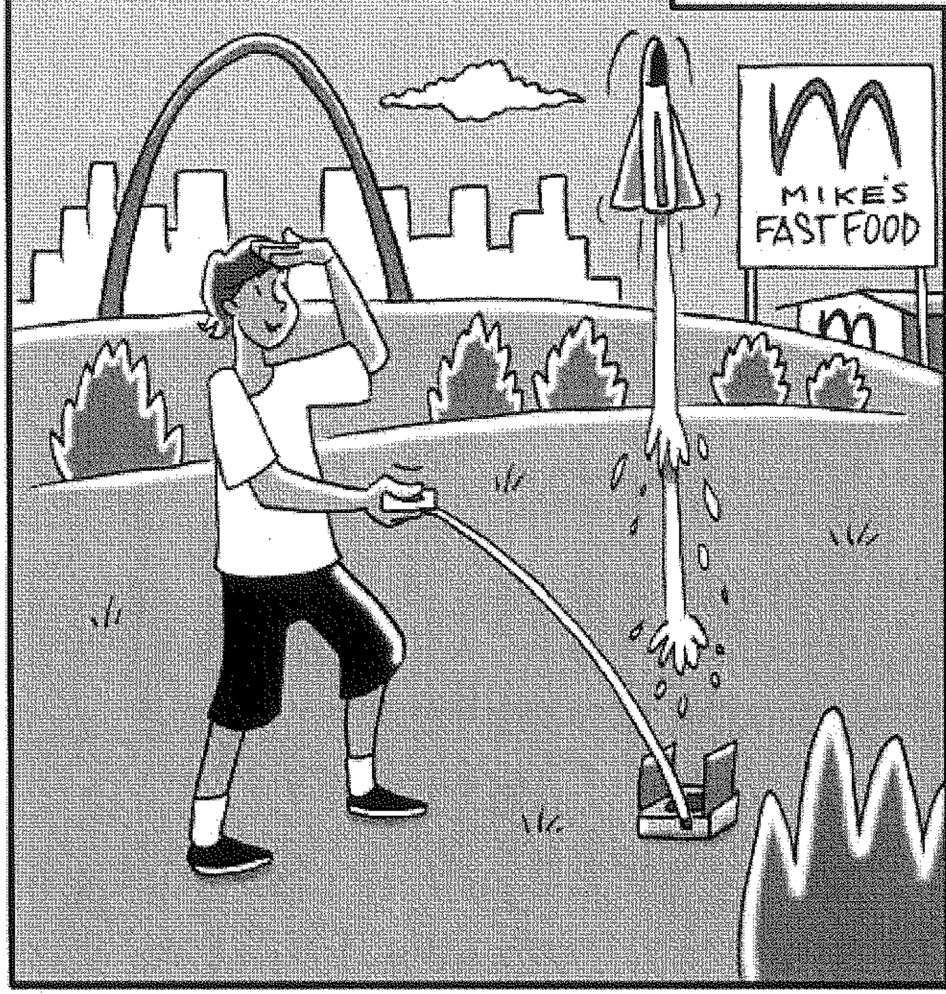
SOLVING QUADRATICS AND INEQUALITIES

9



MATH NOTES

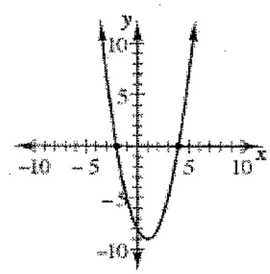
METHODS AND MEANINGS

Zeros and Roots of Quadratics

A **root** or **zero** of a quadratic expression is a value of x that makes the expression equal to zero. For example, the roots or zeros of the quadratic expression x^2-2x-8 are the solutions to the equation $x^2-2x-8=0$.

The x-intercepts of any quadratic function are roots. You find the x-intercepts by setting the function equal to zero and solving for x.

For example, the quadratic function $f(x)=x^2-2x-8=(x+2)(x-4)$ is graphed at right. The x-intercepts are at (0, –2) and (0, 4). The roots or zeros are –2 and 4 because the solutions to the equation $x^2-2x-8=0$ are –2 and 4.



Review & Preview

9-6. Use your generalized process of completing the square to rewrite and solve each quadratic equation below.

a. $w^2 + 28w + 52 = 0$

b. $x^2 + 5x + 4 = 0$

c. $k^2 - 16k - 17 = 0$

d. $z^2 - 1000z + 60775 = 0$

9-7. For each of the following equations, indicate whether its graph would be a line or a parabola.

a. $5x + 2y = 7$

b. $y = 3x^2$

c. $y = 3$

d. $4x^2 + 3x = 7 + y$

9-8. **Multiple Choice:** Which equations below are equivalent to:

$$\frac{1}{2}(6x - 14) + 5x = 2 - 3x + 8 \text{ ?}$$

a. $3x - 7 + 5x = 10 - 3x$

b. $3x - 14 + 5x = 2 - 3x + 8$

c. $8x - 14 = 10 - 3x$

d. $6x - 14 + 10x = 4 - 6x + 16$

9-9. Rewrite each radical expression in exponent form.

a. $\sqrt[3]{10}$

b. $\sqrt{15}$

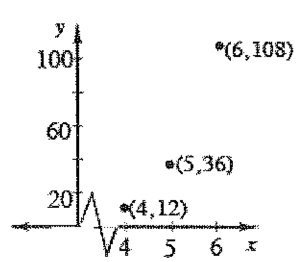
c. $\sqrt[4]{18^3}$

d. $\frac{1}{\sqrt{5}}$

9-10. Examine the two equations below. Where do they intersect?

$$y = 4x - 3$$
$$y = 9x - 13$$

9-11. Find an equation of the sequence represented by this graph.



