
SOLIDS AND CONSTRUCTIONS
9

CHAPTER 9 Solids and Constructions

In your study of geometry so far, you have focused your attention on two-dimensional shapes. You have investigated the special properties of triangles, parallelograms, regular polygons and circles, and have developed tools to help you describe and analyze those shapes. For example, you have tools to find an interior angle of a regular hexagon, to calculate the length of the hypotenuse of a right triangle, and to measure the perimeter of a triangle or the area of a circle.

In Section 9.1, you will turn your focus to three-dimensional shapes (called **solids**), such as cubes and cylinders. You will learn several ways to represent three-dimensional solids and develop methods to measure their volumes and surface areas.

Guiding Question

Mathematically proficient students use appropriate tools strategically.

As you work through this chapter, ask yourself:

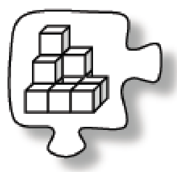
How can I represent it, what tools can I use, and how can I construct it?

Then, in Section 9.2, you will learn how to use special tools to construct accurate diagrams of two-dimensional shapes and geometric relationships. During this investigation, you will revisit many of the geometric conjectures and theorems that you have developed so far.

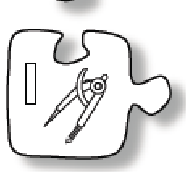
In this chapter, you will learn how to:

- Find the surface area and volume of three-dimensional solids, such as prisms and cylinders.
- Represent a three-dimensional solid with a mat plan, a net, and side and top views.
- Determine the changes to volume when a three-dimensional solid is enlarged proportionally.
- Construct familiar geometric shapes (such as a rhombus or a regular hexagon) using construction tools such as tracing paper, a compass and straightedge, or a dynamic geometry tool.

Chapter Outline



Section 9.1 This section is devoted to the study of three-dimensional solids and their measurement. You will also learn to use a variety of methods to represent the shapes of solids.



Section 9.2 This section will introduce you to the study of constructing geometric shapes and relationships. For example, you will learn how to construct a perpendicular bisector using only a compass and a straightedge.

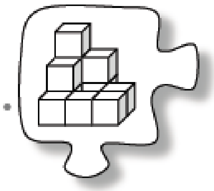
Chapter 9 Teacher Guide

You may wish to omit some of the lessons in this chapter depending on the specific needs of your students. Before skipping any lesson, please refer to the Course Timeline in the "Preparing to Teach This Course" tab in the front matter of this Teacher Edition.

Section	Lesson	Days	Lesson Objectives	Materials	Homework
9.1	9.1.1	1	Three-Dimensional Solids	• Multilink cubes (or alternative) • Index cards, labeled	9-7 to 9-13
	9.1.2	1	Volumes and Surface Areas of Prisms	• Multilink cubes • Index cards, labeled • Lesson 9.1.2 Res. Pg. • Scissors	9-20 to 9-27
	9.1.3	1	Prisms and Cylinders	• Multilink cubes • Deck of cards or ream of paper	9-33 to 9-40
	9.1.4	1	Volumes of Similar Solids	• Graph paper or Lesson 9.1.4 Res. Pg. • Scissors and tape • Multilink cubes	9-45 to 9-52
	9.1.5	1	Ratios of Similarity	• Tetrahedron from problem 7-14 (opt.)	9-56 to 9-63
9.2	9.2.1	1	Introduction to Constructions	• Square tracing paper • Compasses • Straightedges • Lesson 9.2.1 Res. Pg.	9-68 to 9-74
	9.2.2	1	Constructing Bisectors	• Square tracing paper • Compasses • Straightedges • Lesson 9.2.2 Res. Pg.	9-79 to 9-86
	9.2.3	1	More Explorations with Constructions	• Square tracing paper • Compasses • Straightedges • Lesson 9.2.3 Res. Pg.	9-91 to 9-98
	9.2.4	1	Other Constructions	• Compasses • Straightedges • Unlined paper • Cardboard, scissors, and glue • Lesson 9.2.4 Res. Pg.	9-103 to 9-109
Chapter Closure		Various Options			

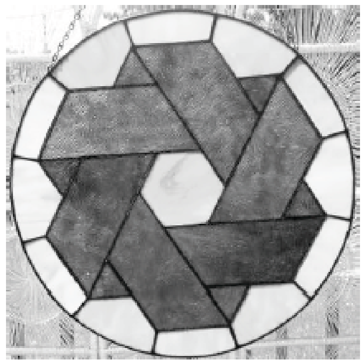
Total: 9 days plus optional time for Chapter Closure and Assessment

9.1.1 How can I build it?



Three-Dimensional Solids

With your knowledge of polygons and circles, you are able to create and explore new, interesting shapes and make elaborate designs such as the one shown in the stained glass window at right. However, in the physical world, the objects you encounter every day are three-dimensional. In other words, physical objects cannot exist entirely on a flat surface, such as a tabletop.



