

CHAPTER 8

Polygons and Circles

In previous chapters, you have extensively studied triangles and quadrilaterals to learn more about their sides and angles. In this chapter, you will broaden your focus to include polygons with 5, 8, 10, and even 100 sides. You will develop a way to find the area and perimeter of a regular polygon and will study how the area and perimeter changes as the number of sides increases.

In Section 8.2, you will re-examine similar shapes to study what happens to the area and perimeter of a shape when the shape is enlarged or reduced.

Finally, in Section 8.3, you will connect your understanding of polygons with your knowledge of the area ratios of similar figures to find the area and circumference of circles of all sizes.

In this chapter, you will learn:

- About special types of polygons, such as regular and non-convex polygons.
- How the measures of the interior and exterior angles of a regular polygon are related to the number of sides of the polygon.
- How the areas of similar figures are related.
- How to find the area and circumference of a circle and parts of circles and use this ability to solve problems in various contexts.

Guiding Question

Mathematically proficient students express regularity in repeated reasoning.

As you work through this chapter, ask yourself:

Can I find the shortcuts and generalize the rules for finding perimeters and areas of polygons?

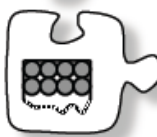
Chapter Outline



Section 8.1 This section begins with an investigation of the interior and exterior angles of a polygon and ends with a focus on the areas and perimeters of regular polygons.



Section 8.2 In this section, similar figures are revisited in order to investigate the ratio of the areas of similar figures.



Section 8.3 While answering the question, “*What if the polygon has an infinite number of sides?*”, a process will be developed to find the area and circumference of a circle.

Chapter 8 Teacher Guide

Section	Lesson	Days	Lesson Objectives	Materials	Homework
8.1	8.1.1	1	Pinwheels and Polygons	<ul style="list-style-type: none"> • Lesson 8.1.1 Res. Pg. • Scissors • Pattern blocks (opt.) 	8-6 to 8-12
	8.1.2	1	Interior Angles of Polygons	• Lesson 8.1.2 Res. Pg.	8-17 to 8-23
	8.1.3	1	Angles of Regular Polygons	<ul style="list-style-type: none"> • Computer and projector (opt.) OR • Lesson 8.1.3 Res. Pg. and tracing paper 	8-29 to 8-35
	8.1.4	1	Regular Polygon Angle Connections	None	8-40 to 8-46
	8.1.5	2	Finding Areas of Regular Polygons	• Presentation materials	8-53 to 8-59 and 8-60 to 8-66
8.2	8.2.1	1	Area Ratios of Similar Figures	<ul style="list-style-type: none"> • Lesson 8.2.1A & B Res. Pgs. • Scissors 	8-71 to 8-77
	8.2.2	1	Ratios of Similarity	None	8-83 to 8-89
8.3	8.3.1	1	A Special Ratio	None	8-93 to 8-99
	8.3.2	1	Area and Circumference of a Circle	None	8-105 to 8-111
	8.3.3	2	Circles in Context	• Box with string (opt.)	8-116 to 8-122 and 8-123 to 8-129
Chapter Closure		Various Options			

Total: 12 days plus optional time for Chapter Closure and Assessment

8.1.1 How can I build it?

Pinwheels and Polygons



In previous chapters, you have studied triangles and quadrilaterals. In Chapter 8, you will broaden your focus to include all polygons and will study what triangles can tell us about shapes with 5, 8, or even 100 sides.

By the end of this lesson, you should be able to answer these questions:

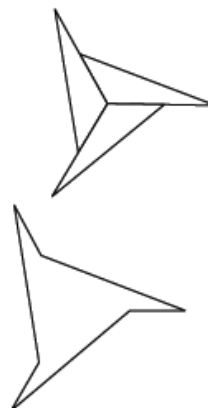
How can you use the number of sides of a regular polygon to find the measure of the central angle?

What type of triangle is needed to form a regular polygon?

8-1. PINWHEELS AND POLYGONS

Inez loves pinwheels. One day in class, she noticed that if she put three congruent triangles together so that one set of corresponding angles are adjacent, she could make a shape that looks like a pinwheel.

- Can you determine any of the angles of her triangles? Explain how you found your answer.
- The overall shape (outline) of Inez's pinwheel is shown at right. How many sides does it have? What is another name for this shape?
- Inez's shape is an **example** of a **polygon** because it is a closed, two-dimensional figure made of non-intersecting straight line segments connected end-to-end. As you study polygons in this course, it is useful to use the names below because they identify how many sides a particular polygon has. Some of these words may be familiar, while others may be new. On your paper, draw an **example** of a *heptagon*.



Name of Polygon	Number of Sides	Name of Polygon	Number of Sides
Triangle	3	Octagon	8
Quadrilateral	4	Nonagon	9
Pentagon	5	Decagon	10
Hexagon	6	11-gon	11
Heptagon	7	<i>n</i> -gon	<i>n</i>

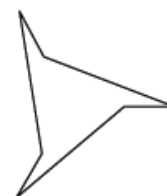
- 8-2. Inez is very excited. She wants to know if you can build a pinwheel using *any* angle of her triangle. Obtain a Lesson 8.1.1 Resource Page from your teacher and cut out Inez's triangles. Then work with your team to build pinwheels and polygons by placing different corresponding angles together at the center. You will need to use the triangles from all four team members together to build one shape. Be ready to share your results with the class.



- 8-3. Jorge likes Inez's pinwheels but wonders, "*Will all triangles build a pinwheel or a polygon?*"
- If you have not already done so, cut out the remaining triangles on the Lesson 8.1.1 Resource Page. Work together to determine which congruent triangles can build a pinwheel (or polygon) when corresponding angles are placed together at the center. For each successful pinwheel, answer the questions below.
 - How many triangles did it take to build the pinwheel?
 - Calculate the measure of a central angle of the pinwheel. (Remember that a central angle is an angle of a triangle with a vertex at the center of the pinwheel.)
 - Is the shape familiar? Does it have a name? If so, what is it?
 - Explain why one triangle may be able to create a pinwheel or polygon while another triangle cannot.
 - Jorge has a triangle with angle measures 32° , 40° , and 108° . Will this triangle be able to form a pinwheel? Explain.

8-4. Jasmine wants to create a pinwheel with equilateral triangles.

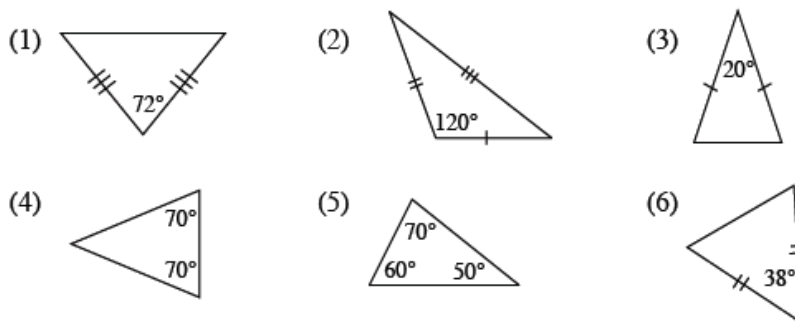
- How many equilateral triangles will she need? Explain how you know.
- What is the name for the polygon she created?
- Jasmine's shape is an **example** of a **convex polygon**, while Inez's shape, shown at right, is **non-convex**. Study the **examples** of **convex** and **non-convex** polygons below and then write a definition of a **convex** polygon on your paper.



Examples of Non-Convex Polygons	Examples of Convex Polygons

8-5. When corresponding angles are placed together, why do some triangles form convex polygons while others result in non-convex polygons? Consider this as you answer the following questions.

- a. Carlisle wants to build a convex polygon using congruent triangles. He wants to select one of the triangles below to use. Which triangle(s) will build a convex polygon if multiple congruent triangles are placed together so that they share a common vertex and do not overlap? Explain how you know.



- b. For each triangle from part (a) that creates a convex polygon, how many sides would the polygon have? What name is most appropriate for the polygon?

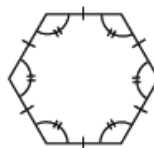


METHODS AND MEANINGS

Convex and Non-Convex Polygons

A **polygon** is defined as a two-dimensional closed figure made up of straight line segments connected end-to-end. These segments may not cross (intersect) at any other points.

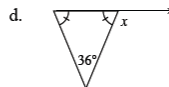
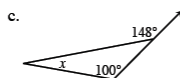
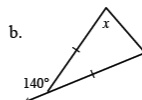
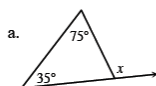
A polygon is referred to as a **regular polygon** if it is equilateral (all sides have the same length) and equiangular (all interior angles have equal measure). For example, the hexagon shown at right is a regular hexagon because all sides have the same length and each interior angle has the same measure.



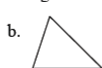
A polygon is called **convex** if each pair of interior points can be connected by a segment without leaving the interior of the polygon. See the example of convex and non-convex shapes in problem 8-4.



8-6. Solve for x in each diagram below.



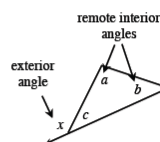
8-7. Using the definition of polygon from the Math Notes box in this lesson determine whether each of the following figures is or is not a polygon. Justify your decisions for each figure.



8-8. After solving for x in each of the diagrams in problem 8-6, Jerome thinks he sees a pattern. He notices that the measure of an exterior angle of a triangle is related to two of the angles of a triangle.

a. Do you see a pattern? To help find a pattern, study the results of problem 8-6.

b. In the example at right, angles a and b are called **remote interior angles** of the given exterior angle because they are not adjacent to the exterior angle. Write a conjecture about the relationships between the remote interior and exterior angles of a triangle.



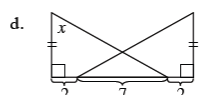
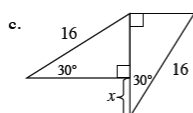
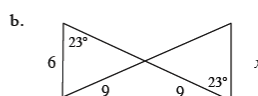
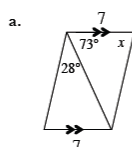
c. Prove that the conjecture you wrote for part (b) is true for all triangles. Your proof can be written in any form, as long as it is convincing and provides reasons for all statements.

8-9. Examine the geometric relationships in the diagram at right. Show all of the steps in your solutions for x and y .



8-10. Steven has 100 congruent triangles that each has an angle measuring 15° . How many triangles would he need to use to make a pinwheel? Explain how you found your answer.

8-11. Find the value of x in each diagram below, if possible. If the triangles are congruent, state which triangle congruence property was used. If the triangles are not congruent or if there is not enough information, state, "Cannot be determined."



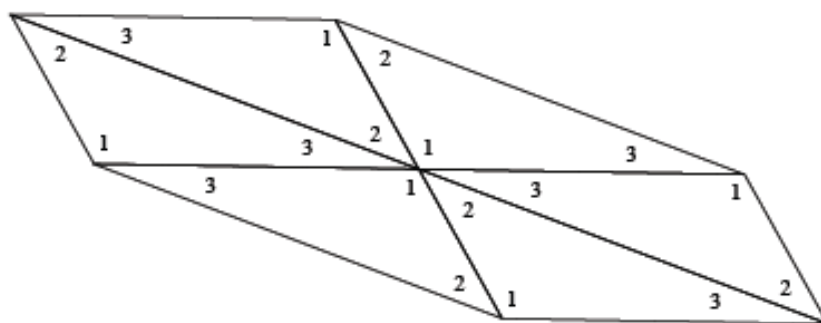
8-12. Decide if the following statements are true or false. If a statement is false, provide a diagram of a counterexample.

- All squares are rectangles.
- All quadrilaterals are parallelograms.
- All rhombi are parallelograms.
- All squares are rhombi.
- The diagonals of a parallelogram bisect the angles.

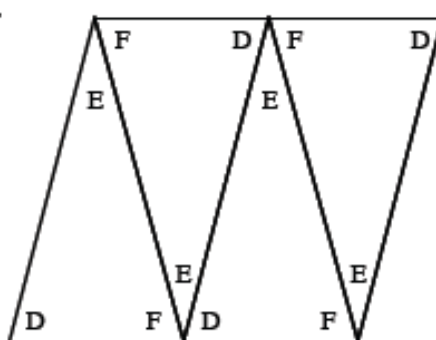
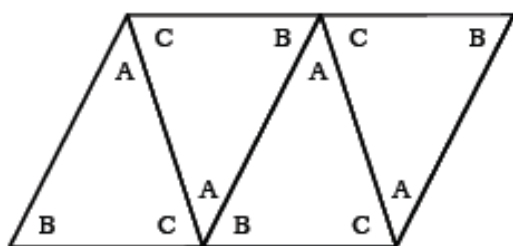
Lesson 8.1.1 Resource Page

Pinwheels and Polygons

- 8-2. **Directions:** Carefully cut out the triangles below. Then work together as a team to build pinwheels by placing different corresponding angles together at the center. You will need to use all of the triangles from all four team members together to build one shape. Be ready to share your results with the class.