METHODS AND MEANINGS

MATH NOTES

The Quadratic Formula

Why is $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ a solution of $ax^2+bx+c=0$? One way to derive this formula is shown below.

1. Begin with the quadratic equation in standard form.

$$ax^2+bx+c=0$$

2. Multiply each side by $4a$.

$$4a(ax^2+bx+c)=4a(0)$$
$$4a^2x^2+4abx+4ac=0$$

3. Add b^2-4ac to each side in order to get a factorable quadratic on the left.

$$4a^2x^2+4abx+b^2=b^2-4ac$$

4. The left side can be factored as $(2ax+b)^2$, which is demonstrated in the generic rectangle shown at right.

$$(2ax+b)^2=b^2-4ac$$

b	$2abx$	b^2
$2ax$	$4a^2x^2$	$2abx$
	$2ax$	b

5. Take the square root of each side. Since a square root refers to the *positive* root, the absolute value of $2ax+b$ is used. Then by "looking inside" there are two possible values for $2ax+b$: $+\sqrt{b^2-4ac}$ and $-\sqrt{b^2-4ac}$.

$$|2ax+b|=\sqrt{b^2-4ac}$$

6. Now continue to solve for x by subtracting b from both sides and dividing by $2a$. Notice that a cannot equal zero or else you will get an error! However, if $a=0$, then this equation would not be quadratic and you would not use this formula.

$$2ax+b=\pm\sqrt{b^2-4ac}$$
$$2ax=-b\pm\sqrt{b^2-4ac}$$
$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Thus, $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ are solutions of the equation $ax^2+bx+c=0$.

Review & Preview

9-17. Solve the following quadratic equations by factoring and using the Zero Product Property. Be sure to check your solutions.

a. $x^2 - 13x + 42 = 0$

b. $0 = 3x^2 + 10x - 8$

c. $2x^2 - 10x = 0$

d. $4x^2 + 8x - 60 = 0$

9-18. Use the Quadratic Formula to solve $x^2 - 13x + 42 = 0$. Did your solution match the solution from part (a) of problem 9-17?

9-19. Does a quadratic equation always have two solutions? That is, does a parabola always intersect the x-axis twice?

a. If possible, draw an example of a parabola that only intersects the x-axis once.

b. What does it mean if the quadratic equation has no solution? Draw a possible parabola that would cause this to happen.

9-20. Find the equation of the line through the point $(-2, 8)$ with slope $\frac{1}{2}$.

9-21. For each quadratic function below, use the idea of completing the square to write it in graphing form. Then state the vertex of each parabola.

a. $f(x) = x^2 + 4x + 5$

b. $f(x) = x^2 - 6x$

c. In both graphs, does the vertex represent the maximum or minimum value of the parabola?

9-22. The increased demand for vegetarian meals has caused an increase in the price of tofu. If the current cost of \$2.99 per pound is increasing 6% per year, what will it cost in 5 years?

9-23. Decide if the statements below are true or false. If necessary, review the descriptions for the inequality symbols $<$, $\le$, $>$, and $\ge$ in the Lesson 9.2.1 Math Notes box.

a. $11 < -13$

b. $5(2) \ge 10$

c. $13 > -3(2-6)$

d. $4 \le 4$

e. $9 \ge -9$

f. $-2 > -2$

g. $-16 < -15$

h. $0 > 6$

MATH NOTES

METHODS AND MEANINGS

Solving a Quadratic Equation

So far in this course, you have learned three algebraic methods to solve a quadratic equation of the form $ax^2+bx+c=0$.

Example 1: Solve $3x^2+x-14=0$ for x using the Zero Product Property.

Solution: First, factor the quadratic so it is written as a product: $(3x+7)(x-2)=0$. (If factoring is not possible, one of the other methods of solving must be used.) The Zero Product Property states that if the product of two terms is 0, then at least one of the factors must be 0. Thus, $3x+7=0$ or $x-2=0$. Solving these equations for x reveals that $x=-\frac{7}{3}$ or that $x=2$.

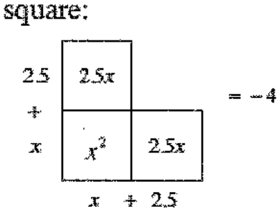
Example 2: Solve $3x^2+x-14=0$ for x using the Quadratic Formula.