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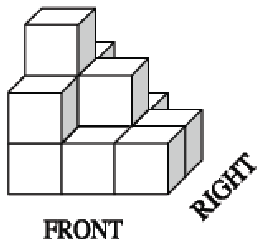
To understand the shapes that you encounter daily, you will need to learn more about how three-dimensional shapes, called **solids**, can be created, described, and measured.

As you work with your team today, be especially careful to explain to your teammates how you "see" each solid. Remember that spatial visualization takes time and effort, so be patient with your teammates and help everyone understand how each solid is built.

9-1. Using blocks provided by your teacher, work with your team to build the three-dimensional solid at right. Assume that blocks cannot hover in midair. That is, if a block is on the second level, assume that it has a block below it to prop it up.



a. Is there more than one arrangement of blocks that could look like the solid drawn at right? Why or why not?

b. To avoid confusion, a **mat plan** can be used to show how the blocks are arranged in the solid. The number in each square represents the number of the blocks stacked in that location if you are looking from above. For example, in the right-hand corner, the solid is only 1 block tall, so there is a "1" in the corresponding corner of its mat plan.

2	1	0	RIGHT
3	2	1	
2	1	1	
FRONT			

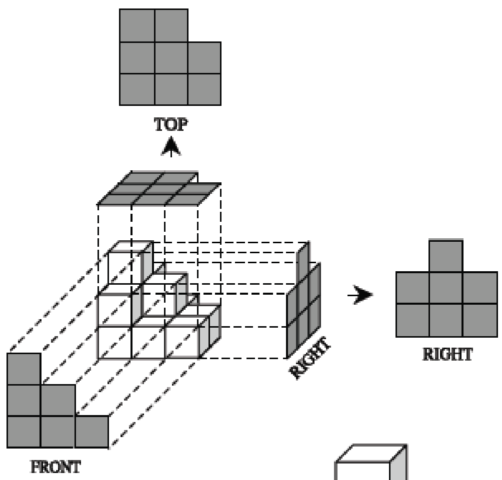
Mat Plan

Verify that the solid your team built matches the solid represented in the mat plan above.

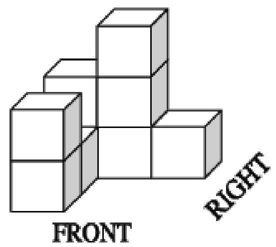
c. What is the **volume** of the solid? That is, if each block represents a cubic unit, how many blocks (cubic units) make up this solid?

9-2. Another way to represent a three-dimensional solid is by its **side** and **top views**.

For example, the solid from problem 9-1 can also be represented by a top, front, and right-hand view, as shown at right. Each view shows *all* of the blocks that are visible when looking directly at the solid from that direction.



Examine the diagram of blocks at right. On graph paper, draw the front, right, and top views of this solid. Assume that there are no hidden blocks.



9-3. For each of the mat plans below:

- Build the three-dimensional solid with the blocks provided by your teacher.
- Find the volume of the solid in cubic units.
- Draw the front, right, and top views of the solid on a piece of graph paper.

a.

0	3	0
2	3	1
0	2	0

RIGHT

FRONT

b.

0	2	1
0	3	0
3	2	1

RIGHT

FRONT

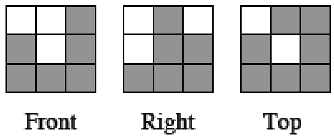
c.

1	1	3
2	1	2
0	0	1

RIGHT

FRONT

9-4. Meagan built a shape with blocks and then drew the views shown at right.



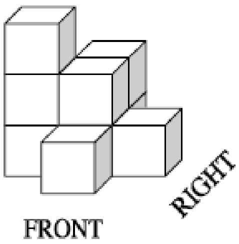
a. Build Meagan's shape using blocks provided by your teacher. Use as few blocks as possible.

b. What is the volume of Meagan's shape?

