

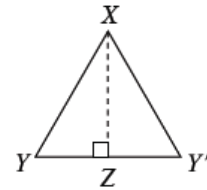
$$\cong \Delta s \rightarrow \cong \text{parts}$$

For example, if $\triangle ABC \cong \triangle PQR$, then it follows that $\angle A \cong \angle P$, $\angle B \cong \angle Q$, and $\angle C \cong \angle R$. Also, $\overline{AB} \cong \overline{PQ}$, $\overline{AC} \cong \overline{PR}$, and $\overline{BC} \cong \overline{QR}$.



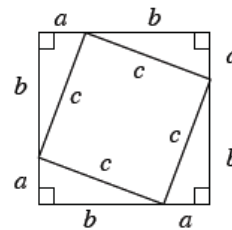
- 7-28. $\triangle XYZ$ is reflected across \overline{XZ} , as shown at right.

- How can you justify that the points Y , Z , and Y' all lie on a straight line?
- What is the relationship between $\triangle XYZ$ and $\triangle X'Y'Z'$? Why?
- Read the Math Notes box for this lesson. Then make all the statements you can about the corresponding parts of these two triangles.

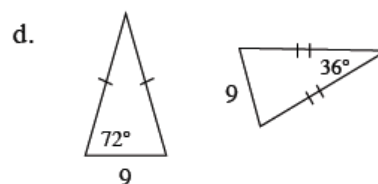
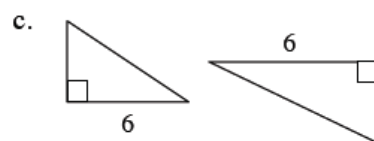
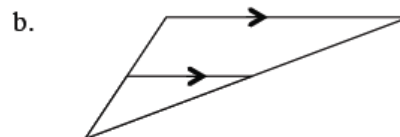
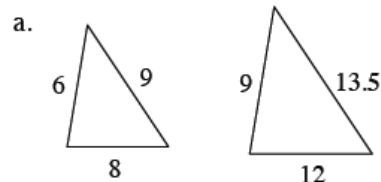


- 7-29. Remember that a midpoint of a line segment is the point that divides the segment into two segments of equal length. On graph paper, plot the points $P(0, 3)$ and $Q(0, 11)$. Where is the midpoint M if $PM = MQ$? Explain how you found your answer.

- 7-30. Examine the diagram at right. Find two equivalent expressions that represent the area of the *inner* square.



- 7-31. For each pair of triangles below, decide whether the triangles are similar and/or congruent. Justify each conclusion.



- 7-32. On graph paper, plot and connect the points $A(1, 1)$, $B(2, 3)$, $C(5, 3)$, and $D(4, 1)$ to form quadrilateral $ABCD$.

- What is the best name for quadrilateral $ABCD$? Justify your answer.
- Find and compare $m\angle DAB$ and $m\angle BCD$. What is their relationship?
- Find the equations of diagonals \overline{AC} and \overline{BD} . Are the diagonals perpendicular?
- Find the point where diagonals \overline{AC} and \overline{BD} intersect.

- 7-33. Solve each system of equations below, if possible. If it is not possible, explain what having "no solution" tells you about the graphs of the equations. Write each solution in the form (x, y) . Show all work.

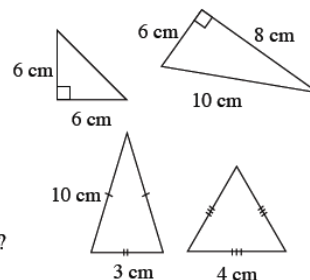
a. $y = -\frac{1}{3}x + 7$
 $y = -\frac{1}{3}x - 2$

b. $y = 2x + 3$
 $y = x^2 - 2x + 3$

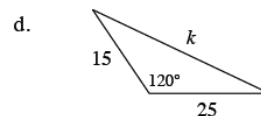
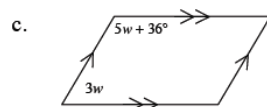
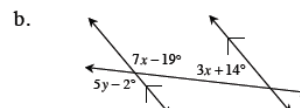
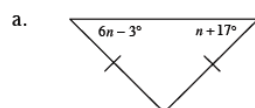
- 7-34. How long is the longest line segment that will fit inside a square of area 50 square units? Show all work.
- 7-35. Graph and connect the points $G(-2, 2)$, $H(3, 2)$, $I(6, 6)$, and $J(1, 6)$ to form $GHIJ$.
- What specific type of shape is quadrilateral $GHIJ$? Justify your conclusion.
 - Find the equations of the diagonals \overline{GI} and \overline{HJ} .
 - Compare the slopes of the diagonals. How do the diagonals of a rhombus appear to be related?
 - Find J' if quadrilateral $GHIJ$ is rotated 90° clockwise (\odot) about the origin.
 - Find the area of quadrilateral $GHIJ$.

- 7-36. The four triangles at right are placed in a bag. If you reach into the bag without looking and pull out one triangle at random, what is the probability that:

- The triangle is scalene?
- The triangle is isosceles?
- at least one side of the triangle is 6 cm?



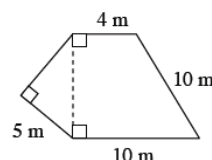
- 7-37. Examine the relationships in the diagrams below. For each one, write an equation and solve for the given variable(s). Show all work.



- 7-38. Copy the table below on your paper and complete it for the equation $y = x^2 + 2x - 3$. Then graph and connect the points on graph paper. Name the roots (x -intercepts).

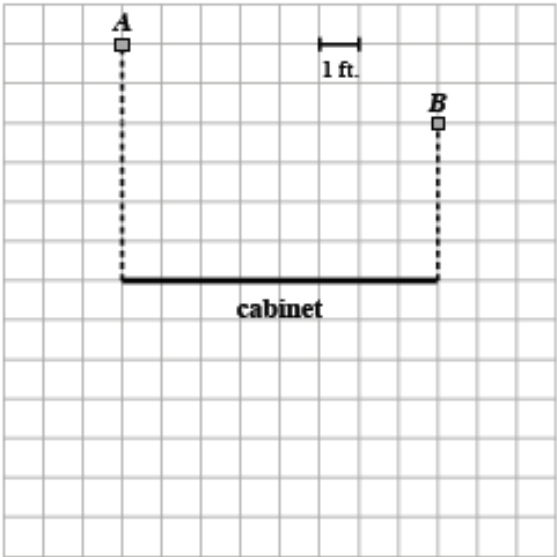
x	-4	-3	-2	-1	0	1	2
y							

- 7-39. Find the perimeter of the shape at right. Show all work.

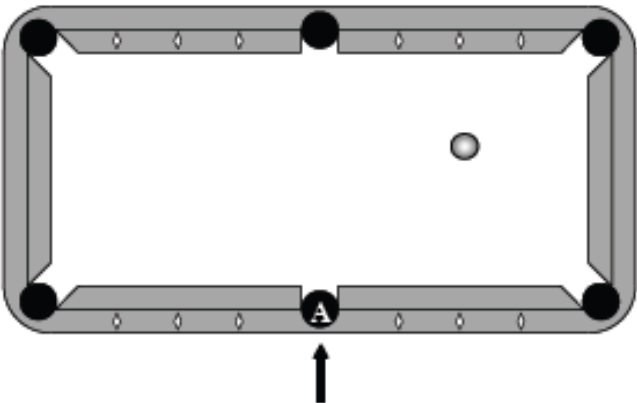


Lesson 7.1.3 Resource Page

7-21. INTERIOR DESIGN

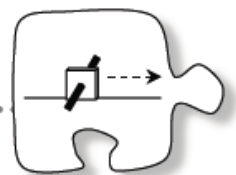


7-26. TAKE THE SHOT



7.1.4 How can I create it?

Using Symmetry to Study Polygons



In Chapter 1, you used a hinged mirror to study the special angles associated with regular polygons. In particular, you investigated what happens as the angle formed by the sides of the mirror is changed. Today, you will use a hinged mirror to determine if there is more than one way to build each regular polygon using the principles of symmetry. And what about other types of polygons? What can a hinged mirror help you understand about them?

As your work with your study team, keep these focus questions in mind:

Is there another way?

What types of symmetry can I find?

What does symmetry tell me about the polygon?

7-40. THE HINGED MIRROR TEAM CHALLENGE

Obtain a hinged mirror, a piece of unlined colored paper, and a protractor from your teacher.

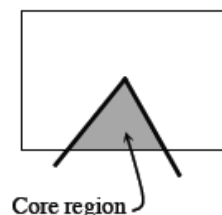
With your team, spend five minutes reviewing how to use the mirror to create regular polygons.

Remember that a **regular polygon** has equal sides and angles. Once everyone remembers how the hinged mirror works, select a team member to read the directions of the task below.



Your Task: Below are four challenges for your team. Each requires you to find a creative way to position the mirror in relation to the colored paper. You can tackle the challenges in any order, but you must work together as a team on each of them. Whenever you successfully create a shape, do not forget to measure the angle formed by the mirror, as well as draw a diagram on your paper of the core region in front of the mirror. If your team decides that a shape is impossible to create with the hinged mirror, explain why.

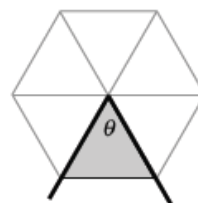
- Create a regular hexagon.
- Create an equilateral triangle at least *two* different ways.
- Create a rhombus that is *not* a square.
- Create a circle.



7-41. ANALYSIS

How can symmetry help you to learn more about shapes? Discuss each question below with the class.

- a. One way to create a regular hexagon with a hinged mirror is with six triangles, as shown in the diagram at right. Note: The gray lines represent reflections of the bottom edges of the mirrors and the edge of the paper, while the core region is shaded.

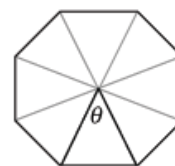


What is special about each of the triangles in the diagram? What is the relationship between the triangles? Support your conclusions. Would it be possible to create a regular hexagon with 12 triangles? Explain.

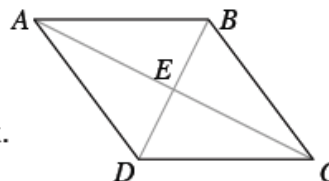
- b. If you have not done so already, create an equilateral triangle so that the core region in front of the mirror is a right triangle. Draw a diagram of the result that shows the different reflected triangles like the one above. What special type of right triangle is the core region? Can all regular polygons be created with a right triangle in a similar fashion?
- c. In problem 7-40, your team formed a rhombus that is not a square. On your paper, draw a diagram like the one above that shows how you did it. How can you be sure your resulting shape is a rhombus? Using what you know about the angle of the mirror, explain what must be true about the diagonals of a rhombus.


7-42. Use what you learned today to answer the questions below.

- a. Examine the regular octagon at right. What is the measure of angle θ ? Explain how you know.



- b. Quadrilateral $ABCD$ at right is a rhombus. If $BD = 10$ units and $AC = 18$ units, then what is the perimeter of $ABCD$? Show all work.




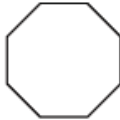





MATH NOTES

METHODS AND MEANINGS

Regular Polygons

A polygon is **regular** if all its sides are congruent and its angles have equal measure. An equilateral triangle and a square are each regular polygons since they are both *equilateral* and *equiangular*. See the diagrams of common regular polygons below.



Equilateral Triangle

Square

Regular Hexagon

Regular Octagon

Regular Decagon



- 7-43. Felipe set his hinged mirror so that its angle was 36° and the core region was isosceles, as shown at right.

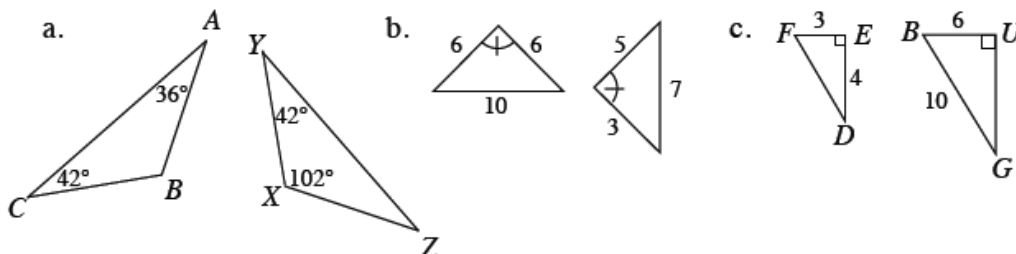
- How many sides did his resulting polygon have? Show how you know.
- What is another name for this polygon?



- 7-44. In problem 7-41 you learned that the diagonals of a rhombus are perpendicular bisectors. If $ABCD$ is a rhombus with side length 15 mm and if $BD = 24$ mm, then find the length of the other diagonal, \overline{AC} . Draw a diagram and show all work.

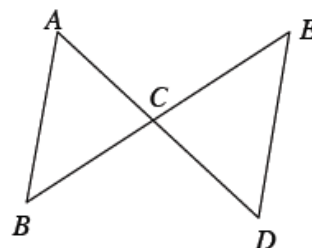
- 7-45. Joanne claims that $(2, 4)$ is the midpoint of the segment connecting the points $(-3, 5)$ and $(7, 3)$. Is she correct? Explain how you know.