Solution: This method works for *any* quadratic. First, identify a, b, and c. a equals the number of x^2 terms, b equals the number of x terms, and c equals the constant. For $3x^2+x-14=0$, $a=3$, $b=1$, and $c=-14$. Substitute the values of a, b, and c into the Quadratic Formula and evaluate the expression twice: once with addition and once with subtraction. Examine this method below:

$$x=\frac{-1+\sqrt{1^2-4(3)(-14)}}{2\cdot 3}=\frac{-1+\sqrt{169}}{6}=\frac{12}{6}=2 \quad \text{or} \quad x=\frac{-1-\sqrt{1^2-4(3)(-14)}}{2\cdot 3}=\frac{-1-\sqrt{169}}{6}=\frac{-14}{6}=-\frac{7}{3}$$

Example 3: Solve $x^2+5x+4=0$ by completing the square.

Solution: This method works most efficiently when the coefficient of x^2 is 1. Rewrite the equation as $x^2+5x=-4$. Rewrite the left side as an incomplete square:



Complete the square and rewrite as $(x+2.5)^2-6.25=-4$ or $(x+2.5)^2=2.25$

Take the square root of both sides, $x+2.5=\pm 1.5$. Solving for x reveals that $x=-1$ or $x=-4$.

9-27. Solve the following quadratic equations by factoring and using the Zero Product Property. Then check your solutions.

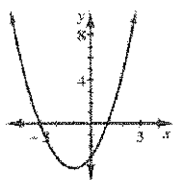
a. $x^2 - 10x + 25 = 0$ b. $0 = 3x^2 + 17x - 6$

c. $3x^2 - 2x = 5$ d. $16x^2 - 9 = 0$

9-28. Use the Quadratic Formula to solve part (b) of problem 9-27 above. Did your solution match the solution you got by factoring and using the Zero Product Property (in part (b) of problem 9-27)?

9-29. Find the equation that represents the information given below.

a.



b.

x	-4	-3	-2	-1	0	1	2	3	4
y	12	5	0	-3	-4	-3	0	5	12

9-30. Solve the following problem using any method. Write your solution as a sentence.

The length of a rectangle is 5 cm longer than twice the length of the width. If the area of the rectangle is 403 square centimeters, what is the width?

9-31. Which of the points below is a solution to $4x - 3y = 10$? Note: More than one point may make this equation true.

a. $(1, 2)$ b. $(4, 2)$ c. $(7, 6)$ d. $(4, -3)$

9-32. In order to quickly get people between terminals in the Minneapolis Airport, long "people mover" conveyor belts were installed. Assume that if someone stood still on a conveyor belt, that person would travel 2 feet per second.

a. Since Jung is in a hurry, he decided to walk on the conveyor belt (in the same direction he would travel standing still). If his terminal is 300 feet away and he wants to get there in 60 seconds, how fast does he need to walk with respect to the conveyor belt? (Assume he can ride the conveyor belt the entire distance.)

b. Jacob, who is four years old, decided it would be fun to walk on the conveyor belt in the "wrong" direction (i.e., in the direction opposite to which he would travel if standing still). If he walks for 18 seconds at a rate of 1 foot per second, how far will he travel? In what direction does he travel? Explain.

9-33. Factor each polynomial completely.

a. $3x^3 - 3x^2$ b. $2x^2 - 10x + 12$

c. $8x^2 - 32$ d. $4x^3 + 10x^2 - 24x$

MATH NOTES

METHODS AND MEANINGS

Simplifying Square Roots

Before calculators were universally available, people who wanted to use approximate decimal values for numbers like $\sqrt{45}$ had a few options:

1. Carry around copies of long square-root tables.
2. Use "guess and check" repeatedly to get desired accuracy.
3. "Simplify" the square roots. A square root is simplified when there are no more perfect square factors (square numbers such as 4, 25, and 81) under the radical sign.

Simplifying square roots was by far the fastest method. People factored the number as the product of integers hoping to find at least one perfect square number. They memorized approximations of the square roots of the integers from one to ten. Then they could figure out the decimal value by multiplying these memorized facts with the roots of the square numbers. Here are some examples of this method.

Example 1: Simplify $\sqrt{45}$.

First rewrite $\sqrt{45}$ in an equivalent factored form so that one of the factors is a perfect square. Simplify the square root of the perfect square. Verify with your calculator that both $3\sqrt{5}$ and $\sqrt{45} \approx 6.71$.

Example 1

$\sqrt{45} = \sqrt{9 \cdot 5}$
$= \sqrt{9} \cdot \sqrt{5}$
$= 3\sqrt{5}$

Examine **Examples 2 and 3** at right. Note that in Example 3, $\sqrt{72}$ was rewritten as $\sqrt{36} \cdot \sqrt{2}$, rather than as $\sqrt{9} \cdot \sqrt{8}$ or $\sqrt{4} \cdot \sqrt{18}$, because 36 is the largest perfect square factor of 72. However, since

Example 2	Example 3
$\sqrt{27}$	$\sqrt{72}$
$= \sqrt{9} \cdot \sqrt{3}$	$= \sqrt{36} \cdot \sqrt{2}$
$= 3\sqrt{3}$	$= 6\sqrt{2}$

$$\sqrt{4} \cdot \sqrt{18} = 2\sqrt{9 \cdot 2} = 2\sqrt{9} \cdot \sqrt{2} = 2 \cdot 3\sqrt{2} = 6\sqrt{2} \quad \text{and}$$
$$\sqrt{9} \cdot \sqrt{8} = 3\sqrt{4 \cdot 2} = 3\sqrt{4} \cdot \sqrt{2} = 3 \cdot 2\sqrt{2} = 6\sqrt{2},$$

you can still get the same answer if you simplify it using different methods.