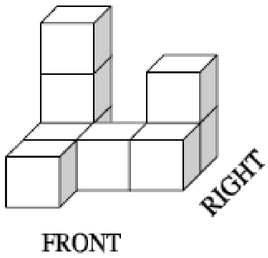
c. Draw a mat plan for her shape.

9-5. Draw a mat plan for each of the following solids. There may be more than one possible answer! Then find the possible volumes of each one.

a.



b.

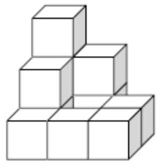


9-6. LEARNING LOG

During this lesson, you have found the volume of several three-dimensional solids. However, what *is* volume? What does it measure? Write a Learning Log entry describing volume. Add at least one example. Title this entry "Volume of a Three-Dimensional Shape" and include today's date.

Review & Preview

9-7. Examine the solid at right.



a. On your paper, draw a possible mat plan for this solid.

b. Find the volume of this solid.

9-8. Assume that two figures on a flat surface, A and B, are similar.

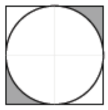
a. If the linear scale factor is $\frac{2}{5}$, then what is the ratio of the areas of A and B?

b. If the ratio of the perimeters of A and B is 14:1, what is the ratio of the areas?

c. If the area of A is 81 times that of B, what is the ratio of the perimeters?

9-9. Find the area of a regular decagon with perimeter 100 units. Show all work.

9-10. The diagram at right shows a circle inscribed in a square. Find the area of the shaded region if the side length of the square is 6 meters.



9-11. Solve each system of equations below. Write your solution in the form (x, y). Check your solution.

a. $3x - y = 14$, $x = 2y + 8$

b. $x = 2y + 2$, $x = -y - 10$

c. $16x - y = -4$, $2x + y = 13$

9-12. Gino looked around at the twelve students in his lunchtime computer science club and wrote down the following descriptions of their sex, clothing, and shoes:

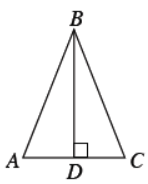
male, long pants, tennis shoes	male, shorts, tennis shoes
female, shorts, tennis shoes	male, shorts, other shoes
female, dress or skirt, other shoes	female, dress or skirt, tennis shoes
female, long pants, tennis shoes	male, long pants, other shoes
male, long pants, other shoes	female, shorts, other shoes
female, long pants, other shoes	male, long pants, tennis shoes

a. Make an area model or tree diagram of all the possible outfits in the sample space. Organize the combinations of sex, clothing, and shoes.

b. In your model or diagram from part (a), indicate the probabilities for each option. What is the probability that a randomly selected student is wearing long pants?

c. Which outcomes are in the event which is the union of {long pants} and {tennis shoes}? Which outcomes are in the intersection of {long pants} and {tennis shoes}?

9-13. **Multiple Choice:** What information would you need to know about the diagram at right in order to prove that $\Delta ABD \cong \Delta CBD$ by SAS $\cong$?



a. $\overline{AD} \cong \overline{CD}$

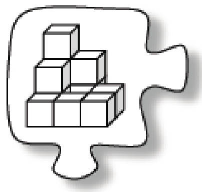
b. $\overline{AB} \cong \overline{CB}$

c. $\angle A \cong \angle C$

d. $\angle ABD \cong \angle CBD$

e. None of these

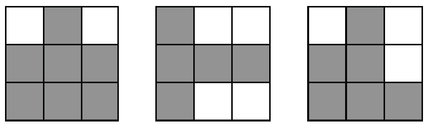
9.1.2 How can I measure it?



Volumes and Surface Areas of Prisms

Today you will continue to study three-dimensional solids and will practice representing a solid using a mat plan and its side and top views. You will also learn a new way to represent a three-dimensional object, called a net. As you work today, you will learn about a special set of solids called prisms and will study how to find the surface area and volume of a prism.

9-14. The front, top, and right-hand views of Heidi's solid are shown at right.