- 7-46. For each pair of triangles below, determine whether or not the triangles are similar. If they are similar, show your reasoning in a flowchart. If they are not similar, explain how you know.



- 7-47. If $\triangle ABC \cong \triangle DEC$, which of the statements below must be true? Justify your conclusion. Note: More than one statement may be true.

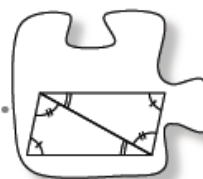
- $\overline{AC} \cong \overline{DC}$
- $m\angle B = m\angle D$
- $\overline{AB} \parallel \overline{DE}$
- $AD = BE$
- None of these are true.



- 7-48. On graph paper, graph the points $A(2, 9)$, $B(4, 3)$, and $C(9, 6)$. Which point (A or C) is closer to point B ? Justify your conclusion.

7.2.1 What can congruent triangles tell me?

Special Quadrilaterals and Proof



In earlier chapters you studied the relationships between the sides and angles of a triangle, and solved problems involving congruent and similar triangles. Now you are going to expand your study of shapes to quadrilaterals. What can triangles tell you about parallelograms and other special quadrilaterals?

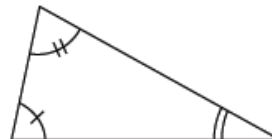
By the end of this lesson, you should be able to answer these questions:

What are the relationships between the sides, angles, and diagonals of a parallelogram?

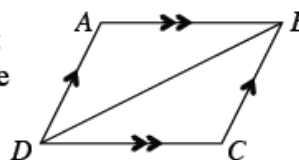
How are congruent triangles useful?

- 7-49. Carla is thinking about parallelograms and wondering if there are as many special properties for parallelograms as there are for triangles. She remembers that it is possible to create a shape that looks like a parallelogram by rotating a triangle about the midpoint of one of its sides.

- a. Carefully trace the triangle at right onto tracing paper. Be sure to copy the angle markings as well. Then rotate the triangle about a midpoint of a side to make a shape that looks like a parallelogram.



- b. Is Carla's shape truly a parallelogram? Use the angles to convince your teammates that the opposite sides must be parallel. Then write a convincing argument.
- c. What else can the congruent triangles tell you about a parallelogram? Look for any relationships you can find between the angles and sides of a parallelogram.
- d. Does this work for all parallelograms? That is, does the diagonal of a parallelogram always split the shape into two congruent triangles? Draw the parallelogram at right on your paper. Knowing only that the opposite sides of a parallelogram are parallel, create a flowchart to show that the triangles are congruent.



7-50. CHANGING A FLOWCHART INTO A PROOF

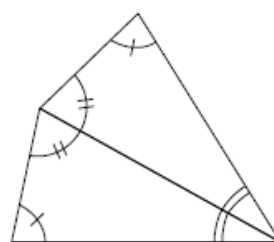
The flowchart you created for part (d) of problem 7-49 shows how you can conclude that if a quadrilateral is a parallelogram, then its each of its diagonals splits the quadrilateral into two congruent triangles.

However, to be convincing, the facts that you listed in your flowchart need to have justifications. This shows the reader how you know the facts are true and helps to prove your conclusion.

Therefore, with the class or your team, decide how to add reasons to each statement (bubble) in your flowchart. You may need to add more bubbles to your flowchart to add justification and to make your proof more convincing.

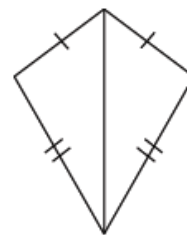
- 7-51. Kip is confused. He put his two triangles from problem 7-49 together as shown at right, but he did not get a parallelogram.

- What shape did he make? Justify your conclusion.
- What transformation(s) did Kip use to form his shape?
- What do the congruent triangles tell you about the angles of this shape?

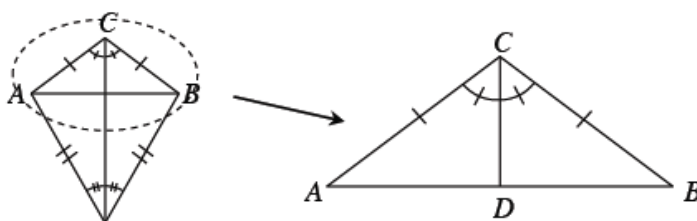


7-52. KITES

Kip shared his findings about his kite with his teammate, Carla, who wants to learn more about the diagonals of a kite. Carla quickly sketched the kite at right onto her paper with a diagonal showing the two congruent triangles.



- EXPLORE:** Trace this diagram onto tracing paper and carefully add the other diagonal. Then, with your team, consider how the diagonals may be related. Use tracing paper to help you explore the relationships between the diagonals. If you make an observation you think is true, move on to part (b) and write a conjecture.
- CONJECTURE:** If you have not already done so, write a conjecture based on your observations in part (a).
- PROVE:** When she drew the second diagonal, Carla noticed that four new triangles appeared. “If any of these triangles are congruent, then they may be able to help us prove our conjecture from part (b),” she said. Examine $\triangle ABC$ below. Are $\triangle ACD$ and $\triangle BCD$ congruent? Create a flowchart proof like the one from problem 7-50 to justify your conclusion.



- Now extend your proof from part (c) to prove your conjecture from part (b).

- 7-53. Reflect on all of the interesting facts about parallelograms and kites you have proven during this lesson. Obtain a Theorem Toolkit (Lesson 7.2.1A Resource Page) from your teacher. On it, record each **theorem** (proven conjecture) that you have proven about the sides, angles, and diagonals of a parallelogram in this lesson. Do the same for a kite. Be sure your diagrams contain appropriate markings to represent equal parts.



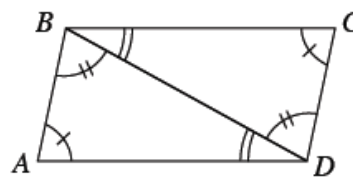


METHODS AND MEANINGS

Reflexive Property of Equality

In this lesson, you used the fact that two triangles formed by the diagonal of a parallelogram share a side of the same length to help show that the triangles were congruent.

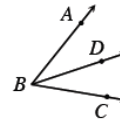
The **Reflexive Property of Equality** states that the measure of any side or angle is equal to itself. For example, in the parallelogram at right, $\overline{BD} \cong \overline{DB}$ because of the Reflexive Property.



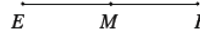
Review & Preview

- 7-54. Use the information given for each diagram below to solve for x . Show all work.

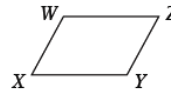
- a. \overline{BD} bisects $\angle ABC$. (Remember that this means it divides the angle into two equal parts.) If $m\angle ABD = 5x - 10^\circ$ and $m\angle ABC = 65^\circ$, solve for x .



- b. Point M is a midpoint of \overline{EF} . If $EM = 4x - 2$ and $MF = 3x + 9$, solve for x .



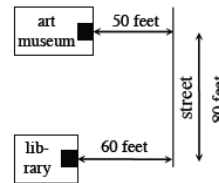
- c. $WXYZ$ at right is a parallelogram. If $m\angle W = 9x - 3^\circ$ and $m\angle Z = 3x + 15^\circ$, solve for x .



- 7-55. Jamal used a hinged mirror to create a regular polygon like you did in Lesson 7.1.4.

- a. If his hinged mirror formed a 72° angle and the core region in front of the mirror was isosceles, how many sides did his polygon have?
- b. Now Jamal has decided to create a regular polygon with 9 sides, called a nonagon. If his core region is again isosceles, what angle is formed by his mirror?

- 7-56. Sandra wants to park her car so that she optimizes the distance she has to walk to the art museum and the library. That is, she wants to park so that her total distance directly to each building is the shortest. Find where she should park.



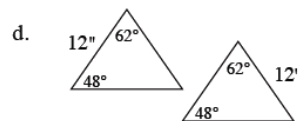
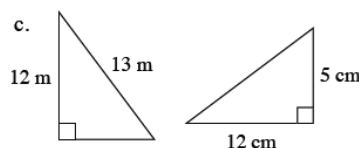
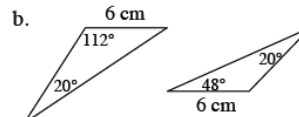
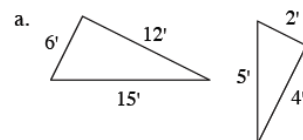
- 7-57. Write an expression for the following sequences.

- a. 40, 60, 80, ... b. $3, \frac{3}{2}, \frac{3}{4}, \dots$

- 7-58. Earl (from Chapter 6) still hates to wash the dishes and take out the garbage. He found his own weighted coin, one that would randomly land on heads 30% of the time. He will flip a coin once for each chore and will perform the chore if the coin lands on heads.

- a. What is the probability that Earl will get out of doing both chores?
- b. What is the probability that Earl will have to take out the garbage, but will not need to wash the dishes?



- 7-59. Which pairs of triangles below are congruent and/or similar? For each part, explain how you know using an appropriate triangle congruence or similarity condition. Note: The diagrams are not necessarily drawn to scale.



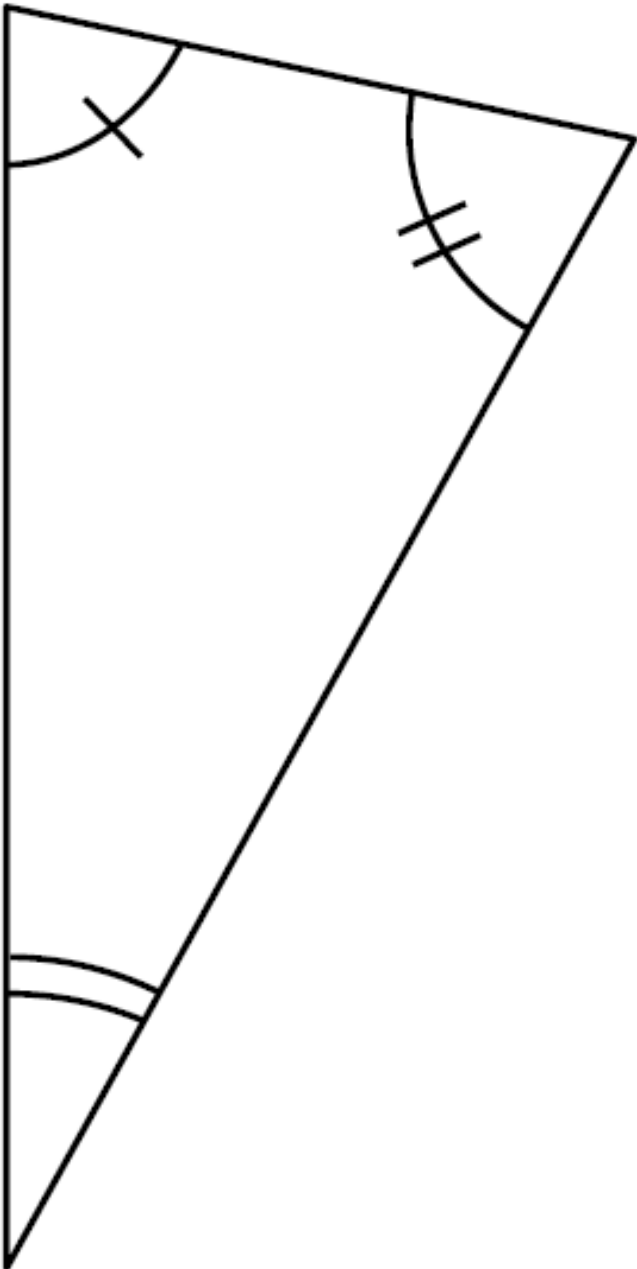
- 7-60. For part (b) of problem 7-59, explain how the triangles are congruent using a sequence of rigid transformations.

Lesson 7.2.1A Resource Page

Theorem Toolkit

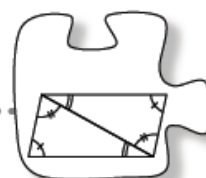
Parallelograms 	Kites 

Lesson 7.2.1B Resource Page



7.2.2 What is special about a rhombus?

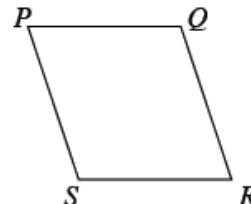
Properties of Rhombi



In Lesson 7.2.1, you learned that congruent triangles can be a useful tool to discover new information about parallelograms and kites. But what about other quadrilaterals? Today you will use congruent triangles to investigate and prove special properties of rhombi (the plural of rhombus). At the same time, you will continue to develop your ability to make conjectures and prove them convincingly.

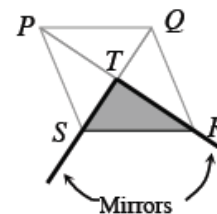
- 7-61. Audrey has a favorite quadrilateral – the rhombus. Even though a rhombus is defined as having four congruent sides, she suspects that the sides of a rhombus have other special properties.