When you take the square root of an integer that is not a perfect square, the result is a decimal that never repeats itself and never ends. It is a number that cannot be written as a fraction using integers. This result is called an **irrational number**. The irrational numbers and the rational numbers together form the **real numbers**.

Generally, since it is now the age of technology, when a **decimal approximation** of an irrational square root is desired, a calculator is used. However for an exact answer, called **exact form** or **radical form**, the number must be written using the $\sqrt{\ }$ symbol.

Review & Preview

9-38. Write and solve an equation (or system of equations) for the situation described below. Define your variable(s) and write your solution as a sentence.

Daria has 18 coins that are all nickels and quarters. The number of nickels is 3 more than twice the number of quarters. If she has $1.90 in all, how many nickels does Daria have?

9-39. Solve the following quadratic equations using any method.

a. $10000x^2 - 64 = 0$

b. $9x^2 - 8 = -34x$

c. $2x^2 - 4x + 7 = 0$

d. $3.2x + 0.2x^2 - 5 = 0$

9-40. Find a rule that represents the number of tiles in Figure x for the tile pattern at right.

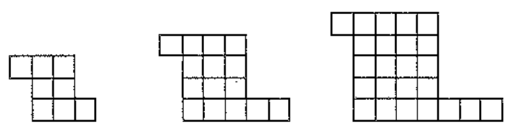


Figure 1 Figure 2 Figure 3

9-41. Solve the equations below for x. Check your solutions.

a. $3x^2 + 3x = 6 + 3x^2$

b. $\frac{5}{x} = \frac{1}{3}$

c. $5 - (2x - 3) = -3x + 6$

d. $6(x - 3) + 2x = 4(2x + 1) - 22$

9-42. Line L passes through the points $(-44, 42)$ and $(-31, 94)$, while line M has the rule $y = 6 + 3x$. Which line is steeper? Justify your answer.

9-43. No Payments For The First Six Months!

You just bought a new tablet computer for $500 including tax. There are no required payments for six months but the company does charge 30% annual interest, compounded monthly, on any unpaid balance.

a. What is the monthly multiplier?

b. If you do not make any payments for six months, how much will you now owe for your new tablet?

METHODS AND MEANINGS

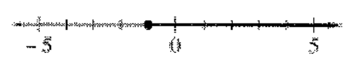
MATH NOTES

Inequality Symbols

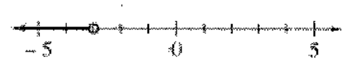
Just as the symbol "=" is used to represent that two quantities are equal in mathematics, the **inequality symbols** at right are used when describing the relationships between quantities that are not necessarily equal.

- $<$ less than
- $\leq$ less than or equal to
- $>$ greater than
- $\geq$ greater than or equal

When graphing an inequality on a number line, such as $x \geq -1$, a filled circle (point) indicates that the value is a solution of the inequality, as shown at right.

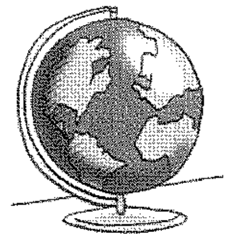


An open circle indicates that the value is not part of the solution, as in $x < -3$, as shown at right.



Review & Preview

9-50. Solve the problem below by writing and solving an equation. Be sure to define your variable.



There are a total of 122 countries in Africa, Europe, and North America (as of 2012). Europe has twice as many countries as North America, and Africa has seven more than Europe. How many countries are in each of these three continents? Write an equation and solve it to answer this question.

9-51. Solve each of the following inequalities for the given variable. Represent your solutions on a number line.

a. $2(3p+1)>-4$ b. $9k-2<3k+10$ c. $5-h\geq 4$

9-52. Solve the following quadratic equations. Check your solutions, if possible.

a. $2k^2+k-6=0$ b. $m^2=9$

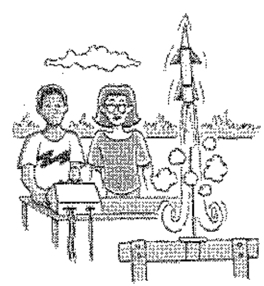
c. $w(2w+8)=24$ d. $3n^2-4n=5$

9-53. Identify the statements below as sometimes true, always true, or never true.

a. $-4\leq 9$ b. $x<1$ c. $-5>-2$

d. $3x+5=2$ e. $61=61$ f. $-6<-6$

9-54. Robbie builds model rockets. One day he sets up a rocket, backs away from the launch pad, and then shoots the rocket off into the air. The rocket's path is represented by the equation $y=-10x^2+130x-400$, where y is the height in meters off the ground and x is the horizontal distance in meters from Robbie.