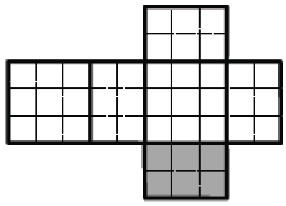
Front Top Right

a. Build Heidi's solid using blocks provided by your teacher. Use the smallest number of blocks possible. What is the volume of her solid?

b. Draw a mat plan for Heidi's solid. Be sure to indicate where the front and right sides are located.

c. Oh no! Heidi accidentally dropped her entire solid into a bucket of paint! What is the **surface area** of her solid? That is, what is the area that is now covered in paint?

9-15. So far, you have studied three ways to represent a solid: a three-dimensional drawing, a mat plan, and its side and top views.

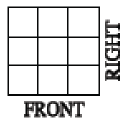


Another way to represent a three-dimensional solid is with a **net**, such as the one shown at right. When folded, a net will form the three-dimensional solid it represents.

a. With your team, predict what the three-dimensional solid formed by this net will look like. Assume the shaded squares make up the base (or bottom) of the solid.

b. Obtain a Lesson 9.1.2 Resource Page and scissors from your teacher and cut out the net. Fold along the solid lines to create the three-dimensional solid. Did the result confirm your prediction from part (a)?

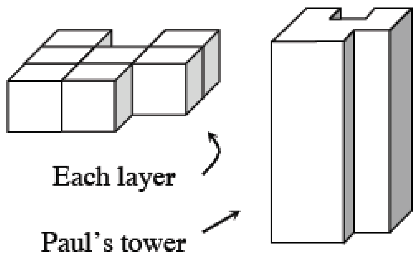
c. Now build the shape with blocks and complete the mat plan at right for this solid.



d. What is the volume of this solid? How did you get your answer?

e. What is the surface area of the solid? How did you find your answer? Be prepared to share any shortcuts with the class.

9-16. Paul built a tower by stacking six identical layers of the shape at right on top of each other.



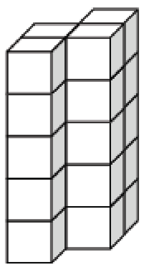
a. What is the volume of his tower? How can you tell without building the shape?

b. What is the surface area of his tower?

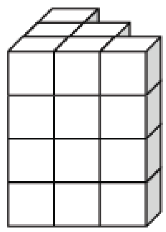
c. Paul's tower is an example of a **prism** because it is a solid and two of its faces (called **bases**) are congruent and parallel. A prism must also have sides that connect the bases (called **lateral faces**). Each lateral face must be a parallelogram (and thus may also be a rectangle, rhombus or a square).

For each of the prisms below, find the volume and surface area.

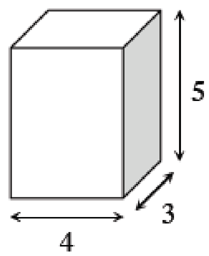
(1)



(2)



(3)



9-17. Heidi created several more solids, represented below. Find the volume of each one.

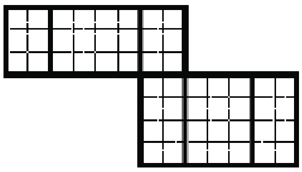
a.

0	3	5
22	10	25
18	15	8
16	12	0

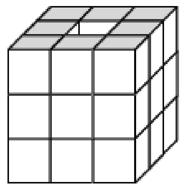
RIGHT

FRONT

b.



c.



9-18. Pilar built a tower by stacking identical layers on top of each other. If her tower used a total of 312 blocks and if the bottom layer has 13 blocks, how tall is her tower? Explain how you know.

9-19. LEARNING LOG

What is the relationship between the area of the base of a prism, its height, and its volume? In a Learning Log entry, summarize how to find the volume of a solid. Be sure to include an example. Title this entry "Finding Volume" and include today's date.

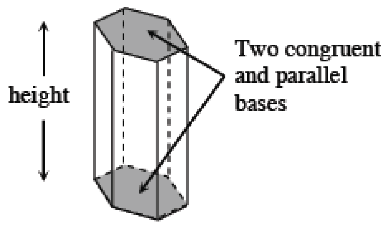
MATH NOTES

METHODS AND MEANINGS

Polyhedra and Prisms

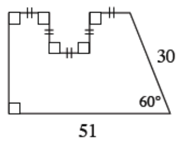
A closed three-dimensional solid that has flat, polygonal faces is called a **polyhedron**. The plural of polyhedron is **polyhedra**. "Poly" is the Greek root for "many," and "hedra" is the Greek root for "faces."