- 8-3.



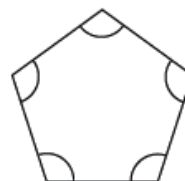
8.1.2 What is its measure?

Interior Angles of Polygons



In an earlier chapter you discovered that the sum of the interior angles of a triangle is always 180° . What about other polygons, such as hexagons or decagons? What about the sum of their interior angles? Do you think it matters if the polygon is convex or not? Consider these questions today as you investigate the angles of a polygon.

- 8-13. Copy the diagram of the regular pentagon at right onto your paper. Then, with your team, find the *sum* of the measures of its interior angles *as many ways as you can*. You may want to use the fact that the sum of the angles of a triangle is 180° . Be prepared to share your team's methods with the class.



8-14. SUM OF THE INTERIOR ANGLES OF A POLYGON

In problem 8-13, you found the sum of the angles of a regular pentagon. But what about other polygons?

- a. Obtain a Lesson 8.1.2 Resource Page from your teacher. Then use one of the methods from problem 8-13 to find the sum of the interior angles of other polygons. Complete the table (also shown below) on the resource page.

Number of Sides of the Polygon	3	4	5	6	7	8	9	10	12
Sum of the Interior Angles of the Polygon	180°								

- b. Does the interior angle sum depend on whether the polygon is convex? Test this idea by drawing a few non-convex polygons (like the one at right) on your paper and determine if it matters whether the polygon is convex. Explain your findings.

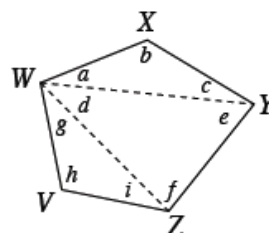


- c. Find the sum of the interior angles of a 100-gon. Explain your reasoning.
- d. LEARNING LOG

In your Learning Log, write an expression that represents the sum of the interior angles of an n -gon. Title this entry "Sum of Interior Angles of a Polygon" and include today's date.

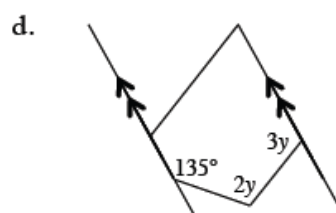
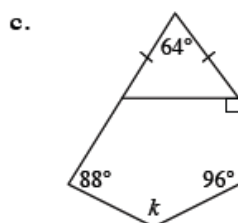
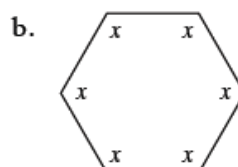
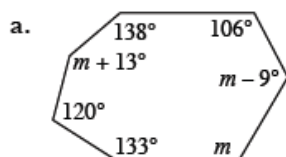


- 8-15. The pentagon at right has been dissected (broken up) into three triangles with the angles labeled as shown. Use the three triangles to prove that the sum of the interior angles of *any* pentagon is always 540° . If you need help doing this, answer the questions below.



- What is the sum of the angles of a triangle?
Use this fact to write three equations based on the triangles in the diagram.
- Add the three equations to create one long equation that represents the sum of all nine angles.
- Substitute the three-letter name for each angle of the pentagon for the lower case letters at each vertex of the pentagon. For example, $m\angle XYZ = c + e$.

- 8-16. Use the angle relationships in each of the diagrams below to solve for the given variables. Show all work.





METHODS AND MEANINGS

Special Quadrilateral Properties

In Chapter 7, you examined several special quadrilaterals and proved conjectures regarding many of their special properties. Review what you learned below.

Parallelogram: Opposite sides of a parallelogram are congruent and parallel. Opposite angles are congruent. Also, since the diagonals create two pair of congruent triangles, the diagonals bisect each other.



Parallelogram

Rhombus: Since a rhombus is a parallelogram, it has all of the properties of a parallelogram. Its diagonals are perpendicular bisectors and bisect the angles of the rhombus. In addition, the diagonals create four congruent triangles.



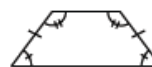
Rhombus

Rectangle: Since a rectangle is a parallelogram, it has all of the properties of a parallelogram. In addition, its diagonals must be congruent.



Rectangle

Isosceles Trapezoid: The base angles (angles joined by a base) of an isosceles trapezoid are congruent.

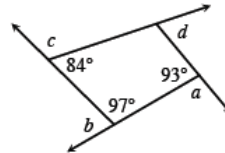


Isosceles Trapezoid

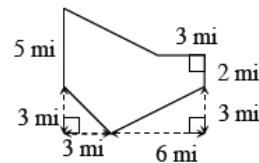


- 8-17. On graph paper, graph $\triangle ABC$ if $A(3, 0)$, $B(2, 7)$, and $C(6, 4)$.
- What is the most specific name for this triangle? Prove your answer is correct using both slope and side length.
 - Find $m\angle A$. Explain how you found your answer.

- 8-18. The exterior angles of a quadrilateral are labeled a , b , c , and d in the diagram at right. Find the measures of a , b , c , and d and then find the sum of the exterior angles.



- 8-19. Find the area and perimeter of the shape at right. Show all work.

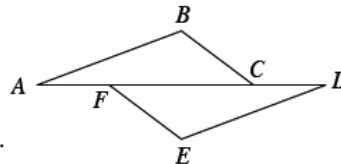


- 8-20. Crystal is amazed! She graphed $\triangle ABC$ using the points $A(5, -1)$, $B(3, -7)$, and $C(6, -2)$. Then she rotated $\triangle ABC$ 90° counterclockwise (↺) about the origin to find $\triangle A'B'C'$. Meanwhile, her teammate took a different triangle ($\triangle TUV$) and rotated it 90° clockwise (↻) about the origin to find $\triangle T'U'V'$. Amazingly, $\triangle A'B'C'$ and $\triangle T'U'V'$ ended up using exactly the same points! Name the coordinates of the vertices of $\triangle TUV$.

- 8-21. Write an equation for each of the following sequences.

- $-30, -15, 0, \dots$
- $9, 3, 1, \dots$

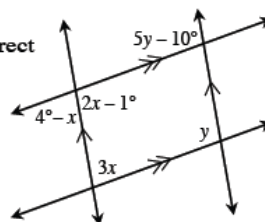
- 8-22. Suzette started to set up a proof to show that if $\overline{BC} \parallel \overline{EF}$, $\overline{AB} \parallel \overline{DE}$, and $AF = DC$, then $\overline{BC} \cong \overline{EF}$. Examine her work below. Then complete her missing statements and reasons.



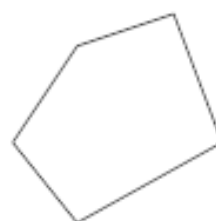
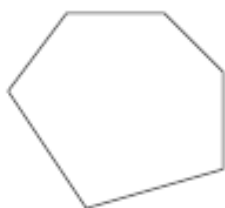
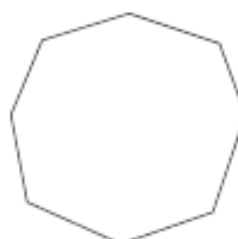
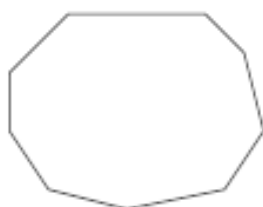
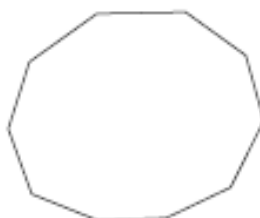
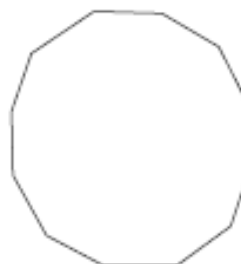
Statements	Reasons
1. $\overline{BC} \parallel \overline{EF}$, $\overline{AB} \parallel \overline{DE}$, and $AF = DC$	1.
2. $m\angle BCF = m\angle EFC$ and $m\angle EDF = m\angle CAB$	2.
3.	3. Reflexive Property
4. $AF + FC + CD = FC$	4. Additive Property of Equality (adding the same amount to both sides of an equation keeps the equation true)
5. $AC = DF$	5. Segment addition
6. $\triangle ABC \cong \triangle DEF$	6.
7.	7. $\cong \Delta s \rightarrow \cong$ parts

- 8-23. **Multiple Choice:** Which equation below is *not* a correct statement based on the information in the diagram?

- $3x + y = 180^\circ$
- $2x - 1^\circ = 4^\circ - x$
- $2x - 1^\circ = 5y - 10^\circ$
- $2x - 1^\circ + 3x = 180^\circ$
- None of these is correct



Lesson 8.1.2 Resource Page

Interior Angle Sum of Polygons $n = 3$  $n = 4$  $n = 5$  $n = 6$  $n = 7$  $n = 8$  $n = 9$  $n = 10$  $n = 12$

Number of Sides of the Polygon	3	4	5	6	7	8	9	10	12
Sum of the Interior Angles of the Polygon	180°								

8.1.3 What if it is a regular polygon?

Angles of Regular Polygons



In Lesson 8.1.2 you discovered how to determine the sum of the interior angles of a polygon with any number of sides. What more can you learn about a polygon? Today you will focus on the interior and exterior angles of regular polygons.

As you work today, keep the following focus questions in mind:

Does it matter if the polygon is regular?

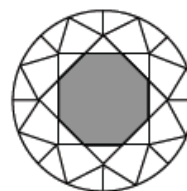
Is there another way to find the answer?

What's the connection?

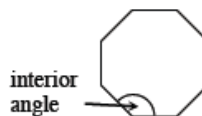
- 8-24. Diamonds, a very valuable naturally-occurring gem, have been popular for centuries because of their beauty, durability, and ability to reflect a spectrum of light. In 1919, a diamond cutter from Belgium, Marcel Tolkowsky, used his knowledge of geometry to design a new shape for a diamond, called the "round brilliant cut" (top view shown at right). He discovered that when diamonds are carefully cut with flat surfaces (called facets or faces) in this design, the angles maximize the brilliance and reflective quality of the gem.



Notice that at the center of this design is a regular octagon with equal sides and equal interior angles. For a diamond cut in this design to achieve its maximum value, the octagon must be cut carefully and accurately. One miscalculation, and the value of the diamond can be cut in half!



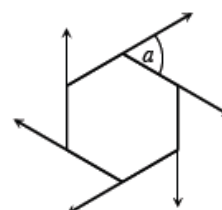
- Determine the measure of each interior angle of a regular octagon. Explain how you found your answer.
- What about the interior angles of other regular polygons? Find the interior angles of a regular nonagon and a regular 100-gon.
- Will the process you used for part (a) work for any regular polygon? Write an expression that will calculate the interior angle of an n -gon.



- 8-25. Fern states, "*If a triangle is equilateral, then all angles have equal measure and it must be a regular polygon.*" Does this reasoning work for polygons with more than three sides? Investigate this idea below.
- If all of the sides of a polygon, such as a quadrilateral, are equal, does that mean that the angles must be equal? If you can, draw a counterexample.
 - What if all of the angles are equal? Does that force a polygon to be equilateral? Explain your thinking. Draw a counterexample on your paper, if possible.

8-26. Jeremy asks, “What about exterior angles? What can we learn about them?”

- a. Examine the regular hexagon shown at right. Angle a is an example of an **exterior angle** because it is formed on the outside of the hexagon by **extending** one of its sides. Are all of the **exterior angles** of a regular polygon equal? Explain how you know.



- b. Find a . Be prepared to share how you found your answer.
- c. This regular hexagon has **six exterior angles**, as shown in the diagram above. What is the sum of the **exterior angles** of a regular hexagon?
- d. What can you determine about the **exterior angles** of other regular polygons? Explore this with your team. Have each team member choose a different shape from the list below to analyze. For each shape:
- Find the measure of one **exterior angle** of that shape, and
 - Find the sum of the **exterior angles**.
- (1) equilateral triangle (2) regular octagon
- (3) regular decagon (4) regular dodecagon (12-gon)
- e. Compare your results from part (d). As a team, write a **conjecture** about the sum of the **exterior angles** of polygons based on your observations. Be ready to share your conjecture with the rest of the class.



- f. Is your conjecture from part (e) true for all polygons or for only regular polygons? Does it matter if the polygon is not **convex**? Explore these questions using a dynamic geometric tool or obtain the Lesson 8.1.3 Resource Page and tracing paper from your teacher. Write a statement **explaining** your findings.

- 8-27. Use your understanding of polygons to answer the questions below, if possible. If there is no solution, explain why not.
- a. Gerardo drew a regular polygon that had exterior angles measuring 40° . How many sides did his polygon have? What is the name for this polygon?
 - b. A polygon has an interior angle sum of 2520° . How many sides does it have?
 - c. A quadrilateral has four sides. What is the measure of each of its interior angles?
 - d. What is the measure of an interior angle of a regular 360-gon? Is there more than one way to find this answer?