- EXPLORE:** Draw a rhombus like the one at right on your paper. Mark the side lengths equal.
- CONJECTURE:** What else might be special about the sides of a rhombus? Write a conjecture.
- PROVE:** Audrey knows congruent triangles can help prove other properties about quadrilaterals. She starts by adding a diagonal \overline{PR} to her diagram so that two triangles are formed. Add this diagonal to your diagram and prove that the created triangles are congruent. Then use a flowchart with reasons to show your logic. Be prepared to share your flowchart with the class.
- How can the triangles from part (c) help you prove your conjecture from part (b) above? Discuss with the class how to extend your flowchart to convince others. Be sure to justify any new statements with reasons.



7-62. Now that you know the opposite sides of a rhombus are parallel, what else can you prove about a rhombus? Consider this as you answer the questions below.

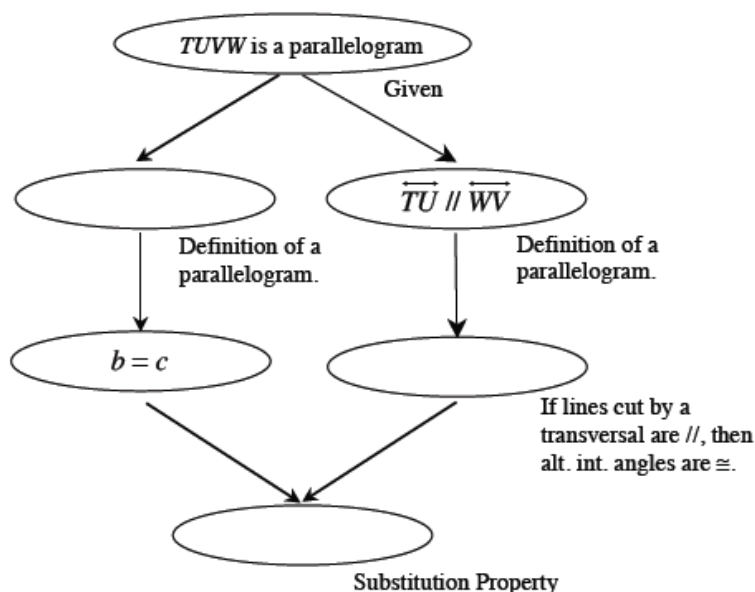
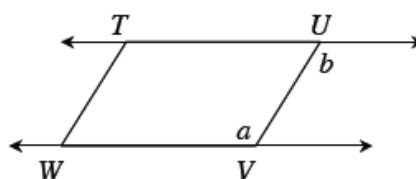
- a. **EXPLORE:** Remember that in Lesson 7.1.4, you explored the shapes that could be formed with a hinged mirror. During this activity, you used symmetry to form a rhombus. Think about what you know about the reflected triangles in the diagram. What do you think is true about the diagonals \overline{SQ} and \overline{PR} ? What is special about \overline{ST} and \overline{QT} ? What about \overline{PT} and \overline{RT} ?



- b. **CONJECTURE:** Use your observations from part (a) to write a conjecture on the relationship of the diagonals of a rhombus.
- c. **PROVE:** Write a flowchart proof that proves your conjecture from part (b). Remember that to be convincing, you need to justify each statement with a reason. To help guide your discussion, consider the questions below. Which triangles should you use? Find two triangles that involve the segments \overline{ST} , \overline{QT} , \overline{PT} , and \overline{RT} .
- How can you prove these triangles are congruent? Create a flowchart proof with reasons to prove these triangles must be congruent.
 - How can you use the congruent triangles to prove your conjecture from part (b)? Extend your flowchart proof to include this reasoning and prove your conjecture.

- 7-63. There are often many ways to prove a conjecture. You have rotated triangles to create parallelograms and used congruent parts of congruent triangles to justify that opposite sides are parallel. But is there another way?

Ansel wants to prove the conjecture “*If a quadrilateral is a parallelogram, then opposite angles are congruent.*” He started by drawing parallelogram $TUVW$ at right. Copy and complete his flowchart. Make sure that each statement has a reason.



- 7-64. Think about the new facts you have proven about rhombi during this lesson. On your Theorem Toolkit (Lesson 7.2.1A Resource Page), record each new theorem you have proven about the angles and diagonals of a rhombus. Include clearly labeled diagrams to illustrate your findings.




MATH NOTES

METHODS AND MEANINGS

Exponential Functions

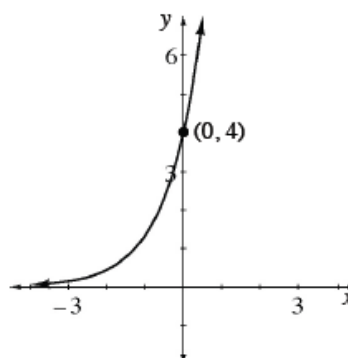
An **exponential function** has the general form $y = a \cdot b^x$, where a is the **initial value** (the y -intercept) and b is the **multiplier** (the growth). Be careful: The independent variable x has to be in the exponent. For example, $y = x^2$ is *not* an exponential equation, even though it has an exponent.

For example, in the multiple representations below, the y -intercept is $(0, 4)$ and the growth factor is 3 because the y -value is increasing by multiplying by 3.

$$y = 4 \cdot 3^x$$

x	y
-3	$\frac{4}{3^3}$ or $\frac{4}{27}$
-2	$\frac{4}{3^2}$ or $\frac{4}{9}$
-1	$\frac{4}{3}$
0	4
1	12
2	36
3	108

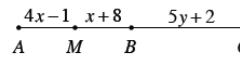
$\times 3$
 $\times 3$



To increase or decrease a quantity by a percentage, use the multiplier for that percentage. For example, the multiplier for an increase of 7% is $100\% + 7\% = 1.07$. The multiplier for a decrease of 7% is $100\% - 7\% = 0.93$.



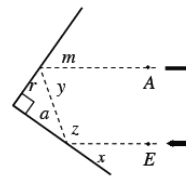
- 7-65. Point M is the midpoint of \overline{AB} and B is the midpoint of \overline{AC} . What are the values of x and y ? Show all work and reasoning.



- 7-66. Read the Math Notes box in this lesson and then answer following questions. The cost of large flat-screen televisions is decreasing 20% per year.

- What is the multiplier?
- If a 50-inch flat-screen now costs \$1200, what will it cost in three years?
- At the same rate, what did it cost two years ago?

- 7-67. A light directed from point E is pointed at a hinged mirror with right angle as shown at right.

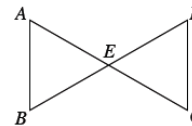


- If $\angle x$ measures 36° , find the measures of $\angle a$, $\angle r$, $\angle m$, $\angle y$, and $\angle z$.
- Why must the arrows at points A and E be parallel?

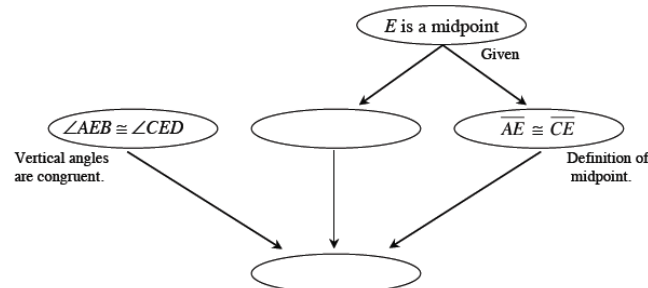
- 7-68. On graph paper, graph quadrilateral $MNPQ$ if $M(-3, 6)$, $N(2, 8)$, $P(1, 5)$, and $Q(-4, 3)$.

- What shape is $MNPQ$? Show how you know.
- Use the function $x \rightarrow x$, $y \rightarrow -y$ to reflect $MNPQ$ across the x -axis and create $M'N'P'Q'$. What are the coordinates of P' ?

- 7-69. Jester started to prove that the triangles at right are congruent. He was only told that point E is the midpoint of segments \overline{AC} and \overline{BD} .



Copy and complete his flowchart below. Be sure that a reason is provided for every statement.



- 7-70. For a school fair, Donny is going to design a spinner with red, white, and blue regions. Since he has a certain proportion of three types of prizes, he wants the $P(\text{red}) = 40\%$ and $P(\text{white}) = 10\%$.



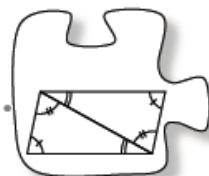
- If the spinner only has red, white, and blue regions, then what is $P(\text{blue})$? Explain how you know.
- Find the central angles of this spinner if it has only three sections. Then draw a sketch of the spinner. Be sure to label the regions accurately.
- Is there a different spinner that has the same probabilities? If so, sketch another spinner that has the same probabilities. If not, explain why there is no other spinner with the same probabilities.

- 7-71. On graph paper, graph and shade the solutions for the inequality below.

$$y < -\frac{2}{3}x + 5$$

7.2.3 What else can be proved?

More Proofs with Congruent Triangles



In Lessons 7.2.1 and 7.2.2, you used congruent triangles to learn more about parallelograms, kites, and rhombi. You now possess the tools to do the work of a geometer (someone who studies geometry): to discover and prove new properties about the sides and angles of shapes.

As you investigate these shapes, focus on proving your ideas. Remember to ask yourself and your teammates questions such as, “*Why does that work?*” and “*Is it always true?*” Decide whether your argument is convincing and work with your team to provide all of the necessary justification.

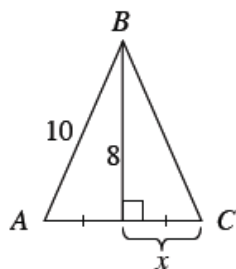
- 7-72. Carla decided to turn her attention to rectangles. Knowing that a rectangle is defined as a quadrilateral with four right angles, she drew the diagram at right.

After some exploration, she conjectured that all rectangles are also parallelograms. Help her prove that her rectangle $ABCD$ must be a parallelogram. That is, prove that the opposite sides must be parallel. Then add this theorem to your Theorem Toolkit (Lesson 7.2.1A Resource Page).

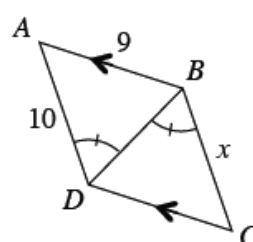


- 7-73. For each diagram below, find the value of x , if possible. If the triangles are congruent, state which triangle congruence condition was used. If the triangles are not congruent or if there is not enough information, state, "Cannot be determined."

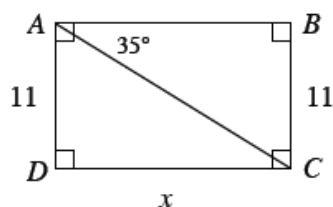
- a. ABC below is a triangle.



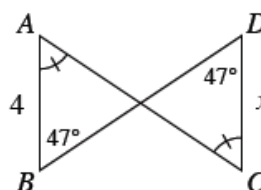
- b.



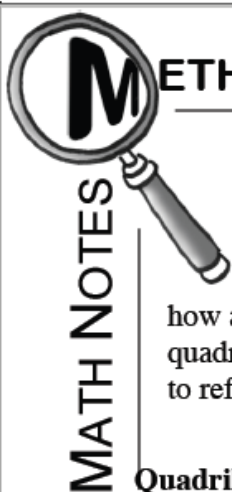
- c.



- d. \overline{AC} and \overline{BD} are straight line segments.



- 7-74. With the class or your team, create a flowchart to prove your answer to part (b) of problem 7-73. That is, prove that $\overline{AD} \cong \overline{CB}$. Be sure to include a diagram for your proof and reasons for every statement. Make sure your argument is convincing and has no “holes.”



METHODS AND MEANINGS

Definitions of Quadrilaterals

When proving properties of shapes, it is necessary to know exactly how a shape is defined. Below are the definitions of several quadrilaterals that you developed in Lesson 1.3.2 and that you will need to refer to in this chapter and the chapters that follow.

Quadrilateral: A closed four-sided polygon.

Kite: A quadrilateral with two distinct pairs of consecutive congruent sides.

Trapezoid: A quadrilateral with at least one pair of parallel sides.

Parallelogram: A quadrilateral with two pairs of parallel sides.

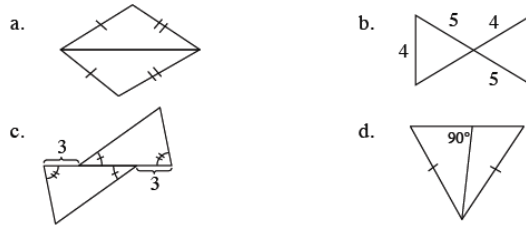
Rhombus: A quadrilateral with four sides of equal length.

Rectangle: A quadrilateral with four right angles.

Square: A quadrilateral with four sides of equal length and four right angles.

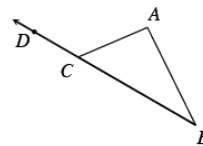


- 7-75. Identify if each pair of triangles below is congruent or not. Remember that the diagram may not be drawn to scale. Justify your conclusion.



- 7-76. For either part (a), (c) or (d) of problem 7-75, create a flowchart to prove your conclusion. Remember to start with the given information and include a reason or justification for each “bubble” in your flowchart.