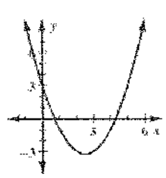


a. Use either the Zero Product Property or the Quadratic Formula to find the x-intercepts of the path of Robbie's rocket. What do the x-intercepts tell you?

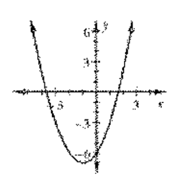
b. When Robbie's rocket lands, how far is it from the launch pad?

9-55. For each parabola graphed below, visually estimate the x-intercepts. Then use the Quadratic Formula to confirm your estimates.

a. $y=x^2-5x+3$



b. $y=x^2+2x-6$



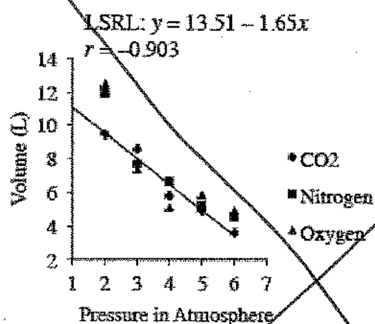
9-59. Solve the inequalities below for the given variables. Represent your solutions on a number line.

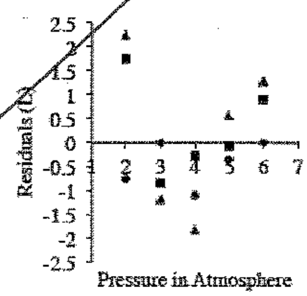
a. $3(2k-1)<9$ b. $\frac{2p}{5}\le 6$

c. $-2+8n>2$ d. $7t-4>2t-4$

9-60. Brett is working in the chemistry lab trying to determine if he can predict the volume of three different gasses (nitrogen, oxygen, and carbon dioxide) based upon the pressure applied to them. He kept all other characteristics of the gasses constant. Brett performed the following analysis:

	Volume		
Pressure	Nitrogen	Oxygen	CO2
2	11.94	12.45	9.45
3	7.71	7.37	8.56
4	6.62	5.09	5.79
5	5.2	5.83	4.9
6	4.51	4.9	3.61





a. Discuss the association, including slope and R-squared.

b. What is the residual with the greatest magnitude and what point does it belong to?

c. Using the LSRL model, estimate the volume of a gas at 2.5, 4.5 and 6.5 atmospheres. Use an appropriate precision.

d. How well would this linear model work in predicting more extreme pressures? Support your answer.

9-62. Find the equation of the line with slope $-\frac{3}{5}$ passing through the point $(-6, 2)$.

9-63. Solve the quadratic equation below. Check your solutions with a calculator.

$$3x^2+2.5x=12.5$$

and

9-64. Factor the expressions below completely, if possible.

a. $4x^2-20x+25$ b. $x^2+11x-2$

c. $3x^2-12x$ d. $10x^2-35x-20$

MATH NOTES

METHODS AND MEANINGS

Curve Fitting an Exponential Function

You can find an exponential function that goes through two points (if both points are above the x-axis). Recall that an exponential function with an asymptote of the x-axis has an equation of the form $y = ab^x$.

To find an exponential function that goes through two given points, create a system of equations by substituting one (x, y) point into $y = ab^x$, then substituting the other point. Rewrite both equations in "$a =$" form. Solve the system with the equal values method to find a and b and now you can write the equation.

For example, find an exponential function that passes through (2, 14) and (5, 112). Create a system of equations by substituting $(x, y) = (2, 14)$ into $y = ab^x$, and then substituting again with $(x, y) = (5, 112)$:

$14 = ab^2$ and $112 = ab^5$ Rewrite as $a = \frac{14}{b^2}$ and $a = \frac{112}{b^5}$.

Use the equal values method to find b:

$$\frac{14}{b^2} = \frac{112}{b^5}$$
$$14b^5 = 112b^2$$
$$14 \cdot \frac{b^5}{b^2} = 112$$
$$14b^3 = 112$$
$$b^3 = \frac{112}{14} = 8$$
$$b = 2$$

Use either original equation to find a:

$$14 = ab^2$$
$$14 = a(2)^2$$
$$\frac{14}{4} = a$$
$$3.5 = a$$

The equation of the line that passes through the two given points is $y = 3.5 \cdot 2^x$.

Review & Preview

9-70. Represent the solutions to the inequalities below on a number line.

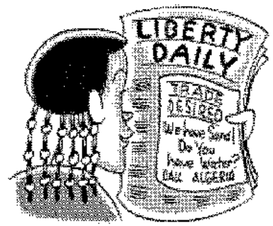
a. $3x-2<10$

b. $5x-1-3x\geq 4x+5$

c. $2(x+2)>10-x$

d. $4(x-3)+5\geq -7$

9-71. Algeria has decided to take out an advertisement in the U.N newspaper, *Liberty Daily*. The newspaper charges a base fee of \$1200 for an ad. There is an additional fee of \$300 for every inch in height. If Algeria is willing to spend any amount up to (and including) \$2700, what choices does the country have for the height of the ad?



9-72. If the graph of an exponential function passes through the points $(1,6)$ and $(4,48)$, find an equation of the function. Refer to the Math Notes box in this lesson if you need help.