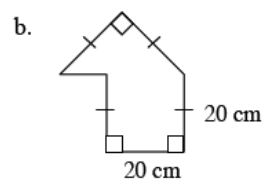
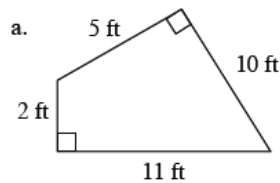
8-28. LEARNING LOG

How can you find the interior angle of a regular polygon?
What is the sum of the exterior angles of a polygon? Write a Learning Log entry about what you learned during this lesson. Title this entry "Interior Angles and Sum of Exterior Angles of a Polygon" and include today's date.

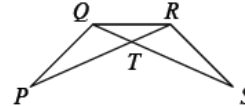




- 8-29. Find the area and perimeter of each shape below. Show all work.



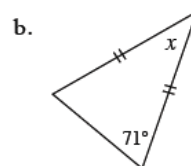
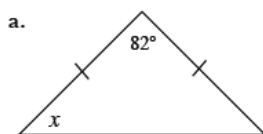
- 8-30. In the figure at right, if $PQ = RS$ and $PR = SQ$, prove that $\angle P \cong \angle S$. Write your proof either in a flowchart or in two-column proof form.



- 8-31. Joey used 10 congruent triangles to create a regular decagon.
- What kind of triangles is he using?
 - Find the three angle measures of one of the triangles. Explain how you know.
 - If the area of each triangle is 14.5 square inches, then what is the area of the regular decagon? Show all work.

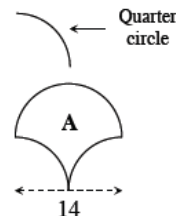
- 8-32. On graph paper, plot $A(2, 2)$ and $B(14, 10)$.
- If C is the midpoint of \overline{AB} , D is the midpoint of \overline{AC} , and E is the midpoint of \overline{CD} , find the coordinates of E .
 - What fraction of the distance from A to B is E ?
 - Use the ratio from part (b) to find the coordinates of E directly as a point a certain ratio between A and B . Show your work.

- 8-33. Use the geometric relationships in the diagrams below to solve for x .



- 8-34. The arc at right is called a quarter circle because it is one-fourth of a circle.

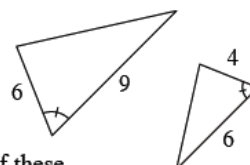
- a. Copy Region A at right onto your paper. If this region is formed using 4 quarter circles, can you find another shape that must have the same area as Region A? Justify your conclusion.



- b. Find the area of Region A. Show all work.

- 8-35. **Multiple Choice:** Which property below can be used to prove that the triangles at right are similar?

- AA ~
- SAS ~
- SSS ~
- HL ~
- None of these



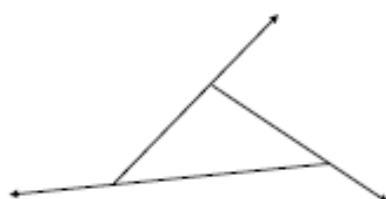
Lesson 8.1.3 Resource Page

Sum of the Exterior Angles of a Polygon

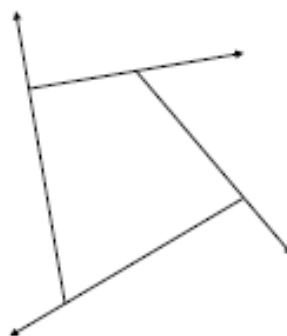
- 8-26. Confirm your conjecture from part (e) by following the directions below.

Directions: For each polygon, start by tracing one exterior angle on tracing paper. Then move the tracing paper to another exterior angle and place it so that the vertices coincide (lie on top of one another) and the angles are adjacent. Then copy the second angle on the tracing paper. Continue to do this until you have copied all adjacent angles. Once you are finished, examine the result. Did the result confirm your conjecture from part (e) of problem 8-26?

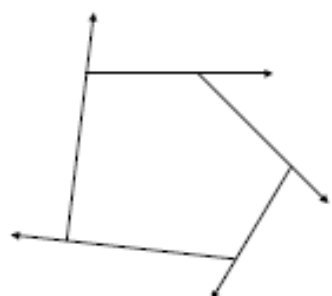
- (1) Triangle



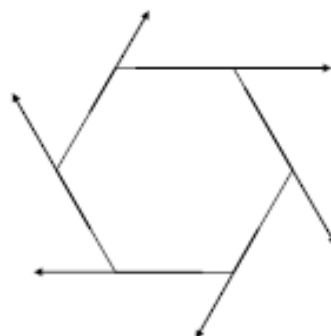
- (2) Quadrilateral



- (3) Pentagon



- (4) Regular Hexagon



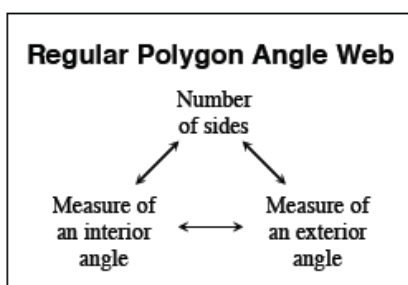
8.1.4 Is there another way?

Regular Polygon Angle Connections



During Lessons 8.1.1 through 8.1.3, you have discovered several ways the number of sides of a regular polygon is related to the measures of the interior and exterior angles of the polygon. These relationships can be represented in the diagram at right.

How can these relationships be useful? What is the most efficient way to go from one measurement to another? This lesson will explore these questions so that you will have a complete set of tools to analyze the angles of a regular polygon.



- 8-36. Which connections in the Regular Polygon Angle Web do you already have? Which do you still need? Explore this as you answer the questions below.
- If you know the number of sides of a regular polygon, how can you find the measure of an interior angle directly? Find the measurements of an interior angle of a 15-gon.
 - If you know the number of sides of a regular polygon, how can you find the measure of an exterior angle directly? Find the measurements of an exterior angle of a 10-gon.
 - What if you know that the measure of an interior angle of a regular polygon is 162° ? How many sides must the polygon have? Show all work.
 - If the measure of an exterior angle of a regular polygon is 15° , how many sides does it have? What is the measure of an interior angle? Show how you know.

- 8-37. Suppose a regular polygon has an interior angle measuring 120° . Find the number of sides using *two* different strategies. Show all work. Which strategy was most efficient?

- 8-38. Use your knowledge of polygons to answer the questions below, if possible.
- How many sides does a polygon have if the sum of the measures of the interior angles is 1980° ? 900° ?
 - If the exterior angle of a regular polygon is 90° , how many sides does it have? What is another name for this shape?
 - Each interior angle of a regular pentagon has measure $2x + 4^\circ$. What is x ? Explain how you found your answer.
 - The measures of four of the exterior angles of a pentagon are 57° , 74° , 56° , and 66° . What is the measure of the remaining angle?
 - Find the sum of the interior angles of an 11-gon. Does it matter if it is regular or not?

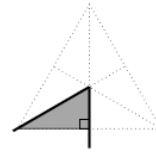
8-39. LEARNING LOG

In a Learning Log entry, copy the Regular Polygon Angle Web that your class created. Explain what it represents and give an **example** of at least two of the connections. Title this entry "Regular Polygon Angle Web" and include today's date.





- 8-40. Esteban used a hinged mirror to create an equilateral triangle, as shown in the diagram at right. If the area of the shaded region is 11.42 square inches, what is the area of the entire equilateral triangle? Justify your solution.

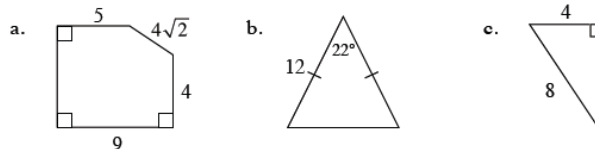


- 8-41. A house purchased for \$135,000 has an annual appreciation of 4%.
- What is the multiplier?
 - Write a function of the form $f(t) = ab^t$ that represents the situation, where t is the time in years after the house was purchased.
 - At the current rate, what will be the value of the house in 10 years?

- 8-42. Copy each shape below on your paper and state if the shape is **convex** or **non-convex**. You may want to compare each figure with the examples provided in problem 8-4.



- 8-43. Find the area of each figure below. Show all work.



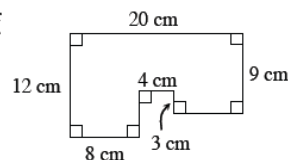
- 8-44. Find the number of sides in a regular polygon if each interior angle has the following measures.

- 60°
- 156°
- 90°
- 140°

- 8-45. Determine if the figures below (not drawn to scale) are similar. Justify your decision.



- 8-46. At right is a scale drawing of the floor plan for Nzinga's dollhouse. The actual dimensions of the dollhouse are 5 times the measurements provided in the floor plan at right.



- Use the measurements provided in the diagram to find the area and perimeter of her floor plan.
- Draw a similar figure on your paper. Label the sides with the actual measurements of Nzinga's dollhouse. What is the perimeter and area of the floor of her actual dollhouse? Show all work.
- Find the ratio of the perimeters of the two figures. What do you notice?
- Find the ratio of the areas of the two figures. How does the ratio of the areas seem to be related to the zoom factor?



8.1.5 What is the area?

Finding Areas of Regular Polygons

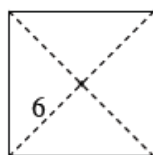


In Lesson 8.1.4, you developed a method to find the measures of the interior and exterior angles of a regular polygon. How can this be useful? Today you will use what you know about the angles of a regular polygon to explore how to find the area of any regular polygon with n sides.

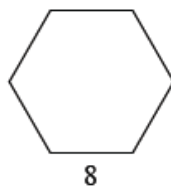
8-47. USING MULTIPLE STRATEGIES

With your team, find the area of each shape below *twice*, each time using a distinctly different method or strategy. Make sure that your results from using different strategies are the same. Be sure that each member of your team understands each method.

a. square



b. regular hexagon



- 8-48. Create a presentation that shows the two different methods that your team used to find the area of the regular hexagon in part (b) of problem 8-47.

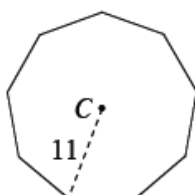
Then, as you listen to other teams present, look for strategies that are different than yours. For each one, consider the questions below.

- *Which geometric tools does this method use?*
- *Would this method help find the area of other regular polygons (like a pentagon or 100-gon)?*

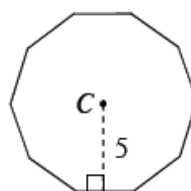


- 8-49. Which method presented by teams in problem 8-48 seemed able to help find the area of other regular polygons? Discuss this with your team. Then find the area of the two regular polygons below. If your method does not work, switch to a different method. Assume C is the center of each polygon.

a.



b.



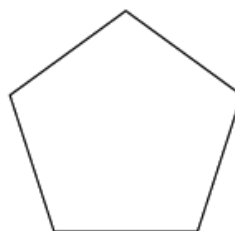
- 8-50. LEARNING LOG

So far, you have found the area of a regular hexagon, nonagon, and decagon. How can you calculate the area of *any* regular polygon? Write a Learning Log entry describing a general process for finding the area of a polygon with n sides. Title this entry "Area of a Regular Polygon" and label it with today's date.



- 8-51. Beth needs to fertilize her flowerbed, which is in the shape of a regular pentagon. A bag of fertilizer states that it can fertilize up to 150 square feet, but Beth is not sure how many bags of fertilizer she should buy.

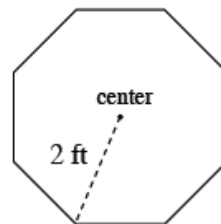
Beth does know that each side of the pentagon is 15 feet long. Copy the diagram of the regular pentagon below onto your paper. Find the area of the flowerbed and tell Beth how many bags of fertilizer to buy. Explain how you found your answer.



15 ft

8-52. GO, ROWDY RODENTS!

Recently, your school ordered a stained-glass window with the design of the school's mascot, the rodent. Your student body has decided that the shape of the window will be a regular octagon, shown at right. To fit in the space, the window must have a radius of 2 feet. The **radius of a regular polygon** is the distance from the center to each vertex.



- a. A major part of the cost of the window is the amount of glass used to make it. The more glass used, the more expensive the window. Your principal has turned to your class to determine how much glass the window will need. Copy the diagram onto your paper and find its area. Explain how you found your answer.
- b. The edge of the window will have a polished brass trim. Each foot of trim will cost \$48.99. How much will the trim cost? Show all work.





METHODS AND MEANINGS

Interior and Exterior Angles of a Polygon

The properties of interior and exterior angles in polygons (which includes regular and non-regular polygons), where n represents the number of sides in the polygon (n -gon), can be summarized as follows:

- The sum of the measures of the interior angles of an n -gon is $180^\circ(n - 2)$.
- The sum of the measures of the exterior angles of an n -gon is always 360° .
- The measure of any interior angle plus its corresponding exterior angle is 180° .