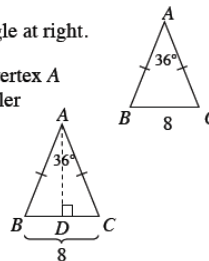
- 7-77. In the diagram at right, $\angle DCA$ is referred to as an **exterior angle** of $\triangle ABC$ because it lies outside the triangle and is formed by extending a side of the triangle.



- If $m\angle CAB = 46^\circ$ and $m\angle ABC = 37^\circ$, what is $m\angle DCA$? Show all work.
- If $m\angle DCA = 135^\circ$ and $m\angle ABC = 43^\circ$, then what is $m\angle CAB$?

- 7-78. Tromika wants to find the area of the isosceles triangle at right.

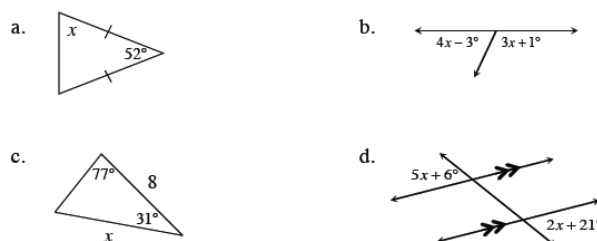
- She decided to start by drawing a height from vertex A to side \overline{BC} as shown below. Will the two smaller triangles be congruent? In other words, is $\triangle ABC \cong \triangle ABD$? Why or why not?
- What is $m\angle DAB$? BD ?
- Find AD . Show how you got your answer.
- Find the area of $\triangle ABC$.



- 7-79. On graph paper, graph quadrilateral $ABCD$ if $A(0, 0)$, $B(6, 0)$, $C(8, 6)$, and $D(2, 6)$.
- What is the best name for $ABCD$? Justify your answer.
 - Find the equation of the lines containing each diagonal. That is, find the equations of lines \overline{AC} and \overline{BD} .
- 7-80. It is often useful to estimate the value of a square root to determine if your answer is reasonable.
- What would be a reasonable estimate of $\sqrt{68}$? Explain your thinking. After you have made an estimate, check your estimation with a calculator.
 - Repeat this process to estimate the values below.

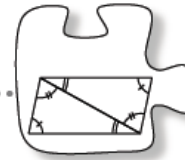
(1) $\sqrt{5}$	(2) $\sqrt{85}$	(3) $\sqrt{50}$	(4) $\sqrt{22}$
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- 7-81. For each diagram below, solve for x . Show all work.



7.2.4 What else can I prove?

More Properties of Quadrilaterals



Today you will work with your team to apply what you have learned to other shapes. Remember to ask yourself and your teammates questions such as, “*Why does that work?*” and “*Is it always true?*” Decide whether your argument is convincing and work with your team to provide all of the necessary justification. By the end of this lesson, you should have a well-crafted mathematical argument proving something new about a familiar quadrilateral.

7-82. WHAT ELSE CAN CONGRUENT TRIANGLES TELL US?

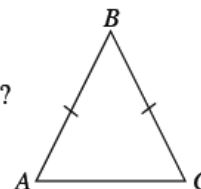
Your Task: For each situation below, determine how congruent triangles can tell you more information about the shape. Then prove your conjecture using a flowchart. Be sure to provide a reason for each statement. For example, stating “ $m\angle A = m\angle B$ ” is not enough. You must give a convincing reason, such as “*Because vertical angles are equal*” or “*Because it is given in the diagram.*” Use your triangle congruence conditions to help prove that the triangles are congruent.



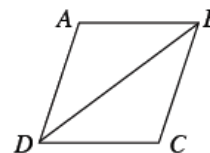
Later, your teacher will select one of these flowcharts for you to place on a poster. On your poster, include a diagram and all of your statements and reasons. Clearly state what you are proving and help the people who look at your poster understand your logic and reasoning.

- a. In Chapter 1, you used the symmetry of an isosceles triangle to show that the base angles must be congruent. How can you prove this result using congruent triangles?

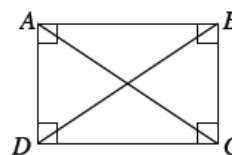
Assume that $\overline{AB} \cong \overline{CB}$ for the triangle at right. With your team, decide how to split $\triangle ABC$ into two triangles that you can show are congruent to show that $\angle BAC \cong \angle BCA$.



- b. What can congruent triangles tell us about the diagonals and angles of a rhombus? Examine the diagram of the rhombus at right. With your team, decide how to prove that the diagonals of a rhombus bisect the angles. That is, prove that $\angle ABD \cong \angle CBD$.



- c. What can congruent triangles tell us about the diagonals of a rectangle? Examine the rectangle at right. Using the fact that the opposite sides of a rectangle are parallel (which you proved in problem 7-72), prove that the diagonals of the rectangle are congruent. That is, prove that $AC = BD$.



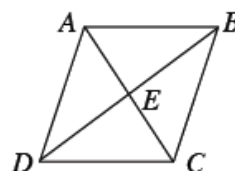


MATH NOTES

METHODS AND MEANINGS

Diagonals of a Rhombus

A **rhombus** is defined as a quadrilateral with four sides of equal length. In addition, you proved in problem 7-62 that the diagonals of a rhombus are perpendicular bisectors of each other.



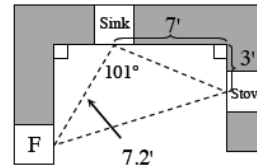
For example, in the rhombus at right, E is a midpoint of both \overline{AC} and \overline{DB} . Therefore, $AE = CE$ and $DE = BE$. Also, $m\angle AEB = m\angle BEC = m\angle CED = m\angle DEA = 90^\circ$.

In addition, you proved in problem 7-82 that the diagonals bisect the angles of the rhombus. For example, in the diagram above, $m\angle DAE = m\angle BAE$.

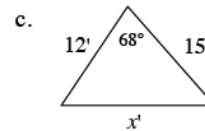
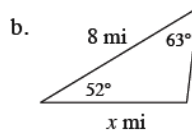
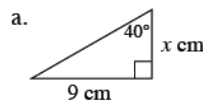


- 7-83. Use Tromika's method from problem 7-78 to find the area of an equilateral triangle with side length 12 units. Show all work.

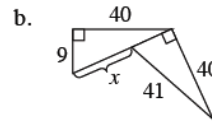
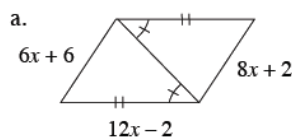
- 7-84. The guidelines set forth by the National Kitchen & Bath Association recommends that the perimeter of the triangle connecting the refrigerator (F), stove, and sink of a kitchen be 26 feet or less. Lashayia is planning to renovate her kitchen and has chosen the design at right. Does her design conform to the National Kitchen and Bath Association's guidelines? Show how you got your answer.



- 7-85. Examine the triangles below. For each, solve for x and name which tool you use. Show all work.



- 7-86. For each figure below, determine if the two smaller triangles in each figure are congruent. If so, create a flowchart to explain why. Then, solve for x . If the triangles are not congruent, explain why not.



- 7-87. The diagonals of a rhombus are 6 units and 8 units long. What is the area of the rhombus? Draw a diagram and show all reasoning.

- 7-88. Kendrick is frantic. He remembers that several years ago he buried his Amazing Electron Ring in his little sister's sandbox, but he cannot remember where. A few minutes ago he heard that someone is willing to pay \$1000 for it. He has his shovel ar



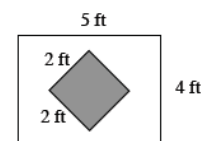
- 7-89. What is the 50th term in this sequence?

- a. The sar
5 feet, ;
time to search in the 2 root-square shaded
region, what is the probability that he will find
the ring?



- b. What is the probability that he will not find the ring?
Explain how you found your answer.

- c. Kendrick decides instead to dig in the square
region shaded at right. Does this improve his
chances for finding the ring? Why or why not?

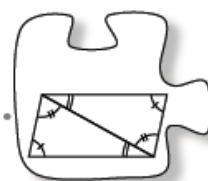


- 7-89. What is the 50th term in this sequence?

17, 14, 11, 8, ...

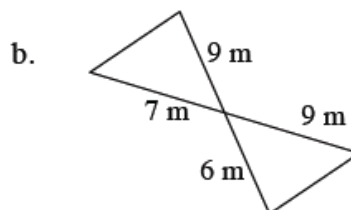
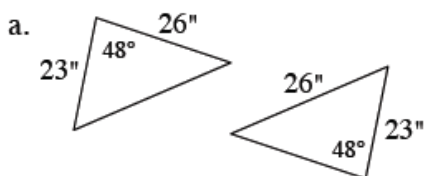
7.2.5 How else can I write it?

Two-Column Proofs

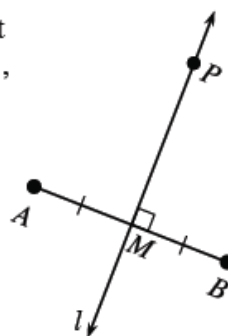


Today you will continue to work with constructing a convincing argument, otherwise known as writing a proof. In this lesson, you will use what you know about flowchart proofs to write a convincing argument using another format, called a two-column proof.

- 7-90. The following pairs of triangles are not necessarily congruent even though they appear to be. Use the information provided in the diagram to show why. Justify your statements.

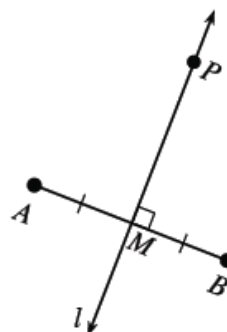


- 7-91. Write a flowchart to prove that if point P on line l (not on \overline{AB}) is a point on the perpendicular bisector of \overline{AB} , then $\overline{PA} \cong \overline{PB}$. That is, point P is the same distance from points A and B (called "*equidistant*" in mathematics). Assume the intersection of \overline{AB} and line l is point M as shown in the diagram.



- 7-92. Another way to organize a proof is called a **two-column proof**. Instead of using arrows to indicate the order of logical reasoning, this style of proof lists statements and reasons in a linear order, first to last, in columns.

The proof from problem 7-91 has been converted to a two-column proof below. Copy and complete the proof on your paper using your statements and reasons from problem 7-91.



If: M is on \overline{AB} and \overline{PM} is the perpendicular bisector of \overline{AB}

Prove: $\overline{PA} \cong \overline{PB}$

Statements	Reasons (This statement is true because...)
Point M is on \overline{AB} and \overline{PM} is the perpendicular bisector of \overline{AB} .	Given
$m\angle PMB = m\angle PMB = 90^\circ$	Definition of perpendicular and angles with the same measure are congruent.
	Definition of a bisector.
$\overline{PM} \cong \overline{PM}$	

- 7-93. Examine the posters of flowchart proofs from problem 7-82. Convert each flowchart proof to a two-column proof. Remember that one column must contain the ordered statements of fact while the other must provide the reason (or justification) explaining why that fact must be true.

- 7-94. So far in Section 7.2, you have proven many special properties of quadrilaterals and other shapes. Remember that when a conjecture is proven, it is called a theorem. For example, once you proved the relationship between the lengths of the sides of a right triangle, you were able to refer to that relationship as the Pythagorean Theorem. Find your Theorem Toolkit (Lesson 7.2.1A Resource Page) and make sure it contains all of the theorems you and your classmates have proven so far about various quadrilaterals. Be sure that your records include diagrams for each statement.



7-95. LEARNING LOG

Reflect on the new proof format you learned today. Compare it to the flowchart proof format that you have used earlier. What are the strengths and weaknesses of each style of proof? Which format is easier for you to use? Which is easier to read? Title this entry “Two-Column Proofs” and label it with today’s date.



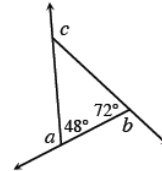


- 7-96. Suppose you know that $\triangle TAP \cong \triangle DOG$ where $TA = 14$, $AP = 18$, $TP = 21$, and $DG = 2y + 7$.

- On your paper, draw a reasonable sketch of $\triangle TAP$ and $\triangle DOG$.
- Find y . Show all work.

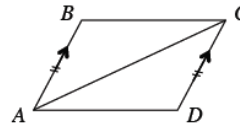
- 7-97. Graph the equation $y = -\frac{3}{2}x + 6$ on graph paper. Label the points where the line intersects the x - and y -axes.

- 7-98. $\angle a$, $\angle b$, and $\angle c$ are exterior angles of the triangle at right. Find $m\angle a$, $m\angle b$, and $m\angle c$. Then find $m\angle a + m\angle b + m\angle c$.



- 7-99. The principal's new car cost \$35,000 but in three years it will only be worth \$21,494. What is the annual percent of decrease?

- 7-100. What else can you prove about parallelograms? Prove that if a pair of opposite sides of a quadrilateral are congruent and parallel, then the quadrilateral must be a parallelogram. For example, for the quadrilateral $ABCD$ at right, given that $\overline{AB} \parallel \overline{CD}$ and $\overline{AB} \cong \overline{CD}$, show that $\overline{BC} \parallel \overline{AD}$. Organize your reasoning in a flowchart. Then record your theorem in your Theorem Toolkit (Lesson 7.2.1A Resource Page).