A **prism** is a special type of polyhedron. It must have two congruent, parallel **bases** that are polygons. Also, its **lateral faces** (the faces connecting the bases) are parallelograms formed by connecting the corresponding vertices of the two bases. Note that lateral faces may be any type of parallelogram, such as rectangles, rhombi, or squares.



Review & Preview

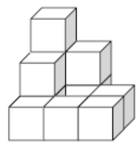
9-20. Mr. Wallis is designing a home. He found the plan for his dream house on the Internet and printed it out on paper.



a. The design of the home is shown at right. If all measurements are in millimeters, find the area of the diagram.

b. Mr. Wallis took his home design to the copier and enlarged it 400%. What is the area of the diagram now? Show how you know.

9-21. At right is the solid from problem 9-7.



a. On graph paper, draw the front, right, and top views.

b. Find the total surface area of the solid.

9-22. Review what you know about the angles of polygons below.

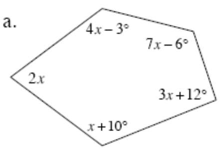
a. If the exterior angle of a polygon is 29°, what is the interior angle?

b. If the interior angle of a polygon is 170°, can it be a regular polygon? Why or why not?

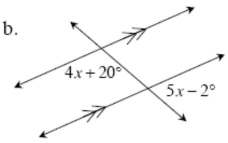
c. Find the sum of the interior angles of a regular 29-gon.

9-23. For each geometric relationship represented below, write and solve an equation for x. Show all work.

a.



b.



9-24. At the band's peak of popularity, a personally signed Black Diamond poster sold for $500. Three years later the band was almost forgotten and the poster was worth only $10. What were the annual multiplier and annual percent of decrease?

9-25. On graph paper, graph ΔABC if $A(-3,-4)$, $B(-1,-6)$, and $C(-5,-8)$.

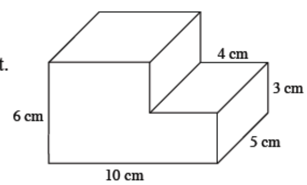
a. What is AB (the length of $\overline{AB}$)?

b. Reflect ΔABC across the x-axis to form $\Delta A'B'C'$. What are the coordinates of B'? Describe the function that would change the coordinates of ΔABC to $\Delta A'B'C'$.

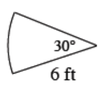
c. Rotate $\Delta A'B'C'$ 90° clockwise (↻) about the origin to form $\Delta A''B''C''$. What are the coordinates of C''?

d. Translate ΔABC so that $(x,y)\rightarrow(x+5,y+1)$. What are the new coordinates of point A?

9-26. Compute the volume of the figure at right.



9-27. **Multiple Choice:** Find the perimeter of the sector at right.