In addition, for *regular* polygons:

- The measure of each interior angle in a regular n -gon is $\frac{180^\circ(n-2)}{n}$.
- The measure of each exterior angle in a regular n -gon is $\frac{360^\circ}{n}$.

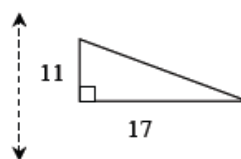


8-53. The exterior angle of a regular polygon is 20° .

- What is the measure of an interior angle of this polygon? Show how you know.
- How many sides does this polygon have? Show all work.

8-54. Examine the triangle and line of reflection at right.

- On your paper, trace the triangle and the line of reflection. Then draw the image of the triangle after it is reflected across the line. Verify your reflection is correct by folding your paper along the line of reflection.



- Find the perimeter of the image. How does it compare with the perimeter of the original figure?
- Find the measure of both acute angles in the original triangle.

8-55. Find the coordinates of the point at which the diagonals of parallelogram $ABCD$ intersect if $B(-3, -17)$ and $D(15, 59)$. Explain how you found your answer.

8-56. Find the area of an equilateral triangle with side length 20 mm. Draw a diagram and show all work.

8-57. A hotel in Las Vegas is famous for its large-scale model of the Eiffel Tower. The model, built to scale, is 128 meters tall and 41 meters wide at its base. If the real tower is 324 meters tall, how wide is the base of the real Eiffel Tower?

8-58. For each equation below, solve for w , if possible. Show all work.

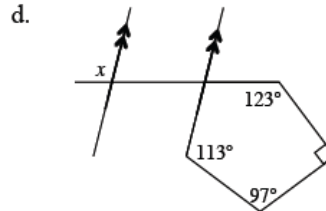
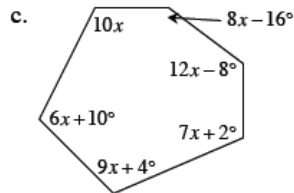
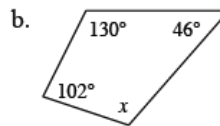
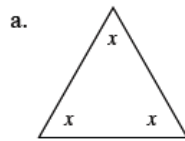
- $5w^2 = 17$
- $5w^2 - 3w - 17 = 0$
- $2w^2 = -3$

8-59. **Multiple Choice:** The triangles at right are congruent because of:

- $SSA \cong$
- $HL \cong$
- $SAS \cong$
- $SSS \cong$
- None of these



8-60. Solve for x in each diagram below.



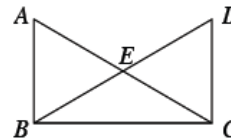
8-61. What is another (more descriptive) name for each polygon described below?

- A regular polygon with an exterior angle measuring 120° .
- A quadrilateral with four equal angles.
- A polygon with an interior angle sum of 1260° .
- A quadrilateral with perpendicular diagonals.

8-62. If $\triangle ABC$ is equilateral and if $A(0, 0)$ and $B(12, 0)$, then what do you know about the coordinates of vertex C ? Prove your answer is true.

8-63. In the figure at right, $\overline{AB} \cong \overline{DC}$ and $\angle ABC \cong \angle DCB$.

- Is $\overline{AC} \cong \overline{DB}$? Prove your answer using a flowchart or two-column proof.
- Do the measures of $\angle ABC$ and $\angle DCB$ make any difference in your solution to part (a)? Explain why or why not.



8-64. On graph paper, graph the parabola $y = 2x^2 - x - 15$.

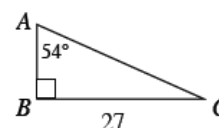
- What are the roots (x -intercepts) of the parabola? Write your points in (x, y) form.
- How would the graph of $y = -(2x^2 - x - 15)$ be the same or different? Can you tell without graphing?

8-65. **Multiple Choice:** To decide if his class would take a quiz today, Mr. Chiu will flip a coin three times. If all three results are heads or all three results are tails, he will give the quiz. Otherwise, his students will not be tested. What is the probability that his class will take the quiz?

- $\frac{1}{8}$
- $\frac{1}{4}$
- $\frac{1}{2}$
- 1

8-66. **Multiple Choice:** Approximate the length of \overline{AB} .

- 15.87
- 21.84
- 37.16
- 19.62
- None of these



8.2.1 How does the area change?

Area Ratios of Similar Figures

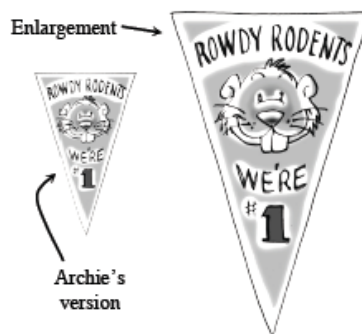


Much of this course has focused on similarity. In Chapter 3, you investigated how to enlarge and reduce a shape to create a similar figure. You also have studied how to use proportional relationships to find the measures of sides of similar figures. Today you will study how the areas of similar figures are related. That is, as a shape is enlarged or reduced in size, how does the area change?

8-67. MIGHTY MASCOT

To celebrate the victory of your school's championship girls' ice hockey team, the student body has decided to hang a giant flag with your school's mascot on the gym wall.

To help design the flag, your friend Archie has created a scale version of the flag measuring 1 foot wide and 1.5 feet tall.



- The student body thinks the final flag should be 3 feet tall. How wide would this enlarged flag be? Justify your solution.
- If Archie used \$2 worth of cloth to create his scale model, then how much will the cloth cost for the full-sized flag? Discuss this with your team. Explain your reasoning.
- Obtain the Lesson 8.2.1A Resource Page and scissors from your teacher. Carefully cut enough copies of Archie's scale version to fit into the large flag. How many did it take? Does this confirm your answer to part (b)? If not, what will the cloth cost for the flag?

d.



The student body is reconsidering the size of the flag. It is now considering enlarging the flag so that it is 3 or 4 times the width of Archie's model. How much would the cloth for a similar flag that is 3 times as wide as Archie's model cost? What if the flag is 4 times as wide?

To answer this question, first *estimate* how many of Archie's drawings would fit into each enlarged flag. Then obtain one copy of the Lesson 8.2.1B Resource Page for your team and confirm each answer by fitting Archie's scale version into the enlarged flags.

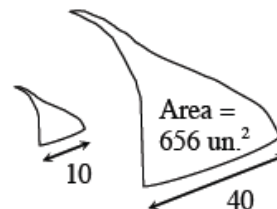
- 8-68. Write down any observations or patterns you found while working on problem 8-61. For **example**, if the area of one shape is 100 times larger than the area of a similar shape, then what is the ratio of the corresponding sides (also called the **linear scale factor**)? And if the linear scale factor is r , then how many times larger is the area of the new shape?

8-69. Use your pattern from problem 8-68 to answer the following questions.

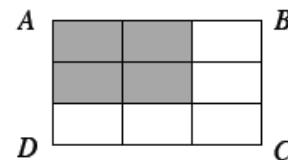
- a. Kelly's shape at right has an area of 17 mm^2 . If she enlarges the shape with a linear scale (zoom) factor of 5, what will be the area of the enlargement? Show how you got your answer.



- b. Examine the two similar shapes at right. What is the linear scale factor? What is the area of the smaller figure?

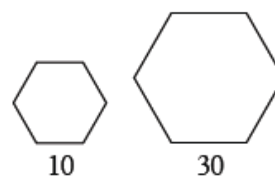


- c. Rectangle $ABCD$ at right is divided into nine smaller congruent rectangles. Is the shaded rectangle similar to $ABCD$? If so, what is the linear scale factor? And what is the ratio of the areas? If the shaded rectangle is not similar to $ABCD$, explain how you know.



- d. While ordering carpet for his rectangular office, Trinh was told by the salesperson that a 16' by 24' piece of carpet costs \$800. Trinh then realized that he read his measurements wrong and that his office is actually 8' by 12'. "Oh, that's no problem," said the salesperson. "That is half the size and will cost \$400 instead." Is that fair? Decide what the price should be.

- 8-70. If the side length of a hexagon triples, how does the area increase? First make a prediction using your pattern from problem 8-68. Then confirm your prediction by calculating and comparing the areas of the two hexagons shown at right.



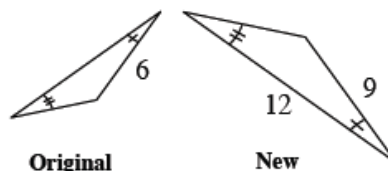


METHODS AND MEANINGS

Ratios of Similarity

Since Chapter 3, you have used the term **zoom factor** to refer to the ratio of corresponding dimensions of two similar figures. However, now that you will be using other ratios of similar figures (such as the ratio of the areas), this ratio needs a more descriptive name. From now on, this text will refer to the ratio of corresponding sides as the **linear scale factor**. The word “linear” is a reference to the fact that the ratio of the lengths of line segments is a comparison of a single dimension of the shapes. Often, this value is represented with the letter r , for ratio.

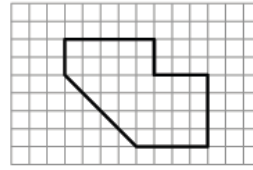
For example, notice that the two triangles at right are similar because of AA \sim . Since the corresponding sides of the new and original shape are 9 and 6, it can be stated that $r = \frac{9}{6} = \frac{3}{2}$.





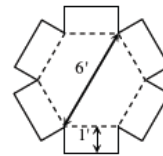
8-71. Examine the shape at right.

- Find the area and perimeter of the shape.
- On graph paper, enlarge the figure so that the linear scale factor is 3. Find the area and perimeter of the new shape.
- What is the ratio of the perimeters of both shapes? What is the ratio of the areas?



8-72. Sandip noticed that when he looked into a mirror that was lying on the ground 8 feet from him, he could see a clock on the wall. If Sandip's eyes are 64 inches off the ground, and if the mirror is 10 feet from the wall, how high above the floor is the clock? Include a diagram in your solution.

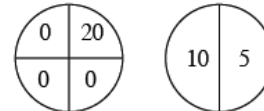
8-73. Mr. Singer has a dining table in the shape of a regular hexagon. While he loves this design, he has trouble finding tablecloths to cover it. He has decided to make his own tablecloth!



In order for his tablecloth to drape over each edge, he will add a rectangular piece along each side of the regular hexagon as shown in the diagram at right. Using the dimensions given in the diagram, find the total area of the cloth Mr. Singer will need.

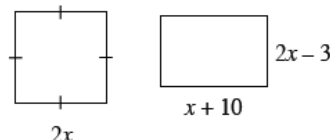
8-74. What is the probability that $x^2 + 7x + k$ is factorable if $0 \leq k \leq 20$ and k is an integer?

8-75. Your teacher has offered your class extra credit. She has created two spinners, shown at right. Your class gets to spin only one of the spinners. The number that the spinner lands on is the number of extra credit points each member of the class will get. Study both spinners carefully.



- Assuming that each spinner is divided into equal portions, which spinner do you think the class should choose to spin and why?
- What if the spot labeled "20" were changed to "100"? Would that make any difference?

8-76. If the rectangles below have the same area, find x . Is there more than one answer? Show all work.



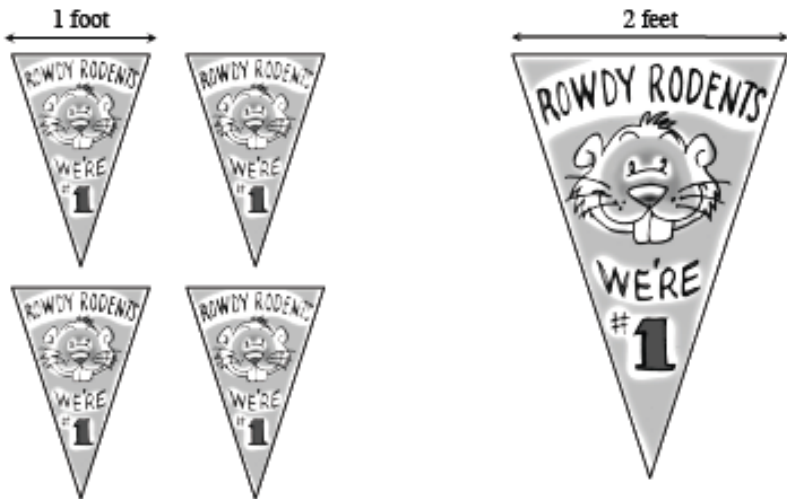
8-77. **Multiple Choice:** A cable 100 feet long is attached 70 feet up the side of a building. If it is pulled taut (i.e., there is no slack) and staked to the ground as far away from the building as possible, approximately what angle does the cable make with the ground?

- 39.99°
- 44.43°
- 45.57°
- 12.22°

Lesson 8.2.1A Resource Page

8-67.