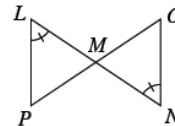
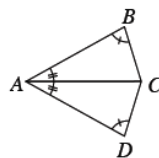
- 7-101. As Ms. Dorman looked from the window of her third-story classroom, she noticed Pam in the courtyard. Ms. Dorman's eyes were 52 feet above ground and Pam was 38 feet from the building. Draw a diagram of this situation. What is the angle at which Ms. Dorman had to look down, that is, what is the angle of depression? (Assume that Ms. Dorman was looking at the spot on the ground below Pam.)

- 7-102. For each pair of triangles below, determine if the triangles are congruent. If the triangles are congruent,

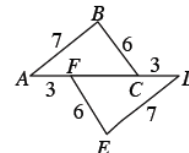
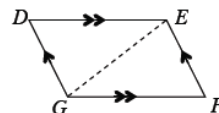
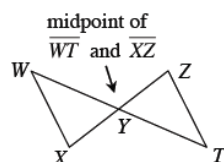
- complete the correspondence statement,
- state the congruence property,
- and record any other ideas you use that make your conclusion true.

Otherwise, explain why you cannot conclude that the triangles are congruent. Note that the figures are not necessarily drawn to scale.

- a. $\triangle ABC \cong \triangle \underline{\hspace{1cm}}$ b. $\triangle SQP \cong \triangle \underline{\hspace{1cm}}$ c. $\triangle PLM \cong \triangle \underline{\hspace{1cm}}$

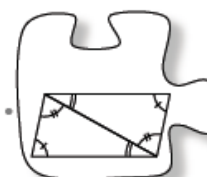


- d. $\triangle WXY \cong \triangle \underline{\hspace{1cm}}$ e. $\triangle EDG \cong \triangle \underline{\hspace{1cm}}$ f. $\triangle ABC \cong \triangle \underline{\hspace{1cm}}$



7.2.6 What can I prove?

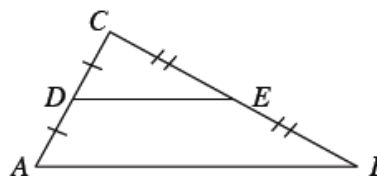
Explore-Conjecture-Prove



So far, congruent triangles have helped you to discover and prove many new facts about triangles and quadrilaterals. But what else can you discover and prove? Today your work will mirror the real work of professional mathematicians. You will investigate relationships, write a conjecture based on your observations, and then prove your conjecture.

7-103. TRIANGLE MIDSEGMENT THEOREM

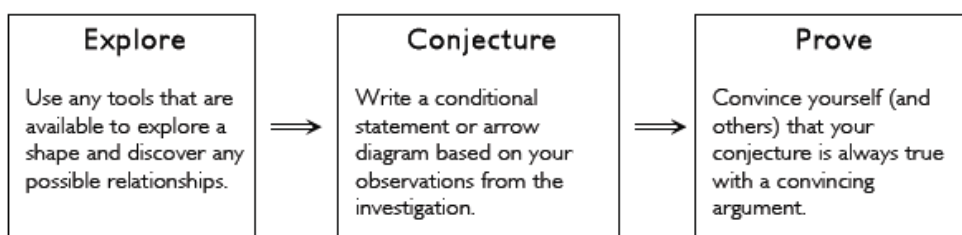
As Sergio was drawing shapes on his paper, he drew a line segment that connected the midpoints of two sides of a triangle. This is called the **midsegment** of a triangle. “*I wonder what we can find out about this midsegment,*” he said to his team. Examine his drawing at right.



- EXPLORE:** Examine the diagram of $\triangle ABC$, drawn to scale above. How do you think \overline{DE} is related to \overline{AB} ? How do their lengths seem to be related?
- CONJECTURE:** Write a conjecture about the relationship between segments \overline{DE} and \overline{AB} .
- PROVE:** Sergio wants to prove that $AB = 2BE$. However, he does not see any congruent triangles in the diagram. How are the triangles in this diagram related? How do you know? Prove your conclusion with a flowchart.
- What is the common ratio between side lengths in the similar triangles? Use this to write a statement relating lengths DE and AB .
- Now Sergio wants to prove that $\overline{DE} \parallel \overline{AB}$. Use the similar triangles to find all the pairs of equal angles you can in the diagram. Then use your knowledge of angle relationships to make a statement about parallel segments.



- 7-104. The work you did in problem 7-103 mirrors the work of many professional mathematicians. In the problem, Sergio examined a geometric shape and thought there might be something new to learn. You then helped him by finding possible relationships and writing a conjecture. Then, to find out if the conjecture was true for all triangles, you wrote a convincing argument (or proof). This process is summarized in the diagram below.

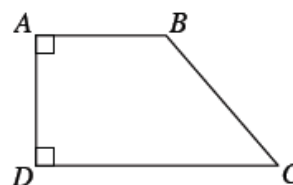


Discuss this process with the class and describe when you have used this process before (either in this class or outside of class). Why do mathematicians rely on this process?

7-105. RIGHT TRAPEZOIDS

Consecutive angles of a polygon occur at opposite ends of a side of the polygon. What can you learn about a quadrilateral with two consecutive right angles?

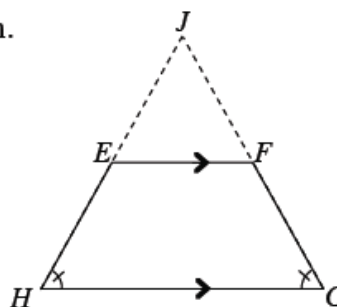
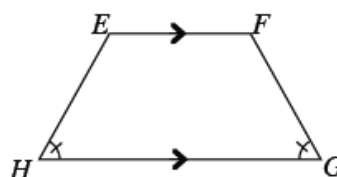
- a. **EXPLORE:** Examine the quadrilateral at right with two consecutive right angles. What do you think is true about \overline{AB} and \overline{DC} ?
- b. **CONJECTURE:** Write a conjecture about what type of quadrilateral has two consecutive right angles. Write your conjecture in conditional ("If..., then...") form.
- c. **PROVE:** Prove that your conjecture from part (b) is true for all quadrilaterals with two consecutive right angles. Write your proof using the two-column format introduced in Lesson 7.2.4. Hint: Look for angle relationships.
- d. The quadrilateral you worked with in this problem is called a **right trapezoid**. Are all quadrilaterals with two right angles a right trapezoid?



7-106. ISOSCELES TRAPEZOIDS

An **isosceles trapezoid** is a trapezoid with a pair of congruent base angles. What can you learn about the sides of an isosceles trapezoid?

- EXPLORE:** Examine trapezoid $EFGH$ at right. How do the non-parallel side lengths appear to be related?
- CONJECTURE:** Write a conjecture about side lengths in an isosceles trapezoid. Write your conjecture in conditional ("If..., then...") form.
- PROVE:** Now prove that your conjecture from part (b) is true for all isosceles trapezoids. Write your proof using the two-column format introduced in Lesson 7.2.5. To help you get started, the isosceles trapezoid is shown at right with its sides extended to form a triangle.



- 7-107. Add the theorems you have proved in this lesson to your Theorem Toolkit (Lesson 7.2.1A Resource Page). Be sure to include diagrams for each statement.



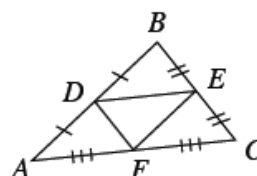


METHODS AND MEANINGS

Triangle Midsegment Theorem

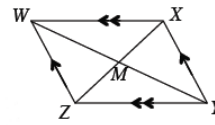
A **midsegment** of a triangle is a segment that connects the midpoints of any two sides of a triangle. Every triangle has three midsegments, as shown at right.

A midsegment between two sides of a triangle is half the length of and parallel to the third side of the triangle. For example, in $\triangle ABC$ at right, \overline{DE} is a midsegment, $\overline{DE} \parallel \overline{AC}$, and $DE = \frac{1}{2}AC$.



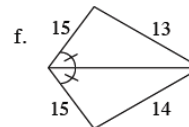
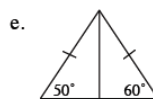
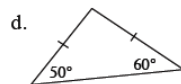
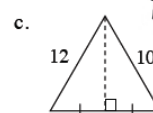
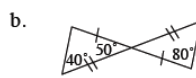
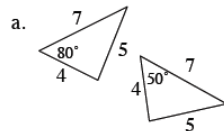


- 7-108. What else can you prove about parallelograms? Prove that the diagonals of a parallelogram bisect each other. For example, assuming that quadrilateral $WXYZ$ at right is a parallelogram, prove that $WM \cong YM$ and $ZM \cong XM$. Organize your reasoning in a flowchart. Then record your theorem in your Theorem Toolkit (Lesson 7.2.1A Resource Page).



- 7-109. One way a shape can be special is to have two congruent sides. For example, an isosceles triangle is special because it has a pair of sides that are the same length. Think about all the shapes you know and list the other special properties shapes can have. List as many as you can. Be ready to share your list with the class at the beginning of Lesson 7.3.1.

- 7-110. Carefully examine each diagram below and explain why the geometric figure cannot exist. Support your statements with reasons. If a line looks straight, assume that it is.



- 7-111. Remember that if a triangle has two equal sides, it is called isosceles. Decide whether each triangle formed by the points below is isosceles. Explain how you decided.

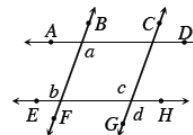
- a. $(6, 0), (0, 6), (6, 6)$ b. $(-3, 7), (-5, 2), (-1, 2)$
c. $(4, 1), (2, 3), (9, 2)$ d. $(1, 1), (5, -3), (1, -7)$

- 7-112. For each pair of numbers, find the number that is exactly halfway between them.

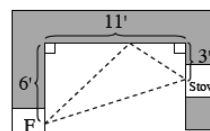
- a. 9 and 15 b. 3 and 27 c. 10 and 21

- 7-113. Penn started the proof below to show that if $\overline{AD} \parallel \overline{EH}$ and $\overline{BF} \parallel \overline{CG}$, then $a = d$. Unfortunately, he did not provide reasons for his proof. Copy his proof and provide a justification for each statement.

Statements	Reasons
1. $\overline{AD} \parallel \overline{EH}$ and $\overline{BF} \parallel \overline{CG}$	
2. $a = b$	
3. $b = c$	
4. $a = c$	
5. $c = d$	
6. $a = d$	

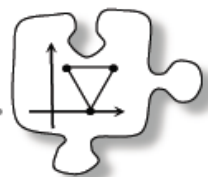


- 7-114. After finding out that her kitchen does not conform to industry standards, Lashayia is back to the drawing board (see problem 7-84). Where can she locate her sink along her top counter so that its distance from the stove and refrigerator is as small as possible? And will this location keep her perimeter below 26 feet? Show all work.



7.3.1 What makes a quadrilateral special?

Studying Quadrilaterals on a Coordinate Grid



In Section 7.2 you investigated special types of quadrilaterals, such as parallelograms, kites, and rhombi. Each of these quadrilaterals has special properties you have proved: parallel sides, sides of equal length, equal opposite angles, bisected diagonals, etc.

But not all quadrilaterals have a special name. How can you tell if a quadrilateral belongs to one of these types? And if a quadrilateral does not have a special name, can it still have special properties? In Section 7.3 you will use both algebra and geometry to investigate quadrilaterals defined on coordinate grids.

7-115. PROPERTIES OF SHAPES

Think about the special quadrilaterals you have studied in this chapter. Each shape has some properties that make it special. For example, a rhombus has two diagonals that are perpendicular. With the class, brainstorm the other types of properties that a shape can have. You may want to refer to your work from problem 7-109. Be ready to share your list with the class.

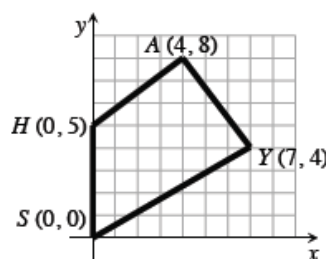


- 7-116. Review some of the algebra tools you already have. Consider two line segments \overline{AB} and \overline{CD} , given $A(0, 8)$, $B(9, 2)$, $C(1, 3)$, and $D(9, 15)$.
- Draw these two segments on a coordinate grid. Find the length of each segment.
 - Find the equation of \overline{AB} and the equation of \overline{CD} . Write both equations in $y = mx + b$ form.
 - Is $\overline{AB} \parallel \overline{CD}$? Is $\overline{AB} \perp \overline{CD}$? Justify your answer.
 - Use algebra to find the coordinates of the point where \overline{AB} and \overline{CD} intersect.

7-117. AM I SPECIAL?

Shayla just drew quadrilateral $SHAY$, shown at right. The coordinates of its vertices are:

$S(0, 0)$ $H(0, 5)$ $A(4, 8)$ $Y(7, 4)$



- Shayla thinks her quadrilateral is a trapezoid. Is she correct? Be prepared to justify your answer to the class.
- Does Shayla's quadrilateral look like it is one of the other kinds of special quadrilaterals you have studied? If so, which one?
- Even if Shayla's quadrilateral does not have a special name, it may still have some special properties like the ones you listed in problem 7-115. Use algebra and geometry tools to investigate Shayla's quadrilateral and see if it has any special properties. If you find any special properties, be ready to justify your claim that this property is present.