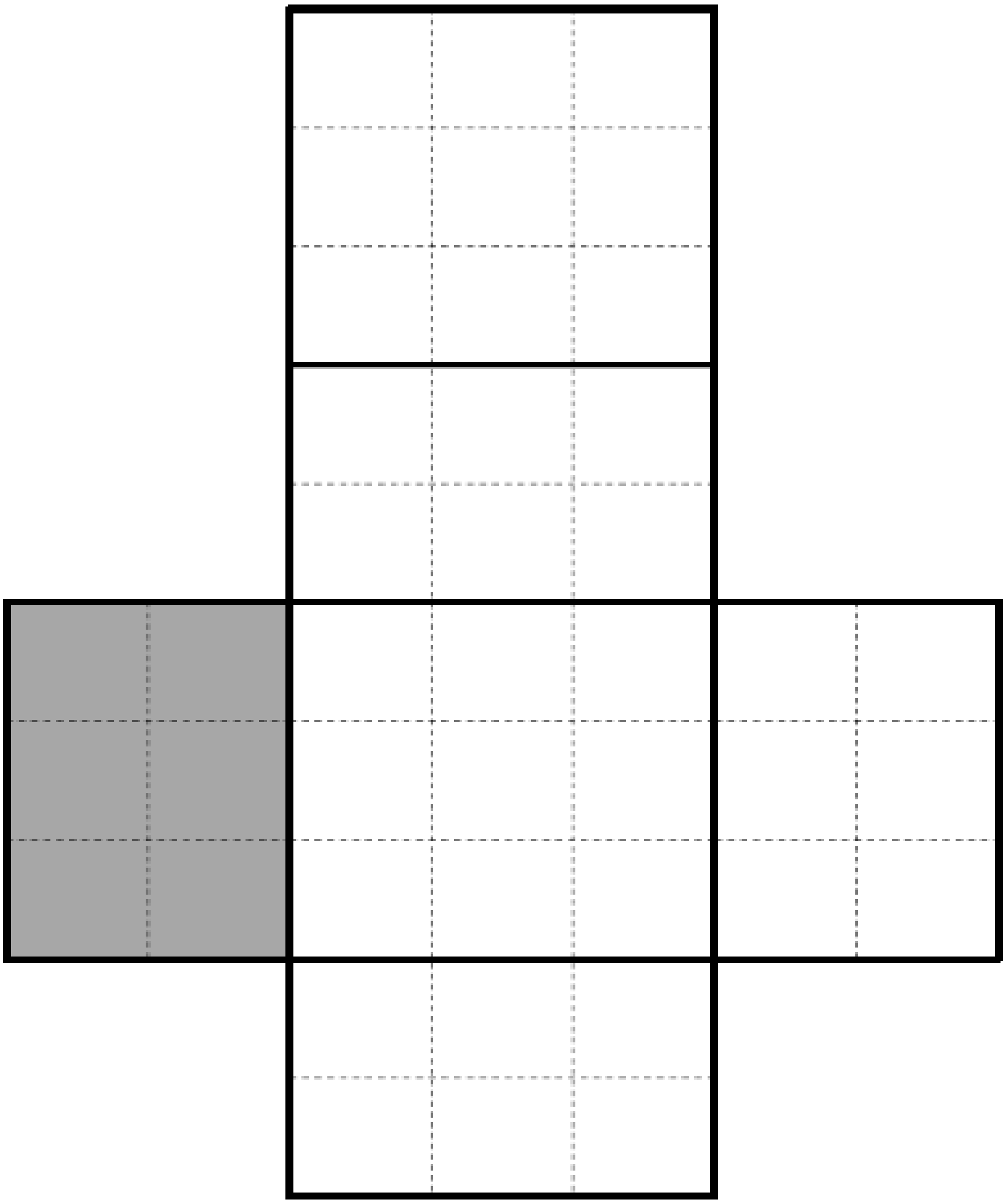


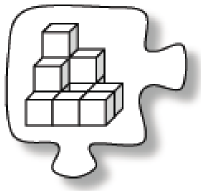
a. 12π ft b. 3π ft c. $6+3\pi$ ft

d. $12+\pi$ ft e. None of these

Lesson 9.1.2 Resource Page

Net for Problem 9-15



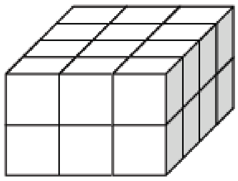


9.1.3 What if the bases are not rectangles?

Prisms and Cylinders

In Lessons 9.1.1 and 9.1.2, you investigated volume, surface area, and special three-dimensional solids called prisms. Today you will explore different ways to find the volume and surface area of a prism and a related solid called a cylinder. You will also consider what happens to the volume of a prism or cylinder if it slants to one side or if it is enlarged proportionally.

9-28. Examine the three-dimensional solid at right.

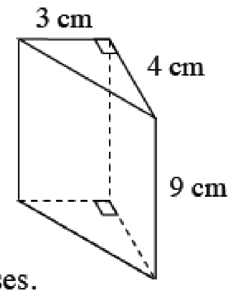


a. On graph paper, draw a net that, when folded, will create this solid.

b. Compare your net with those of your teammates. Is there more than one possible net? Why or why not?

c. Find the surface area and volume of this solid.

9-29. SPECIAL PRISMS



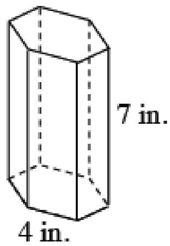
The prism in problem 9-28 is an example of a **rectangular prism**, because its bases are rectangular. Similarly, the prism at right is called a **triangular prism** because the two congruent bases are triangular.

a. Carefully draw the prism at right onto your paper. One way to do this is to draw the two triangular bases first and then to connect the corresponding vertices of the bases. Notice that hidden edges are represented with dashed lines.

b. Find the surface area of the triangular prism. Remember that the surface area includes the areas of *all* surfaces – the sides and the bases. Carefully organize your work and verify your solution with your teammates.