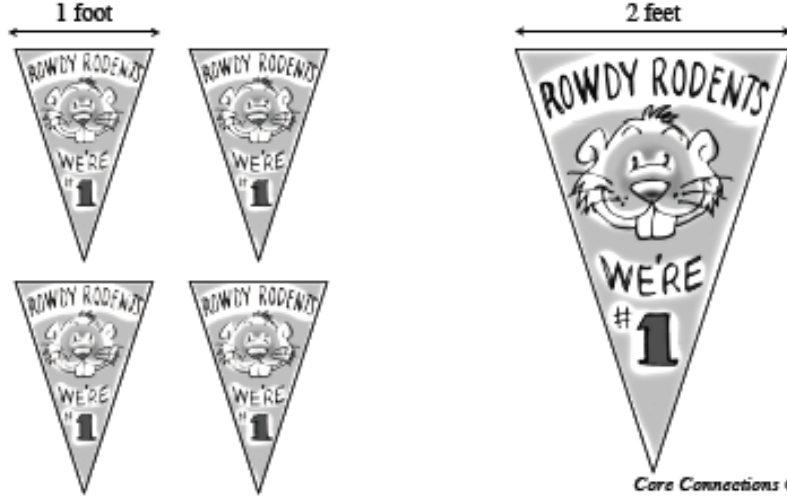
Mighty Mascot



Lesson 8.2.1A Resource Page

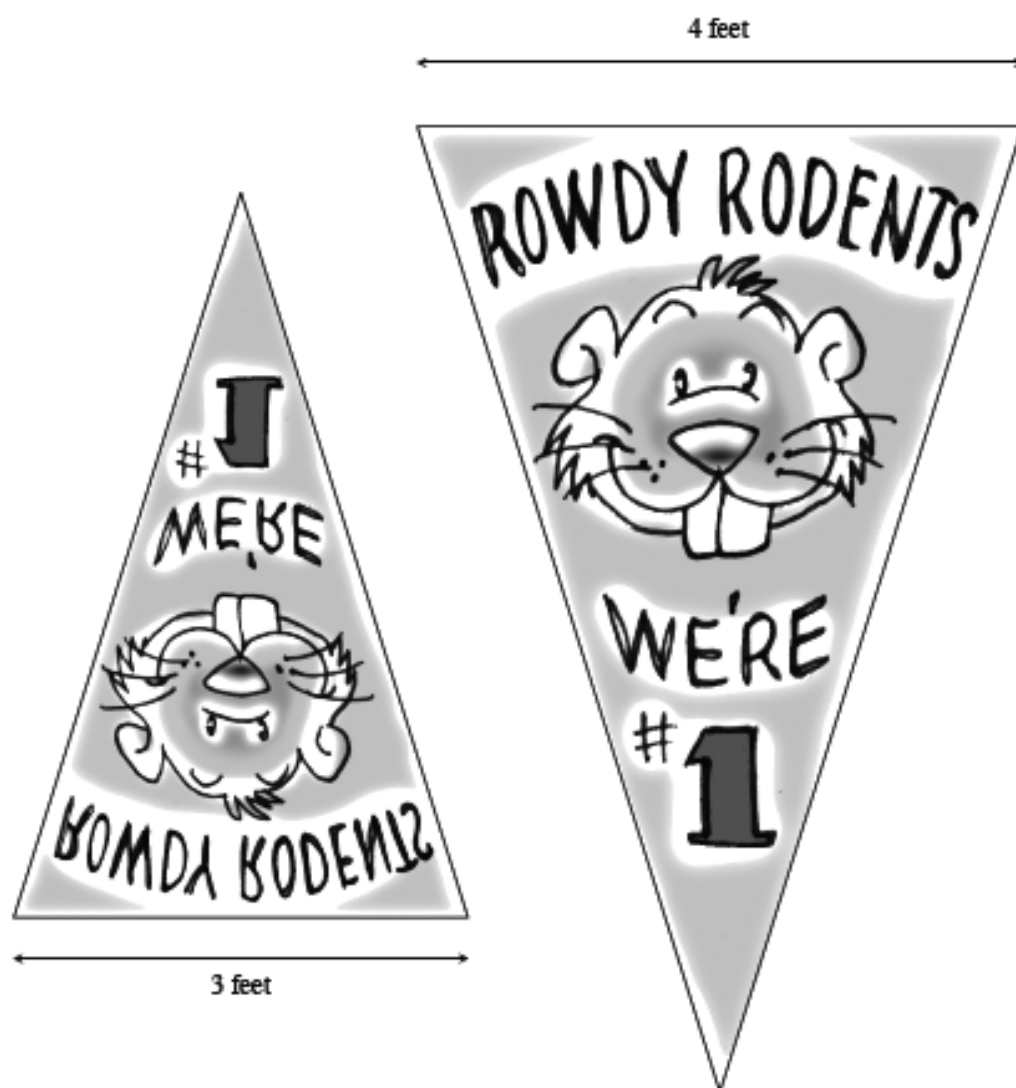
8-67.

Mighty Mascot



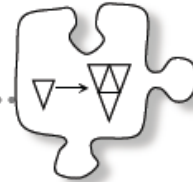
Lesson 8.2.1B Resource Page

8-67.

Enlarged Flags

8.2.2 How does the area change?

Ratios of Similarity



Today you will continue investigating the ratios between similar figures. As you solve today's problems, look for connections between the ratios of similar figures and what you already know about area and perimeter.

8-78. TEAM PHOTO

Alice has a 4-inch by 5-inch photo of your school's championship girls' ice hockey team. To celebrate their recent victory, your principal wants Alice to enlarge her photo for a display case near the main office.



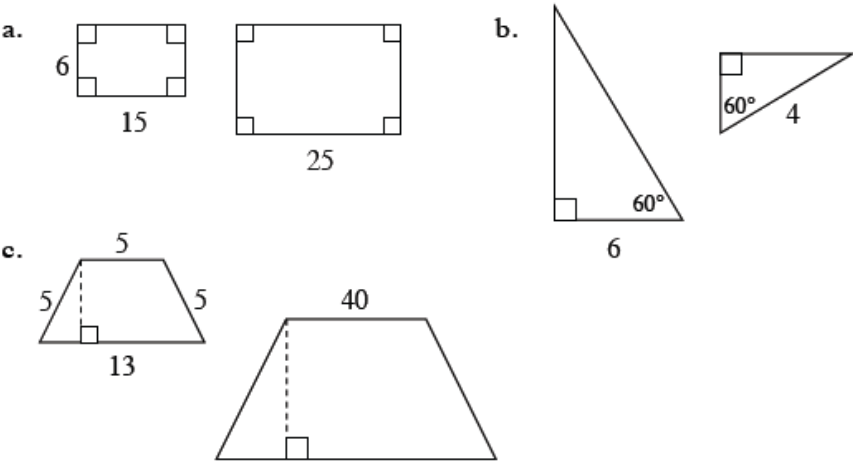
- a. When Alice went to the print shop, she was confronted with many choices of sizes: 7-inch by 9-inch, 8-inch by 10-inch, and 12-inch by 16-inch.

She's afraid that if she picks the wrong size, part of the photo will be cut off. Which size should Alice pick and why?

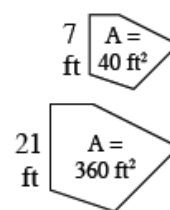
- b. The cost of the photo paper to print Alice's 4-inch-by-5-inch picture is \$0.45. Assuming that the cost per square inch of photo paper remains constant, how much should it cost to print the enlarged photo? Explain how you found your answer.
- c. Unbeknownst to her, the vice-principal also went out and ordered an enlargement of Alice's photo. However, the photo paper for his enlargement cost \$7.20! What are the dimensions of his photo?

8-79. So far, you have discovered and used the relationship between the areas of similar figures. How are the perimeters of similar figures related? Confirm your intuition by analyzing the pairs of similar shapes below. For each pair, calculate the areas and perimeters and complete a table like the one shown below. To help see patterns, reduce fractions to lowest terms or find the corresponding decimal values.

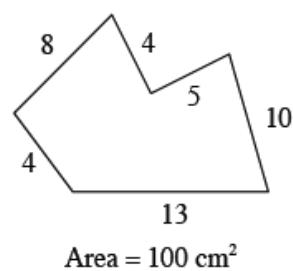
	Ratio of Sides	Perimeter	Ratio of Perimeters	Area	Ratio of Areas
small figure					
large figure					



- 8-80. While Jessie examines the two figures at right, she wonders if they are similar. Decide with your team if there is enough information to determine if the shapes are similar. Justify your conclusion.



- 8-81. Your teacher enlarged the figure at right so that the area of the similar shape is 900 square cm. What is the perimeter of the enlarged figure? Be prepared to explain your method to the class.



8-82. LEARNING LOG

Reflect on what you have learned in Lessons 8.2.1 and 8.2.2. Write a Learning Log entry that **explains** what you know about the areas and perimeters of similar figures.

What connections can you make with other geometric concepts? Be sure to include an **example**. Title this entry "Areas and Perimeters of Similar Figures" and include today's date.



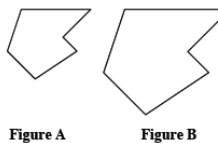


- 8-83. Assume Figure A and Figure B, at right, are similar.

a. If the ratio of similarity is $\frac{3}{4}$, then what is the ratio of the perimeters of Figures A and B?

b. If the perimeter of Figure A is p and the linear scale factor is r , what is the perimeter of Figure B?

c. If the area of Figure A is a and the linear scale factor is r , what is the area of Figure B?



- 8-84. Always a romantic, Marris decided to bake his girlfriend a cookie in the shape of a regular dodecagon (12-gon) for Valentine's Day.

a. If the edge of the dodecagon is 6 cm, what is the area of the top of the cookie?

b. His girlfriend decides to divide the cookie into 12 separate but congruent pieces. After 9 of the pieces have been eaten, what area of cookie is left?

- 8-85. As her team was building triangles with linguini, Karen asked for help building a triangle with sides 5, 6, and 1. "I don't think that's possible," said her teammate, Kelly.

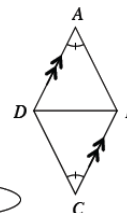
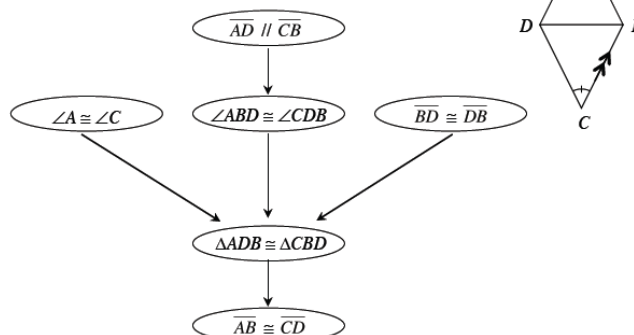
a. Why is this triangle not possible?

b. Change the lengths of one of the sides so that the triangle is possible.

- 8-86. What is the 150th term in this sequence?

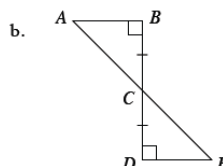
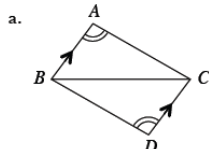
$$17, 16\frac{1}{2}, 16, 15\frac{1}{2}, \dots$$

- 8-87. Callie started to prove that given the information in the diagram at right, then $\overline{AB} \cong \overline{CD}$. Copy her flowchart below on your paper and help her by justifying each statement.



- 8-88. For each pair of triangles below, decide if the triangles are congruent. If the triangles are congruent:

- State which triangle congruence property proves that the triangles are congruent.
- Write a congruence statement (such as $\triangle ABC \cong \triangle \underline{\hspace{1cm}}$).

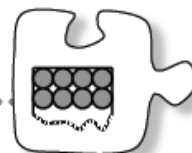


- 8-89. **Multiple Choice:** What is the solution to the system of equations at right?

$$\begin{aligned} y &= \frac{1}{2}x - 4 \\ x - 4y &= 12 \end{aligned}$$

- a. (2, 0) b. (16, 4)
- c. (-2, -5) d. (4, -2)
- e. None of these

8.3.1 What if it has infinitely many sides?



A Special Ratio

In Section 8.1, you developed a method to find the area and perimeter of a regular polygon with n sides. You carefully calculated the area of regular polygons with 5, 6, 8, and even 10 sides. But what if the regular polygon has an infinite number of sides? How can you predict its area?

As you investigate this question today, keep the following focus questions in mind:

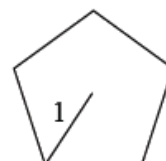
What is the connection?

Do I see any patterns?

How are the shapes related?

8-90. POLYGONS WITH INFINITELY MANY SIDES

In order to predict the area and perimeter of a polygon with infinitely many sides, your team is going to work with other teams to generate data in order to find a pattern.



Your teacher will assign your team three of the regular polygons below. For each polygon, find the area and perimeter if the radius is 1 (as shown in the diagram of the regular pentagon at right). Leave your answer accurate to the nearest 0.01. Place your results into a class chart to help predict the area and perimeter of a polygon with infinitely many sides.