7-118. MUST BE, COULD BE

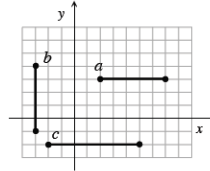
Mr. Quincey likes to play a game with his class. He says, “*My quadrilateral has four right angles.*” His students say, “*Then it **MUST BE** a rectangle*” and “*It **COULD BE** a square.*” For each description of a quadrilateral below, say what special type the quadrilateral *must be* and/or what special type the quadrilateral *could be*. Look out, some descriptions may have no *must be* statements and some descriptions may have many *could be* statements!



- a. My quadrilateral has four equal sides.
- b. My quadrilateral has two pairs of opposite parallel sides.
- c. My quadrilateral has two consecutive right angles.
- d. My quadrilateral has two pairs of equal sides.

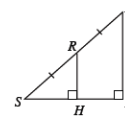


- 7-119. The diagram at right shows three bold segments. Find the coordinates of the midpoint of each segment.



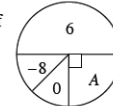
- 7-120. Examine the diagram at right.

- Are the triangles in this diagram similar? Justify your answer using similarity transformations.
- What is the relationship between the lengths of HR and AK ? Between the lengths of SH and SA ? Between the lengths of SH and HA ?
- If $SK = 20$ units and $RH = 8$ units, what is HA ?

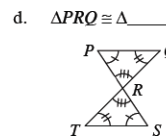
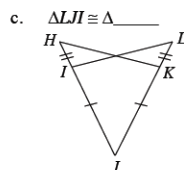
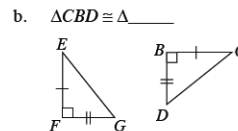
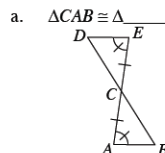


- 7-121. Examine the spinner at right. Assume that the probability of spinning a -8 is equal to that of spinning a 0 .

- Find the spinner's expected value if the value of region A is 8 .
- Find the spinner's expected value if the value of region A is -4 .
- What does the value of region A need to be so that the expected value of the spinner is 0 ?



- 7-122. For each pair of triangles below, determine if the triangles are congruent. If the triangles are congruent, state the triangle congruence condition that justifies your conclusion. If you cannot conclude that the triangles are congruent, explain why not.

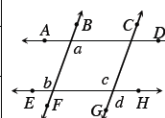


- 7-123. For one of parts (a), (b) or (c) of problem 7-122, create a flowchart or a two-column proof for your conclusion. Remember to start with the given facts and to provide a justification for each step.

- 7-124. Carolina compared her proof to Penn's work in problem 7-113. Like him, she wanted to prove that if $AD \parallel EH$ and $BF \parallel CG$, then $a = d$. Unfortunately, her statements were in a different order. Examine her proof below and help her decide if her statements are in a logical order in order to prove that $a = d$.



Statements	Reasons
1. $AD \parallel EH$ and $BF \parallel CG$	Given
2. $a = b$	If lines are parallel, alternate interior angles are equal.
3. $a = c$	Substitution
4. $b = c$	If lines are parallel, corresponding angles are equal.
5. $c = d$	Vertical angles are equal.
6. $a = d$	Substitution

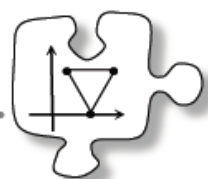


- 7-125. Describe the minimum information you would need to know about the shapes below in order to identify it correctly. For example, to know that a shape is a square, you must know that it has four sides of equal length and at least one right angle. Be as thorough as possible.

- rhombus
- trapezoid

7.3.2 How can I find the midpoint?

Coordinate Geometry and Midpoints



In Lesson 7.3.1, you applied your existing algebraic tools to analyze geometric shapes on a coordinate grid. What other algebraic processes can help us analyze shapes? And what else can be learned about geometric shapes?

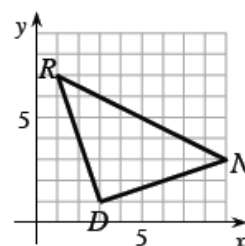
- 7-126. Cassie wants to confirm her theorem on triangle midsegments (from Lesson 7.2.6) using a coordinate grid. She started with $\triangle ABC$ with $A(0, 0)$, $B(2, 6)$, and $C(7, 0)$.
- Graph $\triangle ABC$ on graph paper.
 - With your team, find the coordinates of P , the midpoint of \overline{AB} . Likewise, find the coordinates of Q , the midpoint of \overline{BC} .
 - Prove that the length of the midsegment, \overline{PQ} , is half the length of \overline{AC} . Also verify that \overline{PQ} is parallel to \overline{AC} .

- 7-127. As Cassie worked on problem 7-126, her teammate, Esther, had difficulty finding the midpoint of \overline{BC} . The study team decided to try to find another way to find the midpoint of a line segment.
- To help Cassie, draw \overline{AM} with $A(3, 4)$ and $M(8, 11)$ on graph paper. Then extend the line segment to find a point B so that M is the midpoint of \overline{AB} . Justify your location of point B by drawing and writing numbers on the graph.
 - Esther thinks she understands how to find the midpoint on a graph. *"I always look for the middle of the line segment. But what if the coordinates are not easy to graph?"* she asks. With your team, find the midpoint of \overline{KL} if $K(2, 125)$ and $L(98, 15)$. Be ready to share your method with the class.
 - Test your team's method by verifying that the midpoint between $(-5, 7)$ and $(9, 4)$ is $(2, 5.5)$.



7-128. Randy has decided to study the triangle graphed at right.

- Consider all the special properties this triangle can have. Without using any algebra tools, predict the best name for this triangle.
- For your answer to part (a) to be correct, what is the minimum amount of information that must be true about $\triangle RND$?
- Use your algebra tools to verify each of the properties you listed in part (b). If you need, you may change your prediction of the shape of $\triangle RND$.
- Randy wonders if there is anything special about the midpoint of \overline{RN} . Find the midpoint M , and then find the lengths of \overline{RM} , \overline{DM} , and \overline{MN} . What do you notice?



- 7-129. On a map, Cary drew a set of coordinate axes. He noticed that the town of Coyner is located at the point (3, 1) and Woottonville is located at (15, 7), where 1 grid unit represents 1 mile.
- If the towns want to place a school at the midpoint between the towns, where should it be located? How far would it be from each town?
 - Woottonville argues that since it will have twice as many students attending the school, it should be closer to Woottonville. Where should the school be if it is:
 - $\frac{1}{3}$ of the way from Woottonville to Coyner?
 - $\frac{1}{4}$ of the way from Woottonville to Coyner?
 - $\frac{2}{3}$ of the way from Coyner to Woottonville?
 - Cary's brother is confused and needs help. Describe how he can find a point a certain fraction of the distance from one point to another?

7-130. LEARNING LOG

In your Learning Log, explain what a midpoint is and the method you prefer for finding midpoints of a line segment when given the coordinates of its endpoints. Include any diagram or example that helps explain why this method works. Title this entry "Finding a Midpoint" and label it with today's date.





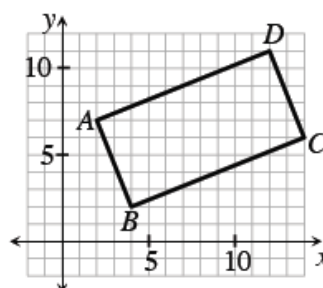
MATH NOTES

METHODS AND MEANINGS

Coordinate Geometry

Coordinate geometry is the study of geometry on a coordinate grid. Using common algebraic and geometric tools, you can learn more about a shape, such as, “Does it have a right angle?” or “Are there two sides with the same length?”

One useful tool is the Pythagorean Theorem. For example, the Pythagorean Theorem could be used to determine the length of side \overline{AB} of $ABCD$ at right. By drawing the slope triangle between points A and B , the length of \overline{AB} can be found to be $\sqrt{2^2 + 5^2} = \sqrt{29}$ units.



Similarly, slope can help analyze the relationships between the sides of a shape. If the slopes of two sides of a shape are equal, then those sides are **parallel**. For example, since the slope of $\overline{BC} = \frac{2}{5}$ and the slope of $\overline{AD} = \frac{2}{5}$, then $\overline{BC} \parallel \overline{AD}$.

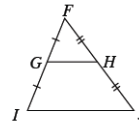
Also, if the slopes of two sides of a shape are opposite reciprocals, then the sides are **perpendicular** (meaning they form a 90° angle). For example, since the slope of $\overline{BC} = \frac{2}{5}$ and the slope of $\overline{AB} = -\frac{5}{2}$, then $\overline{BC} \perp \overline{AB}$.

By using multiple algebraic and geometric tools, you can identify shapes. For example, further analysis of the sides and angles of $ABCD$ above shows that $AB = DC$ and $BC = AD$. Furthermore, all four angles measure 90° . These facts together indicate that $ABCD$ must be a rectangle.



- 7-131. Tomika remembers that the diagonals of a rhombus are perpendicular to each other.
- Graph on $ABCD$ if $A(1, 4)$, $B(6, 6)$, $C(4, 1)$, and $D(-1, -1)$. Is $ABCD$ a rhombus? Show how you know.
 - Find the equation of the lines on which the diagonals lie. That is, find the equations of \overline{AC} and \overline{BD} .
 - Compare the slopes of \overline{AC} and \overline{BD} . What do you notice?
- 7-132. Find another valid, logical order for the statements for Penn's proof from problem 7-113. Explain how you know that changing the order the way you did does not affect the logic.
- 7-133. Each of these number lines shows a segment in bold. Find the midpoint of the segment in bold. Note that the diagrams are *not* drawn to scale.
- -
 -

- 7-134. Examine the diagram at right.
- Are the triangles in this diagram similar? If they are, use a flowchart with justifications or a two-column proof to prove similarity. If they are not similar, explain why not. Explain.
 - Name all the pairs of congruent angles in this diagram you can.
 - Are \overline{GH} and \overline{IJ} parallel? Explain how you know.
 - If $GH = 4x - 3$ and $IJ = 3x + 14$, find x . Then find the length of \overline{GH} .

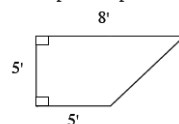


- 7-135. Consider $\triangle ABC$ with vertices $A(2, 3)$, $B(6, 6)$, and $C(8, -5)$.
- Draw $\triangle ABC$ on graph paper. What kind of triangle is $\triangle ABC$? Prove your result.
 - Reflect $\triangle ABC$ across \overline{AC} . Find the location of B' . What name best describes the resulting figure? Prove your claim.

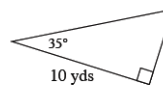
- 7-136. This problem is a checkpoint for solving with trigonometric ratios and the Pythagorean Theorem. It will be referred to as Checkpoint 7.



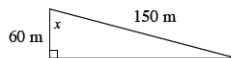
- a. Compute the perimeter.



- b. Solve for x .



- c. Solve for x .



- d. Juanito is flying a kite at the park and realizes that all 500 feet of string are out. Margie measures the angle of the string with the ground using her clinometer and finds it to be 42° . How high is Juanito's kite above the ground? Draw a diagram and use the appropriate trigonometric ratio.

Check your answers by referring to the Checkpoint 7 materials located at the back of your book.

If you needed help solving these problems correctly, then you need more practice. Review the Checkpoint 7 materials and try the practice problems. Also, consider getting help outside of class time. From this point on, you will be expected to do problems like these quickly and easily.

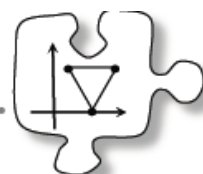
- 7-137. MUST BE, COULD BE

Here are some more challenges from Mr. Quincey. For each description of a quadrilateral below, say what special type the quadrilateral *must be* and/or what special type the quadrilateral *could be*. Look out: Some descriptions may have no *must be* statements, and some descriptions may have many *could be* statements!

- My quadrilateral has a pair of equal sides and a pair of parallel sides.
- The diagonals of my quadrilateral bisect each other.

7.3.3 What kind of quadrilateral is it?

Identifying Quadrilaterals on a Coordinate Grid



Today you will use algebra tools to investigate the properties of a quadrilateral and then will use those properties to identify the type of quadrilateral it is.

7-138. MUST BE, COULD BE

Mr. Quincey has some new challenges for you! For each description below, decide what special type the quadrilateral *must be* and/or what special type the quadrilateral *could be*. Look out: Some descriptions may have no *must be* statements, and some descriptions may have many *could be* statements!

- My quadrilateral has three right angles.
- My quadrilateral has a pair of parallel sides.
- My quadrilateral has two consecutive equal angles.

7-139. THE SHAPE FACTORY

You just got a job in the Quadrilaterals Division of your uncle's Shape Factory. In the old days, customers called up your uncle and described the quadrilaterals they wanted over the phone: "*I'd like a parallelogram with...*".