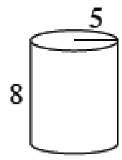
c. Find the volume of the triangular prism. Be prepared to share your team's method with the class.

d. Does your method for finding surface area and volume work on other prisms? For example, what if the bases are hexagonal, like the one shown at right? Work with your team to find the surface area and volume of this hexagonal prism. Assume that the bases are regular hexagons with side length 4 inches.



9-30. CYLINDERS

Carter wonders, "*What if the bases are circular?*" Copy the **cylinder** at right onto your paper. Discuss with your team how to find its surface area and volume if the radius of the base is 5 units and the height of the cylinder is 8 units.



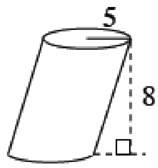
9-31. CAVALIERI'S PRINCIPLE

Bonaventura Cavalieri (1598-1647) was a mathematician who helped to develop calculus, but is best remembered today for a principle named for him. Cavalieri's Principle can be thought of as a way of finding volumes in a relatively easy way.

a. Suppose you have a stack of 25 pennies piled one on top of the other. You decide to slant the stack by sliding some of the pennies over. Does the volume of the 25 pennies change even though they are no longer stacked one on top of another.

b. Would the same thing be true of a stack of 15 books that you slide to the side or twist some of them? What about a stack of 1000 sheets of paper?

c. The idea of viewing solids as slices that can be moved around without affecting the volume is called **Cavalieri's Principle**. Use this principle to find the volume of the cylinder at right. Note that when the lateral faces of a prism or cylinder are not perpendicular to its base, the solid is referred to as an **oblique** cylinder or prism. How is the volume of this prism related to the one in problem 9-30?

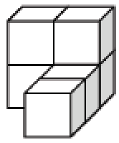


9-32. Hernando needs to replace the hot water tank at his house. He estimates that his family needs a tank that can hold at least 75 gallons of water. His local water tank supplier has a cylindrical model that has a diameter of 2 feet and a height of 3 feet. If 1 gallon of water is approximately 0.1337 cubic feet, determine if the supplier's tank will provide enough water.

MATH NOTES

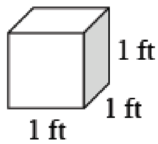
METHODS AND MEANINGS

Volume and Total Surface Area of a Solid



Volume measures the size of a three-dimensional space enclosed within an object. It is expressed as the number of $1 \times 1 \times 1$ cubes (or parts of cubes) that fit inside a solid.

For example, the solid shown above right has a volume of 6 cubic units.



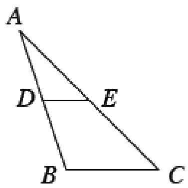
Since volume reflects the number of cubes that fit within a solid, it is measured in **cubic units**. For example, if the dimensions of a solid are measured in feet, then the volume would be measured in cubic feet (a cube with dimensions $1' \times 1' \times 1'$).

On the other hand, the total **surface area** of a solid is the area of all of the external faces of the solid. For example, the total surface area of the solid above is 24 square units.

Cavalieri's Principle states that when the corresponding slices of two solids (with equal heights) have equal area, then the solids have equal volume. One way to think about Cavalieri's Principle is to think about how the volume of a stack of identical books would not change when you slide or twist some of them in the stack.

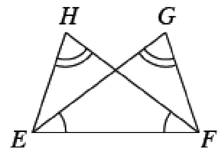
Review & Preview

9-33. In the diagram at right, $\overline{DE}$ is a midsegment of ΔABC. If the area of ΔABC is 96 square units, what is the area of ΔADE? Explain how you know.



9-34. A regular hexagonal prism has a volume of 2546.13 cm^3 and the base has an edge length of 14 cm. Find the height and surface area of the prism.

9-35. Are ΔEHF and ΔFGE congruent? If so, explain how you know. If not, explain why not.



9-36. A sandwich shop delivers lunches by bicycle to nearby office buildings. Unfortunately, sometimes the delivery is made later than promised. A delay can occur either because food preparation took too long, or because the bicycle rider got lost. Last month the food preparation took too long or the rider got lost, 7% of the time. During the same month, the food preparation took longer than expected 11 times and the bicycle rider got lost 4 times. There were 200 deliveries made during the month. For a randomly selected delivery last month, find the probability that both the food preparation took too long and the rider got lost.