- | | | |
|-------------------------|--------------------|--------------------|
| a. equilateral triangle | b. regular octagon | c. regular 30-gon |
| d. square | e. regular nonagon | f. regular 60-gon |
| g. regular pentagon | h. regular decagon | i. regular 90-gon |
| j. regular hexagon | k. regular 15-gon | l. regular 180-gon |

8-91. ANALYSIS OF DATA

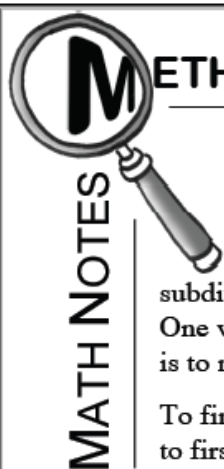
With your team, analyze the chart created by the class.

- a. What do you predict the area will be for a regular polygon with infinitely many sides? What do you predict its perimeter will be?
- b. What is another name for a regular polygon with infinitely many sides?
- c. Does the number $3.14\dots$ look familiar? If so, share what you know with your team. Be ready to share your idea with the class.

8-92. LEARNING LOG

Record the area and circumference of a circle with radius 1 in your Learning Log. Then, include a brief description of how you “discovered” π . Title this entry “Pi” and include today’s date.





METHODS AND MEANINGS

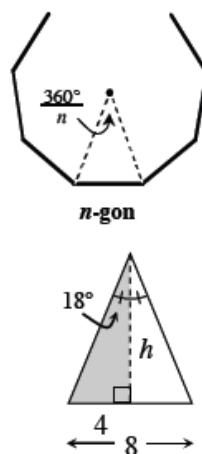
Area of a Regular Polygon

If a polygon is regular with n sides, it can be subdivided into n congruent isosceles triangles. One way to calculate the area of a regular polygon is to multiply the area of one isosceles triangle by n .

To find the area of the isosceles triangle, it is helpful to first find the measure of the polygon's central angle by dividing 360° by n . The height of the isosceles triangle divides the top vertex angle in half.

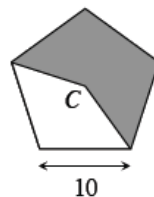
For example, suppose you want to find the area of a regular decagon with side length 8 units. The central angle is $\frac{360^\circ}{10} = 36^\circ$. Then the top angle of the shaded right triangle at right would be $36^\circ \div 2 = 18^\circ$.

Use right triangle trigonometry to find the measurements of the right triangle, then calculate its area. For the shaded triangle above, $\tan 18^\circ = \frac{4}{h}$ and $h \approx 12.311$. Use the height and the base to find the area of the isosceles triangle: $\frac{1}{2}(8)(12.311) \approx 49.242$ sq. units. Then the area of the regular decagon is approximately $10 \cdot 49.242 \approx 492.42$ sq. units.

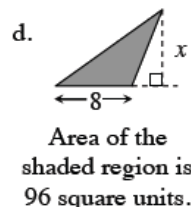
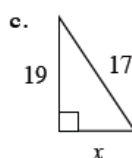
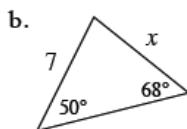
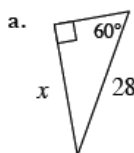




- 8-93. Find the area of the shaded region for the regular pentagon at right if the length of each side of the pentagon is 10 units. Assume that point C is the center of the pentagon.

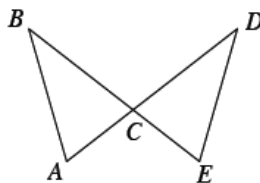


- 8-94. For each triangle below, find the value of x , if possible. Name the triangle tool that you used. If the triangle cannot exist, explain why.

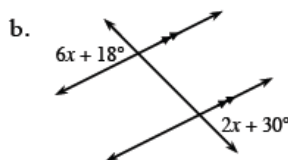
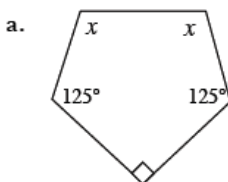


- 8-95. Find the measure of each interior angle of a regular 30-gon using two different methods.

- 8-96. Examine the diagram at right. Assume that \overline{AD} and \overline{BE} are line segments, and that $\overline{BC} \cong \overline{DC}$ and $\angle A \cong \angle E$. Prove that $\overline{AB} \cong \overline{ED}$. Use the form of proof that you prefer (such as the flowchart or two-column proof format). Be sure to copy the diagram onto your paper and add any appropriate markings.



- 8-97. For each diagram below, write and solve an equation to find x .

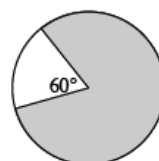


- 8-98. On graph paper, plot the points $A(-3, -1)$ and $B(6, 11)$.

- Find the midpoint M of \overline{AB} .
- Find the point P , on \overline{AB} , that is $\frac{2}{3}$ of the way from A to B .
- Find the equation of the line that passes through points A and B .
- Find the distance between points M and P .

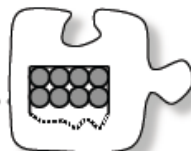
- 8-99. **Multiple Choice:** What fraction of the circle at right is shaded?

- $\frac{60}{360}$
- $\frac{300}{360}$
- $\frac{60}{180}$
- $\frac{120}{180}$
- None of these



8.3.2 What is the relationship?

Area and Circumference of a Circle



In Lesson 8.3.1, your class discovered that the area of a circle with radius 1 unit is π units² and that the circumference is 2π units. But what if the radius of the circle is 5 units or 13.6 units? Today you will develop a method to find the area and circumference of circles when the radius is not 1. You will also explore parts of circles (called sectors and arcs) and learn about their measurements.

As you and your team work together, remember to ask each other questions such as:

Is there another way to solve it?

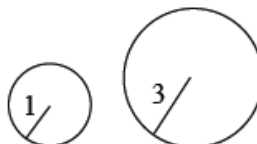
What is the relationship?

What is area? What is circumference?

8-100. AREA AND CIRCUMFERENCE OF A CIRCLE

Now that you know the area and circumference (perimeter) of a circle with radius 1, how can you find the area and circumference of a circle with any radius?

- a. First **examine** how the circles at right are related. Since circles always have the same shape, what is the relationship between any two circles?



- b. What is the ratio of the circumferences (perimeters)? What is the ratio of the areas? **Explain.**
- c. If the area of a circle with radius of 1 is π square units, what is the area of a circle with radius 3 units? With radius 10 units? With radius r units?
- d. Likewise, if the circumference (perimeter) of a circle is 2π units, what is the circumference of a circle with radius 3? With radius 7? With radius r ?

- 8-101. Read the definitions of radius and diameter in the Math Notes box for this lesson. Then answer the questions below.
- a. Find the area of a circle with radius 10 units.
 - b. Find the circumference of a circle with diameter 7 units.
 - c. If the area of a circle is 121π square units, what is its diameter?
 - d. If the circumference of a circle is 20π units, what is its area?

- 8-102. The giant sequoia trees in California are famous for their immense size and old age. Some of the trees are more than 2500 years old and tourists and naturalists often visit to admire their size and beauty. In some cases, you can even drive a car through the base of a tree!

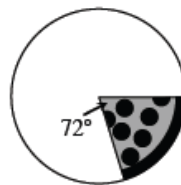
One of these trees, the General Sherman tree in Sequoia National Park, is the largest living thing on the earth. The tree is so gigantic, in fact, that the base has a circumference of 102.6 feet! Assuming that the base of the tree is circular, how wide is the base of the tree? That is, what is its diameter? How does that diameter compare with the length and width of your classroom?



- 8-103. To celebrate their victory, the girls' ice-hockey team went out for pizza.




- a. The goalie ate half of a pizza that had a diameter of 20 inches! What was the area of pizza that she ate? What was the length of crust that she ate? Leave your answers in **exact** form. That is, do not convert your answer to decimal form.
- b. Sonya chose a slice from another pizza that had a diameter of 16 inches. If her slice had a central angle of 45° , what is the area of this slice? What is the length of its crust? Show how you got your answers.
- c. As the evening drew to a close, Sonya noticed that there was only one slice of the goalie's pizza remaining. She measured the central angle and found out that it was 72° . What is the area of the remaining slice? What is the length of its crust? Show how you got your answer.
- d. A portion of a circle (like the crust of a slice of pizza) is called an **arc**. This is a set of connected points a fixed distance from a central point. The length of an arc is a part of the circle's circumference. If a circle has a radius of 6 cm, find the length of an arc with a central angle of 30° .
- e. A region that resembles a slice of pizza is called a **sector**. It is formed by two radii of a central angle and the arc between their endpoints on the circle. If a circle has radius 10 feet, find the area of a sector with a central angle of 20° .



8-104. LEARNING LOG

Reflect on what you have learned today. How did you use similarity to find the areas and circumferences of circles? How are the radius and diameter of a circle related? Write a Learning Log entry about what you learned today. Title this entry "Area and Circumference of a Circle" and include today's date.





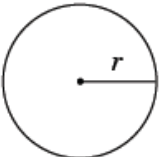
MATH NOTES

METHODS AND MEANINGS

The area of a circle with radius $r = 1$ unit is π units². (Remember that $\pi \approx 3.1415926\dots$)

Since all circles are similar, their areas increase by a square of the linear scale (zoom) factor. That is, a circle with radius 6 has an area that is 36 times the area of a circle with radius 1. Thus, a circle with radius 6 has an area of 36π units², and a circle with radius r has **area** $A = \pi r^2$ units².

Circle Facts





$\text{Area} = \pi r^2$

$\text{Circumference} = 2\pi r = \pi d$

The **circumference** of a circle is its perimeter. It is the distance around a circle. The circumference of a circle with radius $r = 1$ unit is 2π units. Since the perimeter ratio is equal to the ratio of similarity, a circle with radius r has circumference $C = 2\pi r$ units. Since the diameter of a circle is twice its radius, another way to calculate the circumference is $C = \pi d$ units.

A part of a circle is called an **arc**. This is a set of points a **fixed** distance from a center and is defined by a **central angle**. Since a circle does not include its interior region, an arc is like the edge of a crust of a slice of pizza.

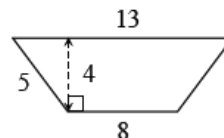
A region that resembles a slice of pizza is called a **sector**. It is formed by two radii of a central angle and the arc between their endpoints on the circle.



- 8-105. A regular hexagon with side length 4 has the same area as a square. What is the length of the side of the square? Explain how you know.

- 8-106. Use what you know about similar figures to complete the following tasks.

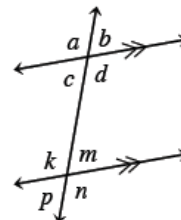


- Find the area and perimeter of the trapezoid at right.
- Find the area and perimeter of the trapezoid that is similar to this one, but has been reduced by a linear scale factor of $\frac{1}{3}$.

- 8-107. An exterior angle of a regular polygon measures 18° .

- How many sides does the polygon have?
- If the length of a side of the polygon is 2, what is the area of the polygon?

- 8-108. Find the missing angle(s) in each problem below using the geometric relationships shown in the diagram at right. Be sure to write down the conjecture that justifies each calculation. Remember that each part is a separate problem.



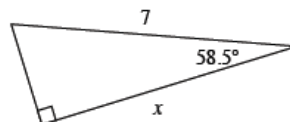
- If $d = 110^\circ$ and $k = 5x - 20^\circ$, write an equation and solve for x .
- If $b = 4x - 11^\circ$ and $n = x + 26^\circ$, write an equation and solve for x . Then find the measure of $\angle n$.

- 8-109. Reynaldo has a stack of blocks on his desk, as shown below at right.



- If his stack is 2 blocks wide, 2 blocks long, and 2 blocks tall, how many blocks are in his stack?
- What if his stack instead is 3 blocks wide, 3 blocks long, and 2 blocks tall? How many blocks are in this stack?
- What if his stack contains 99 blocks, is 3 blocks tall, x blocks long, and y blocks wide? What could x and y be?

- 8-110. Krista is trying to solve for x in the triangle at right.



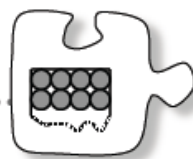
- What equation would Krista write?
- Krista does not have a calculator, but she remembered something funny her friend Juanisha told her. Juanisha's favorite sine ratio is $\sin 31.5^\circ \approx 0.522$ because $5/22$ is Juanisha's birthday! Without a calculator, use Juanisha's favorite ratio to solve your equation from part (a).

- 8-111. **Multiple Choice:** Which type of quadrilateral below does not necessarily have diagonals that bisect each other?