"*But nowadays,*" your uncle says, "*customers using computers have been emailing orders in lots of different ways.*" Your uncle needs your team to help analyze his most recent orders listed below to identify the quadrilaterals and help the shape-makers know what to produce.

Your Task: For each of the quadrilateral orders listed below,

- Create a diagram of the quadrilateral on graph paper.
- Decide if the quadrilateral ordered has a special name. To help the shape-makers, your name must be as specific as possible. (For example, do not just call a shape a rectangle when it is also a square!)
- Record and be ready to present a proof that the quadrilateral ordered must be the kind you say it is. It is not enough to say that a quadrilateral *looks* like it is of a certain type or *looks* like it has a certain property. Customers will want to be sure they get the type of quadrilateral they ordered!

Discussion Points

What special properties might a quadrilateral have?

What algebra tools could be useful?

What types of quadrilaterals might be ordered?

The orders:

- a. A quadrilateral formed by the intersection of these lines:

$$y = -\frac{3}{2}x + 3$$

$$y = \frac{3}{2}x - 3$$

$$y = -\frac{3}{2}x + 9$$

$$y = \frac{3}{2}x + 3$$

- b. A quadrilateral with vertices at these points:

$$A(0, 2)$$

$$B(1, 0)$$

$$C(7, 3)$$

$$D(4, 4)$$

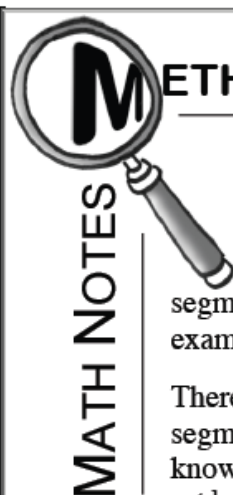
- c. A quadrilateral with vertices at these points:

$$W(0, 5)$$

$$X(2, 7)$$

$$Y(5, 7)$$

$$Z(5, 1)$$

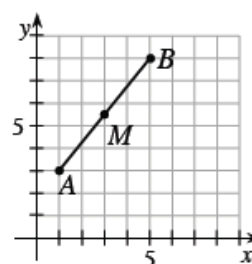


METHODS AND MEANINGS

Finding a Midpoint

A **midpoint** is a point that divides a line segment into two parts of equal length. For example, M is the midpoint of \overline{AB} at right.

There are several ways to find the midpoint of a line segment if the coordinates of the endpoints are known. One way is to add half the change in x ($\frac{1}{2} \Delta x$) and half of the change in y ($\frac{1}{2} \Delta y$) to the x - and y -coordinates of the starting point, respectively.



Thus, if $A(1, 3)$ and $B(5, 8)$, then $\Delta x = 5 - 1 = 4$ and $\Delta y = 8 - 3 = 5$. Then the x -coordinate of M is $1 + \frac{1}{2}(4) = 3$ and the y -coordinate is $3 + \frac{1}{2}(5) = 5.5$. So point M is at $(4, 5.5)$.

This strategy can be used to find other points between A and B that are a proportion of the way from a starting point. For example, if you wanted to find a point $\frac{4}{5}$ of the way from point A to point B , then this could be found by adding $\frac{4}{5}$ of Δx to the x -coordinate of point A and adding $\frac{4}{5}$ of Δy to the y -coordinate of point A . This would be the point $((1 + \frac{4}{5}(4), 3 + \frac{4}{5}(5))$ which is $(4.2, 7)$. Generally, a point a ratio r from $A(x_0, y_0)$ to $B(x_1, y_1)$ is at $(x_0 + r(x_1 - x_0), y_0 + r(y_1 - y_0))$.



- 7-140. Each problem below gives the endpoints of a segment. Find the coordinates of the midpoint of the segment. If you need help, consult the Math Notes box for this lesson.

- a. (5, 2) and (11, 14) b. (3, 8) and (10, 4)

- 7-141. Below are the coordinates of three points and the equations of two lines. For each line, determine which of the points, if any, lie on that line. (There may be more than one!)

X(0, 15) Y(3, 16) Z(7, 0)

- a. $y = \frac{1}{3}x + 15$ b. $y - 16 = -4(x - 3)$

- 7-142. MUST BE, COULD BE

Here are some more challenges from Mr. Quincey. For each description of a quadrilateral below, say what special type the quadrilateral *must be* and/or what special type the quadrilateral *could be*. Look out: Some descriptions may have no *must be* statements, and some descriptions may have many "*could be*" statements!

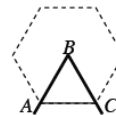
- a. My quadrilateral has two right angles.
b. The diagonals of my quadrilateral are perpendicular.

- 7-143. Examine the arrow diagram below.

Polygon is a parallelogram \rightarrow *area of the polygon equals base times height.*

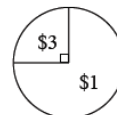
- a. Write this conjecture as a conditional ("If, then") statement.
b. Write the converse of this conditional statement. Is the converse statement a true statement?
c. Write a similar conjecture about triangles. Write it once as a conditional statement and once as an arrow diagram.
d. Write the converse of your triangle conjecture from part (c). Is the converse statement true?

- 7-144. The angle created by a hinged mirror when forming a regular polygon is called a **central angle**. For example, $\angle ABC$ in the diagram at right is the central angle of the regular hexagon.



- a. If the central angle of a regular polygon measures 18° , how many sides does the polygon have?
b. Can a central angle measure 90° ? 180° ? 13° ? For each angle measure, explain how you know.

- 7-145. Jamika designed a game that allows some people to win money and others to lose money, but overall Jamika will neither win nor lose money. Each player will spin the spinner at right and will win the amount of money shown in the result. How much should each player pay to spin the spinner? Explain your reasoning.



- 7-146. Suppose $ADBC$ is a quadrilateral and the diagonal \overline{AB} lies on $y = -\frac{4}{3}x + 5$ and diagonal \overline{CD} lies on $y = \frac{3}{4}x - 1$. Assume the diagonals intersect at point E .

- a. Without graphing, what is the relationship between the diagonals? How do you know?
b. Graph the lines on graph paper. If E is a midpoint of \overline{CD} , what type of quadrilateral could $ADBC$ be? Is there more than one possible type? Explain how you know.

Chapter 7 Closure What have I learned?

Reflection and Synthesis

The activities below offer you a chance to reflect about what you have learned during this chapter. As you work, look for concepts that you feel very comfortable with, ideas that you would like to learn more about, and topics you need more help with. Look for connections between ideas as well as connections with material you learned previously.

① TEAM BRAINSTORM

What have you studied in this chapter? What ideas were important in what you learned? With your team, brainstorm a list. Be as detailed as you can. To help get you started, a list of Learning Log entries, Toolkit Entries, and Math Notes boxes are below.

What topics, ideas, and words that you learned *before* this chapter are connected to the new ideas in this chapter? Again, be as detailed as you can.

Next consider the Standards for Mathematical Practice that follow Activity ③: Portfolio. What Mathematical Practices did you use in this chapter? When did you use them? Give specific examples.

How long can you make your lists? Challenge yourselves. Be prepared to share your team's ideas with the class.



Learning Log Entries

- Lesson 7.1.1 – Shapes of Wheels
- Lesson 7.1.3 – Shortest Distance
- Lesson 7.2.5 – Two-Column Proofs
- Lesson 7.3.2 – Finding a Midpoint

Toolkit Entries

- Theorem Toolkit (Lesson 7.2.1A Resource Page and problems 7-53, 7-64, 7-72, 7-94, 7-100, 7-107, and 7-108.)

Math Notes

- Lesson 7.1.2 – Parts of a Circle
- Lesson 7.1.3 – Congruent Triangles → Congruent Corresponding Parts
- Lesson 7.1.4 – Regular Polygons
- Lesson 7.2.1 – Reflexive Property of Equality
- Lesson 7.2.2 – Exponential Functions
- Lesson 7.2.3 – Definitions of Quadrilaterals
- Lesson 7.2.4 – Diagonals of a Rhombus
- Lesson 7.2.6 – Triangle Midsegment Theorem
- Lesson 7.3.2 – Coordinate Geometry
- Lesson 7.3.3 – Finding a Midpoint

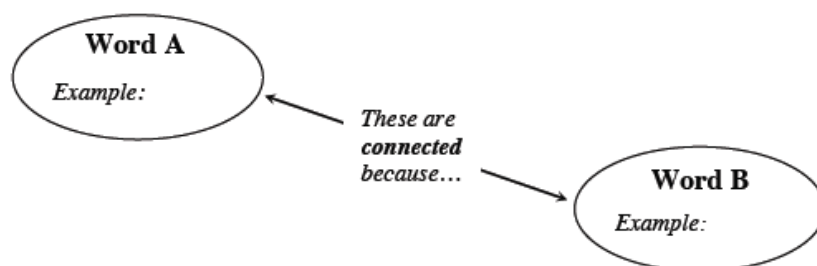


② MAKING CONNECTIONS

Below is a list of the vocabulary used in this chapter. Make sure that you are familiar with all of these words and know what they mean. Refer to the glossary or index for any words that you do not yet understand.

bisect	center	central angle
circle	congruent	conjecture
consecutive angles	coordinate geometry	diagonal
diameter	edge	exterior angle
face	flowchart	isosceles trapezoid
kite	midpoint	midsegment
opposite	parallel	parallelogram
perpendicular	proof	quadrilateral
radius	rectangle	Reflexive Property
regular polygon	rhombus	right trapezoid
square	tetrahedron	theorem
three-dimensional	trapezoid	two-column proof
two-dimensional	vertex	

Make a concept map showing all of the connections you can find among the key words and ideas listed above. To show a connection between two words, draw a line between them and explain the connection, as shown in the model below. A word can be connected to any other word as long as you can justify the connection. For each key word or idea, provide an example or sketch that shows the idea.



Your teacher may provide you with vocabulary cards to help you get started. If you use the cards to plan your concept map, be sure either to re-draw your concept map on your paper or to glue the vocabulary cards to a poster with all of the connections explained for others to see and understand.

While you are making your map, your team may think of related words or ideas that are not listed here. Be sure to include these ideas on your concept map.

③ PORTFOLIO: EVIDENCE OF MATHEMATICAL PROFICIENCY



Re-create your proof from problem 7-100 to showcase your current understanding of formal proof. Make sure your flowchart includes given information, bubbles that contain statements, and justifications for the statements. Do *not* use quadrilateral theorems from your Theorem Toolkit or elsewhere in this chapter as part of your flowchart. In other words, start with the given information and make statements based on angle relationships, shape definitions, triangle properties, and triangle similarity, as needed, but do not use quadrilateral theorems (such as “diagonals of a parallelogram bisect each other”).

Then rewrite your proof, showcasing your ability to write a two-column proof.

Your teacher may give you the Chapter 7 Closure Resource Page: Quadrilaterals Graphic Organizer to record your work. A Graphic Organizer is a tool you can use to organize your thoughts, showcase your knowledge, and communicate your ideas clearly.

Next, consider the Standards for Mathematical Practice that follow. What Mathematical Practices did you use in this chapter? When did you use them? Give specific examples.

<p>Chapter 7 Closure Resource Page</p> <p>Quadrilaterals GO (Graphic Organizer)</p> <div style="border: 1px solid black; padding: 5px; margin: 5px;"> <p>Parallelogram</p> <p>Diagram:</p> <p>Special properties:</p> </div> <p><i>If a parallelogram has all equal sides, then it is a rhombus.</i></p> <div style="border: 1px solid black; padding: 5px; margin: 5px;"> <p>Rhombus</p> <p>Diagram:</p> <p>Special properties:</p> </div>

BECOMING MATHEMATICALLY PROFICIENT
The Common Core State Standards For Mathematical Practice

This book focuses on helping you use some very specific Mathematical Practices. The Mathematical Practices describe ways in which mathematically proficient students engage with mathematics everyday.



Make sense of problems and persevere in solving them:

Making sense of problems and persevering in solving them means that you can solve problems that are full of different kinds of mathematics. These types of problems are not routine, simple, or typical. Instead, they combine lots of math ideas and everyday situations. You have to stick with challenging problems, try different strategies, use multiple representations, and use a different method to check your results.

Reason abstractly and quantitatively:

Throughout this course, everyday situations are used to introduce you to new math ideas. Seeing mathematical ideas within a context helps you make sense of the ideas. Once you learn about a math idea in a practical way, you can “**reason abstractly**” by thinking about the concept more generally, representing it with symbols, and manipulating the symbols. **Reasoning quantitatively** is using numbers and symbols to represent an everyday situation, taking into account the units involved, and considering the meaning of the quantities as you compute them.

Construct viable arguments and critique the reasoning of others:

To **construct a viable argument** is to present your solution steps in a logical sequence and to justify your steps with conclusions, relying on number sense, facts and definitions, and previously established results. You communicate clearly, consider the real-life context, and provide clarification when others ask. In this course, you regularly share information, opinions, and expertise with your study team. You **critique the reasoning of others** when you analyze the approach of others, build on each other's ideas, compare the effectiveness of two strategies, and decide what makes sense and under what conditions.