9-73. Line m has intercepts $(-7,0)$ and $(0,-2)$.

a. Find the equation of line m.

b. Is the point $(49,-16)$ also on line m? How do you know?

9-74. Thui made the following hypotheses: $2n-1<5$ and $n+1\leq 2n$. Which of the following conclusions can she make?

a. $n\geq 1$ and $n\leq 3$

b. $n\geq 1$ and $n<3$

c. $n>1$ and $n\leq 3$

d. $n>1$ and $n<3$

9-75. **Multiple Choice:** Which of the expressions below is a factor of $6m^2+7m-5$?

a. $2m+1$ b. $m+5$ c. $2m-5$ d. $3m+5$

9-76. For the quadratic function $f(x)=x^2+6x+11$:

a. Use the idea of completing the square to write it in graphing form.

b. State the vertex and sketch a graph of the parabola.

c. Use the graph to explain why the equation $x^2+6x+11=0$ has no real solutions.

MATH NOTES

METHODS AND MEANINGS

Solving One-Variable Linear Inequalities

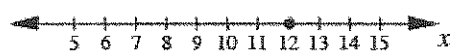
To solve a one-variable linear inequality, first treat the problem as if it were an equality. The solution to the equality is called the **boundary point**. For example, $x = 12$ is the boundary point for the inequality $10 - 2(x-3) \geq -8$, as shown below.

Problem: $10 - 2(x-3) \geq -8$

First change the problem to an equality and solve for x:

$$10 - 2(x-3) = -8$$
$$10 - 2x + 6 = -8$$
$$-2x + 16 = -8$$
$$-2x = -24$$
$$x = 12$$

Since the original inequality is true when $x = 12$, place your boundary point on the number line as a solid point. Then test one value on either side in the *original* inequality to determine which set of numbers makes the inequality true.



Test: $x = 8$	Test: $x = 15$
$10 - 2(8-3) \geq -8$	$10 - 2(15-3) \geq -8$
$10 - 2(5) \geq -8$	$10 - 2(12) \geq -8$
$0 \geq -8$	$-14 \geq 17$
TRUE!	FALSE!

Therefore, the solution is $x \leq 12$.

When the inequality is $<$ or $>$, the boundary point is *not* included in the answer. On a number line, this is indicated with an open circle at the boundary point.

Review & Preview

9-83. **Multiple Choice:** Which of the expressions below is a factor of $6x^2 + 7x - 20$?

a. $3x - 4$ b. $2x - 5$ c. $3x + 4$ d. $4x - 3$

9-84. Graph the inequalities below on graph paper.

a. $y \le -x + 5$ b. $y > \frac{2}{3}x - 1$

9-85. Write and solve an inequality for this situation.

To honor 50 years in business, All Strikes Bowling is having an anniversary special. Shoes rent for \$1.25 and each game is \$0.75. If Charlie has \$20 and needs to rent shoes, how many games can he bowl?

9-86. Solve the following equations and inequalities for x. Check your solution(s), if possible.

a. $\frac{3}{x} = 9$ b. $\sqrt{x} = 4$ c. $x^2 = 25$ d. $2(x - 3) > 4$

9-87. During a race, Bernie ran 9 meters every 4 seconds, while Wendel ran 2 meters every second and got a 9-meter head start. If the race was 70 meters long, did Bernie ever catch up with Wendel? If so, when? Justify your answer.

9-88. Determine the number of times the graph of $y = 5x^2 + 7x - 6$ intersects the x-axis using *two different methods*. The answers from each method should match.

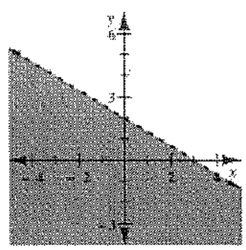
MATH NOTES

METHODS AND MEANINGS

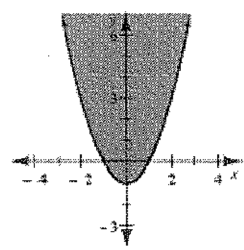
Solving Inequalities with Two Variables

To graph solve an inequality with two variables, first graph the boundary line or curve. If the inequality does not include an equality (that is, if it is > or < rather than ≥ or ≤), then the graph of the boundary is dashed to indicate that it is not included in the solution. Otherwise, the boundary is a solid line or curve.

Once the boundary is graphed, choose a point that does not lie on the boundary to test in the inequality. If that point makes the inequality true, then the entire region in which that point lies is a solution. Examine the two examples below. There are infinite solutions to each of the inequalities. The shaded portion of the graph is a diagram of all of the solutions.



$y < -\frac{2}{3}x + 2$

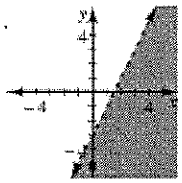


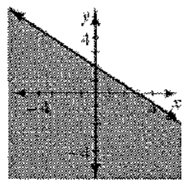
$y \geq x^2 - 1$

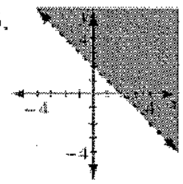
Review & Preview

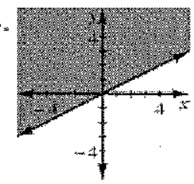
9-94. Match each graph below with the correct inequality.

a. $y > -x + 2$ b. $y < 2x - 3$ c. $y \geq \frac{1}{2}x$ d. $y \leq -\frac{2}{3}x + 2$

1. 