- square
- rectangle
- rhombus
- trapezoid

8.3.3 How can I use it?

Circles in Context

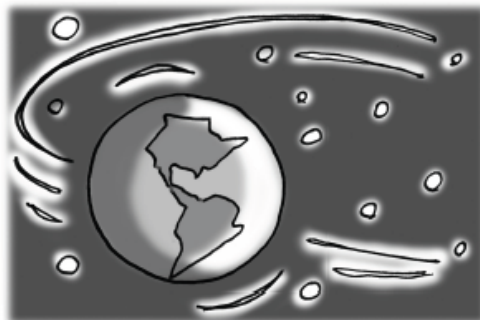


In Lesson 8.3.1, you developed methods to find the area and circumference of a circle with radius r . During this lesson, you will work with your team to solve problems from different contexts involving circles and polygons.

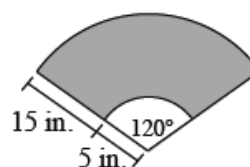
8-112. While the earth's orbit (path) about the sun is slightly elliptical, it can be approximated by a circle with a radius of 93,000,000 miles.

a. How far does the earth travel in one orbit about the sun? That is, what is the approximate circumference of the earth's path?

b. Approximately how fast is the earth traveling in its orbit in space? Calculate your answer in miles per hour.



- 8-113. A certain car's windshield wiper clears a portion of a sector as shown shaded at right. If the angle the wiper pivots during each swing is 120° , find the area of the windshield that is wiped during each swing.



8-114. THE GRAZING GOAT

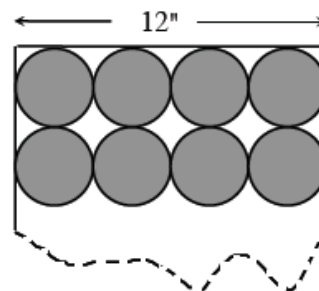
Zoe the goat is tied by a rope to one corner of a 15 meter-by-25 meter rectangular barn in the middle of a large, grassy field. Over what area of the field can Zoe graze if the rope is:

- 10 meters long?
- 20 meters long?
- 30 meters long?
- Zoe is happiest when she has at least 400 m^2 to graze. What possible lengths of rope could be used?



8-115. THE COOKIE CUTTER

A cookie baker has an automatic mixer that turns out a sheet of dough in the shape of a square 12 inches wide. His cookie cutter cuts 3-inch diameter circular cookies as shown at right. The supervisor complained that too much dough was being wasted and ordered the baker to find out what size cookie would have the least amount of waste.

**Your Task:**

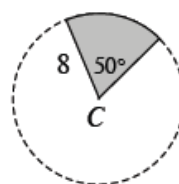
- Analyze this situation and determine how much cookie dough is “wasted” when 3-inch cookies are cut. Then have each team member find the amount of dough wasted when a cookie of a different diameter is used. Compare your results.
- Write a note to the supervisor explaining your results. Justify your conclusion.



METHODS AND MEANINGS

Arc Length and Area of a Sector

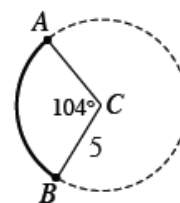
The ratio of the area of a sector to the area of a circle with the same radius equals the ratio of its central angle to 360° . For example, for the sector in circle C at right, the area of the entire circle is $\pi(8)^2 = 64\pi$ square units. Since the central angle is 50° , then the area of the sector can be found with the proportional equation:



$$\frac{50^\circ}{360^\circ} = \frac{\text{area of sector}}{64\pi}$$

Thus, the area of the sector is $\frac{50^\circ}{360^\circ} (64\pi) = \frac{80\pi}{9} \approx 27.93$ sq. units.

The length of an arc can be found using a similar process. The ratio of the length of an arc to the circumference of a circle with the same radius equals the ratio of its central angle to 360° . To find the length of \widehat{AB} at right, first find the circumference of the entire circle, which is $2\pi(5) = 10\pi$ units. Then:



$$\frac{104^\circ}{360^\circ} = \frac{\text{arc length}}{10\pi}$$

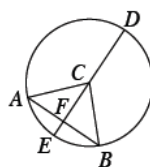
Multiplying both sides of the equation by 10π , the arc length is $\frac{104^\circ}{360^\circ} (10\pi) = \frac{26\pi}{9} \approx 9.08$ units.

You may be surprised to learn that there are other units of measure (besides degrees) to measure an angle. A very special angle measure is called a **radian** and is defined as the measure of a central angle when the length of the arc equals the length of the radius. $1 \text{ radian} = \frac{360^\circ}{2\pi} \approx 57.296^\circ$.



- 8-116. Use what you know about the area and circumference of circles to answer the questions below. Show all work. Leave answers in terms of π .
- If the radius of a circle is 14 units, what is its circumference? What is its area?
 - If a circle has diameter 10 units, what is its circumference? What is its area?
 - If a circle has circumference 100π units, what is its area?
 - If a circle has circumference C , what is its area in terms of C ?
- 8-117. In 2000 a share of Jiffy Stock was worth \$20 and in 2010 it was worth \$50. What was the annual multiplier and the annual percent of increase?

- 8-118. Larry started to set up a proof to show that if $\overline{AB} \perp \overline{DE}$ and \overline{DE} is a diameter of $\odot C$, then $\overline{AF} \cong \overline{FB}$. Examine his work below. Then complete his missing statements and reasons.

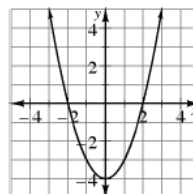


Statements	Reasons
1. $\overline{AB} \perp \overline{DE}$ and \overline{DE} is a diameter of $\odot C$.	1.
2. $\angle AFC$ and $\angle BFC$ are right angles.	2.
3. $FC = FC$	3.
4. $\overline{AC} = \overline{BC}$	4. Definition of a Circle (radii must be equal)
5.	5. HL \cong
6. $\overline{AF} \cong \overline{FB}$	6.

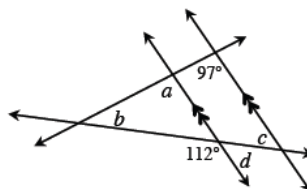
- 8-119. Match each regular polygon named on the left with a statement about its qualities listed on the right.
- regular hexagon
 - regular decagon
 - equilateral triangle
 - square
- Central angle of 36°
 - Exterior angle measure of 90°
 - Interior angle measure of 120°
 - Exterior angle measure of 120°

- 8-120. Examine the graph of $f(x)$ at right. Use the graph to find the following values.

- $f(1)$
- $f(0)$
- x if $f(x) = 4$
- x if $f(x) = 0$

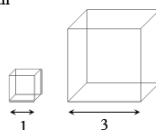


- 8-121. Examine the relationships that exist in the diagram at right. Find the measures of angles a , b , c , and d . Remember that you can find the angles in any order, depending on the angle relationships you use.

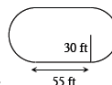


- 8-122. **Multiple Choice:** How many cubes with edge length 1 unit would fit in a cube with edge length 3 units?

- a. 3 b. 9
c. 10 d. 27
e. None of these



- 8-123. The city of Denver wants you to help build a dog park. The design of the park is a rectangle with two semicircular ends. (Note: A semicircle is half of a circle.)



- a. The entire park needs to be covered with grass. If grass is sold by the square foot, how much grass should you order?
- b. The park also needs a fence for its perimeter. A sturdy chain-linked fence costs about \$8 per foot. How much will a fence for the entire park cost?
- c. The local design board has rejected the plan because it was too small. "Big dogs need lots of room to run," the president of the board said. Therefore, you need to increase the size of the park with a linear scale factor of 2. What is the area of the new design? What is the perimeter?

- 8-124. This problem is a checkpoint for angle relationships in geometric figures. It will be referred to as Checkpoint 8.



For each diagram, solve for the variable and name the relationship used.

a. $4x - 3^\circ$ and $3x + 1^\circ$

b. $5x + 6^\circ$ and $2x + 21^\circ$

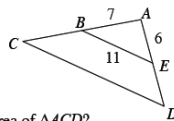
c. $4x + 28^\circ$, $x + 19^\circ$, and $3x + 13^\circ$

d. 40° and $4x - 10^\circ$

Check your answers by referring to the Checkpoint 8 materials located at the back of your book.

If you needed help solving these problems correctly, then you need more practice. Review the Checkpoint 8 materials and try the practice problems. Also, consider getting help outside of class time. From this point on, you will be expected to do problems like these quickly and easily.

- 8-125. \overline{BE} is the midsegment of $\triangle ACD$, shown at right.



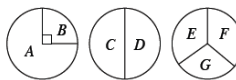
- a. Find the perimeter of $\triangle ACD$.
- b. If the area of $\triangle ABE$ is 54 cm^2 , what is the area of $\triangle ACD$?

- 8-126. Christie has tied a string that is 24 cm long into a closed loop, like the one at right.



- a. She decided to form an equilateral triangle with her string. What is the area of the triangle?
- b. She then forms a square with the same loop of string. What is the area of the square? Is it more or less than the equilateral triangle she created in part (a)?
- c. If she forms a regular hexagon with her string, what would be its area? Compare this area with the areas of the square and equilateral triangle from parts (a) and (b).
- d. What shape do you think that Christie conjectures will enclose the greatest area?

- 8-127. Three spinners are shown at right. If each spinner is randomly spun and if spinners #2 and #3 are each equally divided, find the following probabilities.



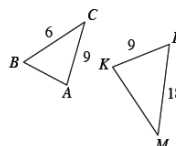
- a. $P(\text{spinning } A, C, \text{ and } E)$
- b. $P(\text{spinning at least one vowel})$

- 8-128. The **Isoperimetric Theorem** states that of all closed figures on a flat surface with the same perimeter, the circle has the greatest area. Use this fact to answer the questions below.

- a. What is the greatest area that can be enclosed by a loop of string that is 24 cm long?
- b. What is the greatest area that can be enclosed by a loop of string that is 18π cm long?

- 8-129. **Multiple Choice:** The diagram at right is not drawn to scale. If $\triangle ABC \sim \triangle KLM$, find KM .

- a. 6 b. 12
c. 15 d. 21
e. None of these



Chapter 8 Closure What have I learned?

Reflection and Synthesis

The activities below offer you a chance to reflect about what you have learned during this chapter. As you work, look for concepts that you feel very comfortable with, ideas that you would like to learn more about, and topics you need more help with. Look for connections between ideas as well as connections with material you learned previously.



① TEAM BRAINSTORM

What have you studied in this chapter? What ideas were important in what you learned? With your team, brainstorm a list. Be as detailed as you can. To help get you started, lists of Learning Log entries and Math Notes boxes are below.

What topics, ideas, and words that you learned *before* this chapter are connected to the new ideas in this chapter? Again, be as detailed as you can.

How long can you make your list? Challenge yourselves. Be prepared to share your team's ideas with the class.



Learning Log Entries

- Lesson 8.1.2 – Sum of Interior Angles of a Polygon
- Lesson 8.1.3 – Interior Angles and Sum of Exterior Angles of a Polygon
- Lesson 8.1.4 – Regular Polygon Angle Web
- Lesson 8.1.5 – Area of a Regular Polygon
- Lesson 8.2.2 – Areas and Perimeters of Similar Figures
- Lesson 8.3.1 – Pi
- Lesson 8.3.2 – Area and Circumference of a Circle

Math Notes

- Lesson 8.1.1 – Convex and Non-Convex Polygons
- Lesson 8.1.2 – Special Quadrilateral Properties
- Lesson 8.1.5 – Interior and Exterior Angles of a Polygon
- Lesson 8.2.1 – Ratios of Similarity
- Lesson 8.3.1 – Area of a Regular Polygon
- Lesson 8.3.2 – Circle Facts
- Lesson 8.3.3 – Arc Length and Area of a Sector

② MAKING CONNECTIONS

Below is a list of the vocabulary used in this chapter. Make sure that you are familiar with all of these words and know what they mean. Refer to the glossary or index for any words that you do not yet understand.

arc	area	central angle
circumference	convex (polygon)	diameter
exterior angle	interior angle	linear scale factor
non-convex (polygon)	perimeter	pi (π)
polygon	radius	radius of a regular polygon
regular polygon	remote interior angle	sector
similar	zoom factor	

Make a concept map showing all of the connections you can find among the key words and ideas listed above. To show a connection between two words, draw a line between them and explain the connection. A word can be connected to any other word as long as you can justify the connection. For each key word or idea, provide an example or sketch that shows the idea.

While you are making your map, your team may think of related words or ideas that are not listed here. Be sure to include these ideas on your concept map.

③ PORTFOLIO: EVIDENCE OF MATHEMATICAL PROFICIENCY

Write down all the ways that you can find the interior and exterior angles of a regular polygon. Make a sketch of a seven-sided regular polygon and any other regular polygon you choose. Showcase your understanding of regular polygons by finding all the interior and exterior angles of these two polygons.



Use problem 8-107 to further showcase your understanding of polygons. Explain clearly and in detail, to a student that does not know anything about the topic, how you found the measurements.