Model with mathematics:

When you **model with mathematics**, you take a complex situation and use mathematics to represent it, often by making assumptions and approximations to simplify the situation. Modeling allows you to analyze and describe the situation and to make predictions. For example, to find the density of your body, you might model your body with a more familiar shape, say, a cylinder of the same diameter and height. Although a model may not be perfect, it can still be very useful for describing data and making predictions. When you interpret the results, you may need to go back and improve your model by revising your assumptions and approximations.

Use appropriate tools strategically:

To **use appropriate tools strategically** means that you analyze the task and decide which tools may help you model the situation or find a solution. Some of the tools available to you include diagrams, graph paper, calculators, computer software, databases, and websites. You understand the limitations of various tools. A result can be checked or estimated by strategically choosing a different tool.

Attend to precision:

To **attend to precision** means that when solving problems, you need to pay close attention to the details. For example, you need to be aware of the units, or how many digits your answer requires, or how to choose a scale and label your graph. You may need to convert the units to be consistent. At times, you need to go back and check whether a numerical solution makes sense in the context of the problem.

You need to **attend to precision** when you communicate your ideas to others. Using the appropriate vocabulary and mathematical language can help make your ideas and reasoning more understandable to others.

Look for and make use of structure:

Looking for and making use of structure is a guiding principal of this course. When you are involved in analyzing the structure and in the actual development of mathematical concepts, you gain a deeper, more conceptual understanding than when you are simply told what the structure is and how to do problems. You often use this practice to bring closure to an investigation.

There are many concepts that you learn by looking at the underlying structure of a mathematical idea and thinking about how it connects to other ideas you have already learned. For example, geometry theorems are developed from the structure of translations.

Look for and express regularity in repeated reasoning:

To **look for and express regularity in repeated reasoning** means that when you are investigating a new mathematical concept, you notice if calculations are repeated in a pattern. Then you look for a way to generalize the method for use in other situations, or you look for shortcuts. For example, the investigations with simple shapes can be applied to more complex shapes using repeated reasoning.

④

WHAT HAVE I LEARNED?

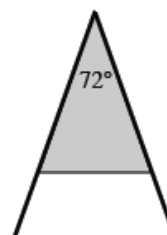
Most of the problems in this section represent typical problems found in this chapter. They serve as a gauge for you. You can use them to determine which types of problems you can do well and which types of problems require further study and practice. Even if your teacher does not assign this section, it is a good idea to try these problems and find out for yourself what you know and what you still need to work on.



Solve each problem as completely as you can. The table at the end of the closure section has answers to these problems. It also tells you where you can find additional help and practice with problems like these.

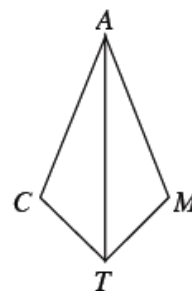
CL 7-147. Julius set his hinged mirror so that its angle was 72° and the core region was isosceles, as shown at right.

- a. How many sides did his resulting polygon have?
Show how you know.
- b. What is another name for this polygon?



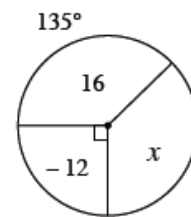
CL 7-148. Kelly started the proof below to show that if $\overline{TC} \cong \overline{TM}$ and \overline{AT} bisects $\angle CTM$, then $\overline{CA} \cong \overline{MA}$. Copy and complete her proof.

Statements	Reasons
1. $\overline{TC} \cong \overline{TM}$ and \overline{AT} bisects $\angle CTM$	
2.	Definition of bisect
3. $\overline{AT} \cong \overline{AT}$	
4.	
5.	$\cong \Delta s \rightarrow \cong$ parts



CL 7-149. Examine the spinner at right.

- Find the expected value of the spinner if $x = 4$.
- Find the expected value of the spinner if $x = -8$.
- Find x so that the expected value of the spinner is 6.

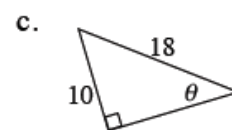
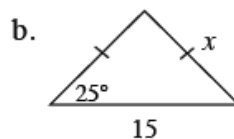
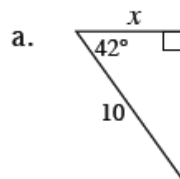


CL 7-150. $ABCD$ is a parallelogram. If $A(3, -4)$, $B(6, 2)$, $C(4, 6)$, then what are the possible locations of point D ? Draw a graph and justify your answer.

CL 7-151. Each problem below gives the endpoints of a segment. Find the coordinates of the midpoint of the segment.

- $(-3, 11)$ and $(5, 6)$
- $(-4, -1)$ and $(8, 9)$

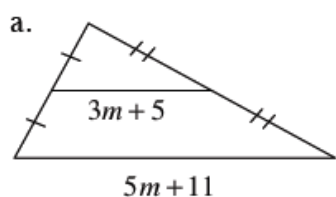
CL 7-152. For each diagram below, solve for the variable.



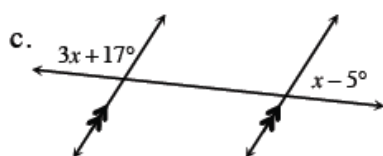
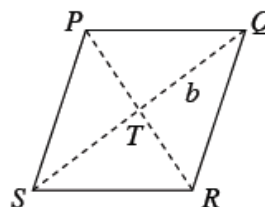
CL 7-153. On graph paper, draw quadrilateral $MNPQ$ if $M(1, 7)$, $N(-2, 2)$, $P(3, -1)$, and $Q(6, 4)$.

- Find the slopes of \overline{MN} and \overline{NP} . What can you conclude about $\angle MNP$?
- What is the best name for $MNPQ$? Justify your answer.
- Which diagonal is longer? Explain how you know your answer is correct.
- Find the midpoint of \overline{MN} .

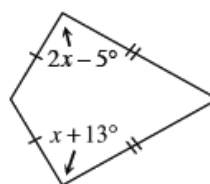
CL 7-154. Examine the geometric relationships in each of the diagrams below. For each one, write and solve an equation to find the value of the variable. Name any geometric property or conjecture that you used.



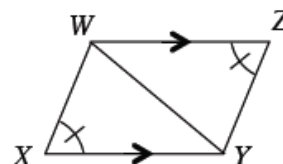
- b. $PQRS$ is a rhombus with perimeter = 28 units and $PR = 8$ units, find b (QT)



d.



CL 7-155. Given the information in the diagram at right, use a flowchart to prove that $\triangle WXY \cong \triangle YZW$.



CL 7-156. MUST BE, COULD BE

Here are some more challenges from Mr. Quincey. For each description of a quadrilateral below, say what special type the quadrilateral *must be* and/or what special type the quadrilateral *could be*. Remember: Some descriptions may have no *must be* statements, and some descriptions may have many *could be* statements!

- a. The diagonals of my quadrilateral are equal.
- b. My quadrilateral has one right angle.
- c. My quadrilateral has one pair of equal adjacent sides.

CL 7-157. Check your answers using the table at the end of the closure section. Which problems do you feel confident about? Which problems were hard? Use the table to make a list of topics you need help on and a list of topics you need to practice more.

Answers and Support for Closure Activity #4
What Have I Learned?

Note: MN = Math Note, LL = Learning Log

Problem	Solution	Need Help?	More Practice												
CL 7-147.	a. $360^\circ \div 72^\circ = 5$ sides b. regular pentagon	Lesson 7.1.4 MN: 7.1.4	Problems 7-43, 7-55, and 7-144												
CL 7-148.	<table><thead><tr><th>Statements</th><th>Reasons</th></tr></thead><tbody><tr><td>1. $\overline{TC} \cong \overline{TM}$ and \overline{AT} bisects $\angle CTM$</td><td>Given</td></tr><tr><td>2. $\angle CTA \cong \angle MTA$</td><td>Definition of bisect</td></tr><tr><td>3. $\overline{AT} \cong \overline{AT}$</td><td>Reflexive Property</td></tr><tr><td>4. $\triangle CAT \cong \triangle MAT$</td><td>SAS \cong</td></tr><tr><td>5. $\overline{CA} \cong \overline{MA}$</td><td>$\cong \triangle s \rightarrow \cong$ parts</td></tr></tbody></table>	Statements	Reasons	1. $\overline{TC} \cong \overline{TM}$ and \overline{AT} bisects $\angle CTM$	Given	2. $\angle CTA \cong \angle MTA$	Definition of bisect	3. $\overline{AT} \cong \overline{AT}$	Reflexive Property	4. $\triangle CAT \cong \triangle MAT$	SAS \cong	5. $\overline{CA} \cong \overline{MA}$	$\cong \triangle s \rightarrow \cong$ parts	Lesson 7.2.5 MN: 7.1.3 and 7.2.1 LL: 7.2.5 Theorem Toolkit	Problems 7-102, 7-113, 7-122, and 7-124
Statements	Reasons														
1. $\overline{TC} \cong \overline{TM}$ and \overline{AT} bisects $\angle CTM$	Given														
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5. $\overline{CA} \cong \overline{MA}$	$\cong \triangle s \rightarrow \cong$ parts														
CL 7-149.	a. 4.5 b. 0 c. $x = 8$	Lesson 4.2.5 MN: 5.2.2	Problems CL 5-141, CL 6-107, 7-121, and 7-145												
CL 7-150.	Point D is at $(1, 0)$, $(5, -8)$, or $(7, 12)$.	Lessons 7.3.1 and 7.3.3 MN: 7.2.3 and 7.3.2	Problems 7-32, 7-35, 7-68, and 7-79												
CL 7-151.	a. $(1, 8.5)$ b. $(2, 4)$	Lesson 7.3.2 MN: 7.3.3 LL: 7.3.2	Problems 7-20, 7-29, 7-45, 7-119, and 7-140												
CL 7-152.	a. $x \approx 7.43$ b. $x \approx 8.28$ c. $\theta \approx 33.7^\circ$	Lesson 2.3.2, Sections 4.1 and 5.1 MN: 2.3.2, 4.1.2, 4.1.4, 5.1.2, and 5.1.4 LL: 2.3.2, 4.1.3, 4.1.4, 5.1.2, 5.1.3, and 5.3.1 Triangle Toolkit	Problems CL 3-119(a), CL 4-122, CL 5-139, 7-10, 7-85, and 7-136												
CL 7-153.	a. Slope of $\overline{MN} = \frac{5}{3}$ and $\overline{NP} = -\frac{3}{5}$, $\angle MNP$ is a right angle. b. It is a square because all sides are equal and all angles are right angles. c. The diagonals have equal length. Each is $\sqrt{68}$ units long. d. $(-\frac{1}{2}, \frac{9}{2})$	Section 7.3 MN: 7.2.3, 7.3.2, and 7.3.3 LL: 7.3.3	Problems 7-20, 7-32, 7-35, 7-48, 7-79, and 7-131												
CL 7-154.	a. $2(3m + 5) = 5m + 11$, $m = 1$ b. $b^2 + 4^2 = 7^2$, so $b = \sqrt{33} \approx 5.74$ units c. $3x + 17^\circ + x - 5^\circ = 180^\circ$, so $x = 42^\circ$ d. $2x - 5^\circ = x + 13^\circ$, so $x = 18^\circ$	Section 2.1, Lessons 2.3.2 and 7.2.6 MN: 2.1.1, 2.1.4, 2.3.2, 7.2.4, and 7.2.6 LL: 2.1.1, 2.3.2 Angle Relationships Toolkit Theorem Toolkit	Problems 7-7, 7-18, 7-37, and 7-81												
CL 7-155.	<pre>graph TD A([WZ // YX]) -- Given --> B([∠Z = ∠X]) A -- "If lines are //, then alt. int. angles are" --> C([∠ZWY = ∠XYW]) D([WY = YW]) -- Reflexive Property --> E([ΔXYW ≅ ΔZWY]) B -- Given --> E C -- "If lines are //, then alt. int. angles are" --> E E -- AAS ≅ --> E</pre>	Section 3.2 and Lessons 6.1.1 through 6.1.4 MN: 3.2.2, 3.2.4, 6.1.4, and 7.2.1 LL: 3.2.2	Problems CL 3-121, CL 4-123, CL 5-140, CL 6-101, 7-6, 7-15, 7-46, 7-69, 7-76, 7-86, 7-100, 7-108, 7-123, and 7-134(a)												
CL 7-156.	a. Must be: rectangle; Could be: square b. Must be: none; Could be: any quadrilateral c. Must be: none; Could be: kite, rhombus, square	Lessons 7.3.1 and 7.3.3 MN: 7.2.3	Problems 7-9, 7-125, 7-137, and 7-142												

Bisect	Center
Central angle	Circle
Congruent	Conjecture
Consecutive angles	Coordinate geometry
Diagonal	Diameter

Edge	Exterior angle
Face	Flowchart
Isosceles trapezoid	Kite
Midpoint	Midsegment
Opposite	Parallel

Parallelogram	Perpendicular
Proof	Quadrilateral
Radius	Rectangle
Reflexive Property	Regular polygon
Rhombus	Right trapezoid

Square	Tetrahedron
Theorem	Three-dimensional
Trapezoid	Two-column proof
Two-dimensional	Vertex

Chapter 7 Closure Resource Page

Quadrilateral Graphic Organizer