2. 

3. 

4. 

9-95. Solve each inequality below. Represent the solutions on number lines.

a. $7x - 2 < 3 + 2x$ b. $\frac{1}{3}x \geq 2$

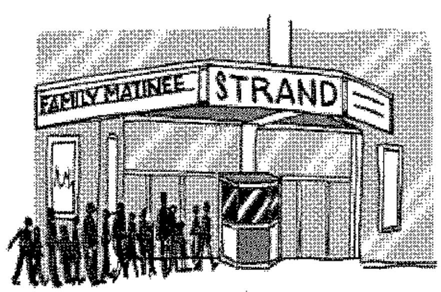
c. $3(2m - 1) - 5m \leq -1$ d. $2k + 3 \leq 2k + 1$

9-96. Three years ago the average price of a movie ticket was \$8.75 and now it is \$11.00. What was the annual multiplier and the percent increase?

9-97. Factor the following quadratics completely.

a. $5x^3 + 13x^2 - 6x$ b. $6t^2 - 26t + 8$ c. $6x^2 - 24$

9-98. When a family with two adults and three children bought tickets for a movie, they paid a total of \$27.75. The next family in line, with two children and three adults, paid \$32.25 for the same movie. Find the adult and child ticket prices by writing a system of equations with two variables.



9-99. Solve the equation below by completing the square. Give you answer in exact (radical) form.

$$x^2 - 6x + 3 = 0$$

9-100. **Multiple Choice:** Which of the points below is a solution of $y < |x - 3|$?

a. (2, 1) b. (−4, 5) c. (−2, 8) d. (0, 3)

Review & Preview

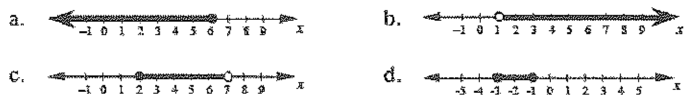
9-105. Graph and shade the solution for the inequality below.

$$y \le x^2 + 2x - 8$$

9-106. Graph and shade the solution for the system of inequalities below.

$$y \ge \frac{3}{4}x - 2$$
$$y < -\frac{1}{2}x + 3$$

9-107. Write the inequality that represents the x-values highlighted on each number line below.



9-108. Determine if the following statements are true or false.

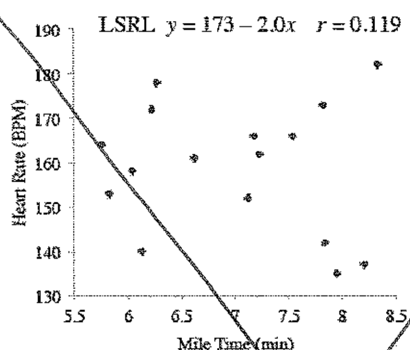
a. $|-6| < 4$

b. $|-3+5| > 2.5$

c. $4 \ge |0|$

d. $|-4+3| > 1$

9-109. Coach Ron has 15 athletes trying out for 12 openings on the varsity basketball team. As part of the tryout he has each player run a mile and Ron records their time and heart rate as they finish. Coach Ron made a scatterplot as follows.

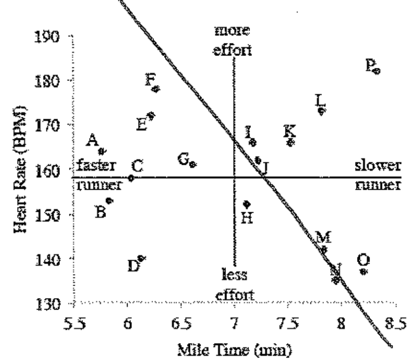


	Min.	BPM
A	5.76	164
B	5.83	153
C	6.04	158
D	6.13	140
E	6.22	172
F	6.27	178
G	6.62	161
H	7.12	152
I	7.18	166
J	7.23	162
K	7.53	166
L	7.82	173
M	7.84	142
N	7.95	135
O	8.2	137
P	8.33	182

a. Describe the association to Coach Ron in detail.

Ron's assistant Cheryl believes they can still find something useful in the results. She proposes that if heart rates could be considered the level of effort given, some athletes may stand out as more or less desirable for the team.

Cheryl divided the scatterplot into four quadrants and put the new labels on the axes. She also labeled each point. She drew the following scatterplot.



b. Coach Cheryl believes that the farther a player is from the center, the more they help or hurt the team. Give specific advice to Coach Ron about players D, F, N, O and P.

Review & Preview

9-114. While setting up a mathematical sentence to solve a problem. Paulina and Aliya came up with the equations below. Since the equations did not look alike, the girls turned to you for help.

$$\text{Paulina: } 4x+2y=6$$

$$\text{Aliya: } 12x+6y=18$$

a. Are these equations equivalent? That is, will the graph of each line be the same? Explain how you know.

b. Find another equation that is equivalent to both of these. How did you find your equation?