Choose one or two problems from Lesson 8.3.3 that you feel best exhibits your understanding of circles and carefully copy your work, modifying and expanding it if needed. Again, make sure your explanation is clear and detailed. Remember you are not only exhibiting your understanding of the mathematics, but you are also exhibiting your ability to communicate your justifications.

④

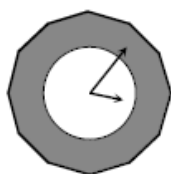
WHAT HAVE I LEARNED?

Most of the problems in this section represent typical problems found in this chapter. They serve as a gauge for you. You can use them to determine which types of problems you can do well and which types of problems require further study and practice. Even if your teacher does not assign this section, it is a good idea to try these problems and find out for yourself what you know and what you still need to work on.



Solve each problem as completely as you can. The table at the end of the closure section has answers to these problems. It also tells you where you can find additional help and practice with problems like these.

CL 8-130.



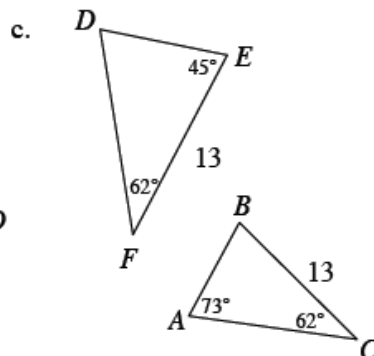
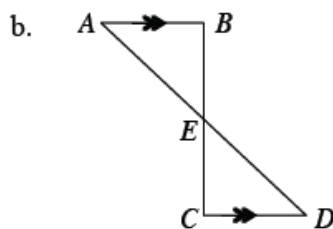
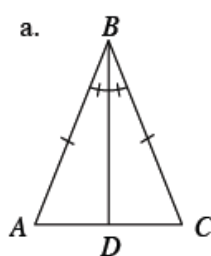
Mrs. Frank loves the clock in her classroom because it has the school colors, green and purple. The shape of the clock is a regular dodecagon with a radius of 14 centimeters. Centered on the clock's face is a green circle of radius 9 cm. If the region outside the circle is purple, which color has more area? (See problem 8-52 in Lesson 8.1.5 for the definition of the radius of a polygon.)

CL 8-131. Graph the quadrilateral $ABCD$ if $A(-2, 6)$, $B(2, 3)$, $C(2, -2)$, and $D(-2, 1)$.

- What is the most descriptive name for this quadrilateral? Justify your conclusion.
- Find the area of $ABCD$.
- Find the slope of the diagonals, \overline{AC} and \overline{BD} . How are the slopes related?
- Find the point of intersection of the diagonals. What is the relationship between this point and diagonal \overline{AC} ?

CL 8-132. Examine the triangle pairs below, which are not necessarily drawn to scale. For each pair, determine:

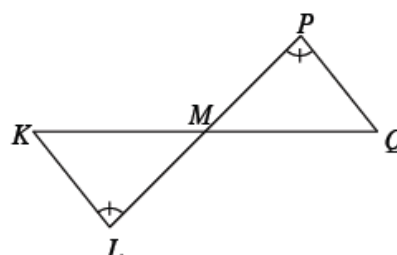
- If they must be congruent. If they are congruent, write a correct congruence statement (such as $\triangle PQR \cong \triangle STU$) and state the congruence property (such as SAS \cong).
- If there is not enough information, explain why you cannot assure congruence.
- If they cannot be congruent, and explain why.



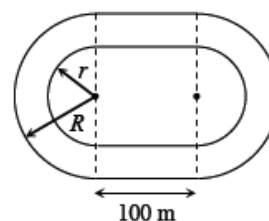
CL 8-133. Complete the following statements.

- If $\triangle YSR \cong \triangle NVD$, then $\overline{DV} \cong \underline{\hspace{1cm}}$ and $m\angle RYS = \underline{\hspace{1cm}}$.
- If \overline{AB} bisects $\angle DAC$, then $\underline{\hspace{1cm}} \cong \underline{\hspace{1cm}}$.
- In $\triangle WQY$, if $\angle WQY \cong \angle QWY$, then $\underline{\hspace{1cm}} \cong \underline{\hspace{1cm}}$.
- If $ABCD$ is a parallelogram, and $m\angle B = 148^\circ$, then $m\angle C = \underline{\hspace{1cm}}$.

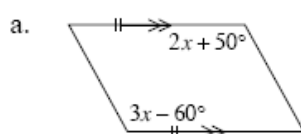
- CL 8-134. Examine the diagram at right. If M is the midpoint of \overline{KQ} and if $\angle P \cong \angle L$, prove that $\overline{KL} \cong \overline{QP}$. Use a flowchart or a two-column proof format.



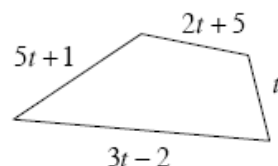
- CL 8-135. A running track design is composed of two half circles connected by two straight-line segments. Garrett is jogging on the inner lane (with radius r) while Devin is jogging on the outer (with radius R). If $r = 30$ meters and $R = 33$ meters, how much longer does Devin have to run to complete one lap?



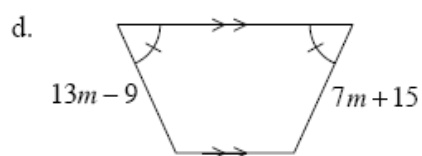
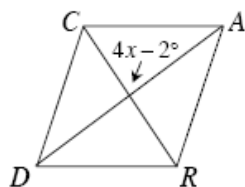
CL 8-136. Use the relationships in the diagrams below to solve for the given variable. Justify your solution with a definition or theorem.



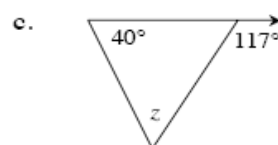
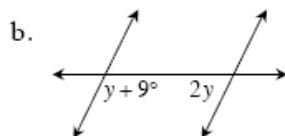
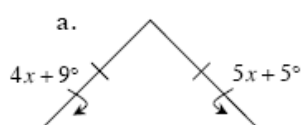
b. The perimeter of the quadrilateral below is 202 units.



c. $CARD$ is a rhombus.



CL 8-137. Use the relationships in the diagrams below to find the values of the variables, if possible. The diagrams are not drawn to scale.



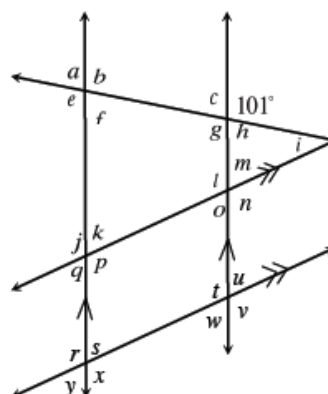
CL 8-138. Answer the following questions about polygons. If there is not enough information or the problem is impossible, explain why.

- a. Find the sum of the interior angles of a dodecagon.
- b. Find the number of sides of a regular polygon if its central angle measures 35° .
- c. If the sum of the interior angles of a regular polygon is 900° , how many sides does the polygon have?
- d. If the exterior angle of a regular polygon is 15° , find its central angle.
- e. Find the exterior angle of a polygon with 10 sides.

CL 8-139. Examine the diagram at right.

- a. Find the measures of each of the angles below, if possible. If it is not possible, explain why it is not possible. If it is possible, state your reasoning.

- | | |
|-----------------|-----------------|
| (1) $m\angle b$ | (2) $m\angle f$ |
| (3) $m\angle m$ | (4) $m\angle g$ |
| (5) $m\angle h$ | (6) $m\angle i$ |

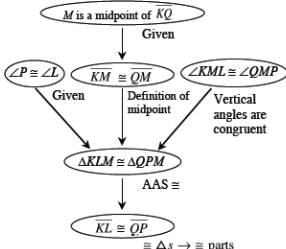


- b. If $m\angle p = 130^\circ$, can you now find the measures of any of the angles from part (a) that you couldn't before? Find the measures for all that you can. Be sure to justify your reasoning.

CL 8-140. Check your answers using the table at the end of the closure section. Which problems do you feel confident about? Which problems were hard? Use the table to make a list of topics you need help on and a list of topics you need to practice more.

Answers and Support for Closure Activity #4
What Have I Learned?

MN: Math Notes, LL: Learning Log

Problem	Solution	Need Help?	More Practice
CL 8-130.	Area of green = $81\pi \approx 254.5 \text{ cm}^2$; area of purple = $588 - 81\pi \approx 333.5 \text{ cm}^2$, so the area of purple is greater.	Lessons 8.1.5 and 8.3.2 MN: 8.1.5, 8.3.1, and 8.3.2 LL: 8.1.4, 8.1.5, and 8.3.2	Problems 8-67, 8-84, 8-93, 8-107(b), and 8-126
CL 8-131.	a. Rhombus. It is a quadrilateral with four equal sides. b. 20 square units c. The slopes are -2 and $\frac{1}{2}$. They are opposite reciprocals. d. The point of intersection is $(0, 2)$. It is the midpoint of the diagonal.	Section 7.3 MN: 7.2.3, 7.3.2, 7.3.3, and 8.1.2 LL: 7.3.3	Problems CL 7-153, 8-17, 8-32, 8-55, and 8-98
CL 8-132.	a. Congruent (SAS \cong), $\triangle ABD \cong \triangle CBD$ b. Not enough information (the triangles are similar (AA \sim), but no side lengths are given to know if they are the same size.) c. Congruent (ASA \cong or AAS \cong), $\triangle ABC \cong \triangle DEF$	Section 3.2 and Lessons 6.1.1 through 6.1.4 MN: 3.2.2 and 6.1.4	Problems CL 3-121, CL 4-123, CL 5-140, CL 6-101, CL 7-155, 8-11, and 8-88
CL 8-133.	a. $\overline{DV} \cong \overline{RS}$, $m\angle RYS = m\angle DNV$ b. $\angle DAB \cong \angle CAB$ c. $\overline{WY} \cong \overline{QY}$ d. $m\angle D = 32^\circ$	MN: 7.1.3	Problems 7-24, 7-38, 7-48, 7-75, 8-22, 8-30, and 8-88
CL 8-134.		Section 3.2 and Lessons 6.1.1 through 6.1.4 MN: 3.2.2, 3.2.4, 6.1.4, and 7.1.3 LL: 3.2.2	Problems CL 3-121, CL 4-123, CL 5-140, CL 6-101, CL 7-155, 8-22, 8-30, 8-63, 8-87, 8-96, and 8-118
CL 8-135.	Devin must run 6π meters farther than Garrett on each lap.	Lesson 8.3.2 MN: 8.3.2 LL: 8.3.2	Problems 8-116 and 8-123(b)
CL 8-136.	a. $x = 110^\circ$ (Opposite angles in a parallelogram are equal.) b. $t = 18$ (Perimeter equals the sum of the sides.) c. $x = 23^\circ$ (Diagonals of a rhombus are perpendicular.) d. $m = 4$ (Nonparallel sides of an isosceles trapezoid are congruent.)	Section 2.1, Lessons 2.3.2 and 7.2.6 MN: 2.1.1, 2.1.4, 2.3.2, 7.2.4, 7.2.6, and 8.1.2 LL: 2.1.1, 2.3.2 Angle Relationships Toolkit	Problems CL 7-154, 8-6, 8-60, 8-97, 8-108, 8-121, and 8-124
CL 8-137.	a. $x = 4^\circ$ b. Cannot determine because the lines are not marked parallel. c. $z = 77^\circ$	Section 2.1 MN: 2.1.1 and 2.1.4 LL: 2.1.1	Problems 8-6, 8-23, 8-33, 8-60, 8-97, 8-108, 8-121, and 8-124
CL 8-138.	a. 1800° b. Impossible. In a regular polygon, the central angle must be a factor of 360° . c. 7 sides d. 15° e. 36°	Lessons 8.1.2, 8.1.3, and 8.1.4 MN: 7.1.4 and 8.1.5 LL: 8.1.2, 8.1.3, and 8.1.4	Problems 8-8, 8-18, 8-44, 8-53, 8-95, 8-107(a), and 8-119
CL 8-139.	a. (1) $b = 101^\circ$, corres. angles are equal (2) $f = 79^\circ$, supplementary angles (3) not enough information (4) $g = 101^\circ$, vertical angles are equal (5) $h = 79^\circ$, supplementary angles (6) not enough information b. $m = 50^\circ$; $v = 130^\circ$; $i = 51^\circ$	Section 2.1 MN: 2.1.1, 2.1.4, and 2.2.1 LL: 2.1.1	Problems 8-6, 8-23, 8-33, 8-60, 8-97, 8-108, 8-121, and 8-124

Arc	Area
Central angle	Circumference
Convex (polygon)	Diameter
Exterior angle	Interior angle
Linear scale factor	Non-convex (polygon)

Perimeter	Pi (π)
Polygon	Radius
Radius of a regular polygon	Remote interior angle
Sector	Similar
Zoom factor	