9-115. The town you live in has decided to limit the amount of trash thrown out each month. Your town, which has 3280 homes, has asked each household to keep track of how many pounds of trash they produce during a month. In addition, the town council has found that other sources of trash, such as local businesses, combine to create 1500 lbs of trash each month. If the town has a goal of creating *less than* 50,000 lbs of trash, how much trash should a household be limited to? Write an inequality for this situation and solve it.

9-116. Solve the following inequalities for the given variable and represent the solutions on a number line.

a. $2<2m-8$

b. $\frac{1}{3}x-1\le -3$

c. $5(2x-8)+24>3(4+2x)$

d. $5+2k<k-2+k$

9-117. Solve the quadratic equation below *twice*, once using the Quadratic Formula and once by completing the square. Which was easier?

$$x^2-10x+21=-4$$

9-118. Read the following problem. Then decide which system of equations below can represent this situation.

Multiple Choice: The length of a rectangle is 4 units longer than twice its width. If the area is 126 square units, find the length and width.

a. $w=2l+4$, $wl=126$

b. $l=2w+4$, $l+w=126$

c. $w=2l+4$, $l+w=126$

d. $l=2w+4$, $wl=126$

9-119. This problem is a checkpoint for writing exponential equations. It will be referred to as Checkpoint 9.

Write an exponential equation to represent the situation and answer the question.

a. If the cost of a loaf of bread is now \$2.75 and is increasing at 5% per year, what will it cost 10 years from now?

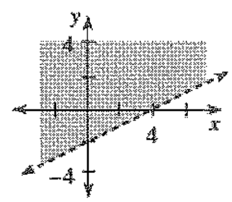
b. The population of Flood River City is now 42,000. Experts predict the population will decrease 25% each year for the next five years. What will be the population in five years?

c. A share of Orange stock that was worth \$25 in 2000 was worth \$60 in 2010. What is the multiplier and percent increase?

Check your answers by referring to the Checkpoint 9 materials located at the back of your book.

If you needed help solving these problems correctly, then you need more practice. Review the Checkpoint 9 materials and try the practice problems. Also, consider getting help outside of class time. From this point on, you will be expected to do like these quickly and easily.

CL 9-120. Write an inequality that represents the graph at right.



CL 9-121. Is the point (0, 4) a solution to the system of inequalities at right? Justify your answer.

$y \le -3x + 4$

$y > x^2 + 3x - 2$

CL 9-122. Factor these quadratic expressions completely, if possible.

a. $x^2 + x - 30$

b. $-3x^3 + 23x^2 - 14x$

c. $2x^2 - 5x + 4$

d. $6x^3 + 10x^2 - 24x$

CL 9-123. Solve each inequality below for the given variable. Then represent each solution on a number line.

a. $4x - 3 \geq 9$

b. $3(t + 4) < 5$

c. $\frac{2y}{7} < 8$

d. $5x + 4 > -3(x - 8)$

CL 9-124. Brian was holding a ballroom dance. He wanted to make sure girls would come, so he charged boys \$5 to get in but girls only \$3. The 45 people who came paid a total of \$175. How many girls came to the dance?

CL 9-125. Solve each quadratic equation using the specified method.

a. The Quadratic Formula

$0 = 3x^2 + 4x - 7$

b. Factoring

$x^2 - 3x - 18 = 0$

c. Completing the square

$x^2 + 4x + 1 = 0$

d. Using a graph

$2x^2 + 5x - 12 = 0$

CL 9-126. Given the quadratic function $f(x)=(x-1)^2-4$:

a. State the location of the vertex.

b. Determine the x-intercepts.

c. Sketch a graph of the function.

CL 9-127. Graph the system of inequalities below on graph paper.

$$y<x^2$$
$$y\geq x+2$$

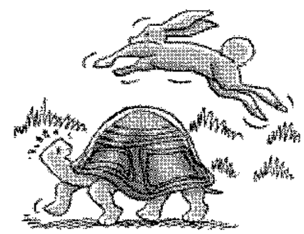
CL 9-128. Lew says to his granddaughter Audrey, "Even if you tripled your age and added 9, you still wouldn't be as old as I am." Lew is 60 years old. Write and solve an inequality to determine the possible ages Audrey could be.

CL 9-129. The cost to rent a DVD has decreased 10% per year over the past several years.

a. If the current cost is $5, write an exponential equation describing this situation.

b. According to the equation, what did it cost to rent a DVD 5 years ago?

CL 9-130. The hare leaps 500 centimeters every 20 seconds. The tortoise crawls 250 centimeters every 50 seconds, but gets a 1000-centimeter head start. Use any method you know to determine how long it takes the hare to catch up to the tortoise.



CL 9-131. Find the equation of an exponential function of the form $y = ab^x$ that passes through the points $(3, 13.5)$ and $(5, 30.375)$.

CL 9-132. Check your answers using the table at the end of the closure section. Which problems do you feel confident about? Which problems were hard? Use the table to make a list of topics you need help on and a list of topics you need to practice more.