9-37. Remember that the absolute value of a number is its positive value. For example, $|-5| = 5$ and $|5| = 5$. Use this understanding to solve the equations below, if possible. If there is no solution, explain how you know.

a. $|x| = 6$ b. $|x| = -2$ c. $|x+7| = 10$

9-38. Cindy's cylindrical paint bucket has a diameter of 12 inches and a height of 14.5 inches. If 1 gallon $\approx$ 231 in.^{3}, how many gallons does her paint bucket hold?

9-39. Write the equation of an exponential function that passes through the points (0, 32) and (3, 4).

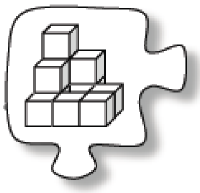
9-40. A cork in the shape of a cylinder has a radius of 2 cm, a height of 5 cm, and weighs 2.5 grams.

a. What is the volume of the cork?

b. What is the cork's density in grams per cubic centimeter? That is, how many grams of cork are there per cubic centimeter?

9.1.4 How does the volume change?

Volumes of Similar Solids

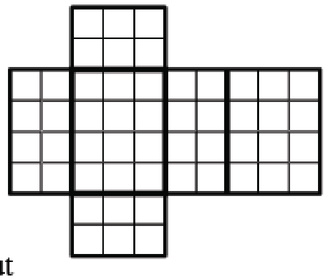


As you continue your study of three-dimensional solids, today you will explore how the volume of a solid changes as the solid is enlarged proportionally.

9-41. HOW DOES THE VOLUME CHANGE?

In Lesson 9.1.3, you began a study of the surface area and volume of solids. Today, you will continue that investigation in order to generalize about the ratios of similar solids.

a. Describe the solid formed by the net at right. What are its dimensions (length, width, and height)?



b. Have each team member select a different enlargement ratio from the list below. On graph paper, carefully draw the net of a similar solid using your enlargement ratio. Then cut out your net and build the solid (so that the gridlines end up on the outside the solid) using scissors and tape.

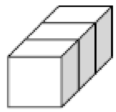
(1) 1 (2) 2 (3) 3 (4) 4

c. Find the volume of your solid and compare it to the volume of the original solid. What is the ratio of these volumes? Share the results with your teammates so that each person can complete a table like the one below.

Linear Scale Factor	Original Volume	New Volume	Ratio of Volumes
1			
2			
3			
4			
r			

d. How does the volume change when a three-dimensional solid is enlarged or reduced to create a similar solid? For example, if a solid's length, width, and depth are enlarged by a linear scale factor of 10, then how many times bigger does the volume get? What if the solid is enlarged by a linear scale factor of r? Explain.

9-42. Examine the $1 \times 1 \times 3$ solid at right.



a. Build this solid with blocks provided by your teacher.

b. If this shape is enlarged by a linear scale factor of 2, how wide will the new shape be? How tall? How deep?

c. How many of the $1 \times 1 \times 3$ solids would you need to build the enlargement described in part (b) above? Use blocks to prove your answer.

d. What if the $1 \times 1 \times 3$ solid is enlarged with a linear scale factor of 3? How many times larger would the volume of the new solid be? Explain how you found your answer.

9-43. 

At the movies, Maurice counted the number of kernels of popcorn that filled his tub and found that it had 320 kernels. He decided that next time, he will get an enlarged tub that is similar, but has a linear scale factor of 1.5. How many kernels of popcorn should the enlarged tub hold?

9-44. LEARNING LOG

In your Learning Log, explain how the volume changes when a solid is enlarged proportionally. That is, if a three-dimensional object is enlarged by a linear scale factor of 2, by what factor does the volume increase? Title this entry "Volumes of Similar Solids" and include today's date.

Review & Preview

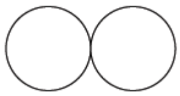
9-45. Koy is inflating a spherical balloon for her brother's birthday party. She has used three full breaths so far and her balloon is only half the width she needs. Assuming that she puts the same amount of air into the balloon with each breath, how many more breaths does she need to finish the task? Explain how you know.

9-46. Draw a cylinder on your paper. Assume the radius of the cylinder is 6 inches and the height is 9 inches.

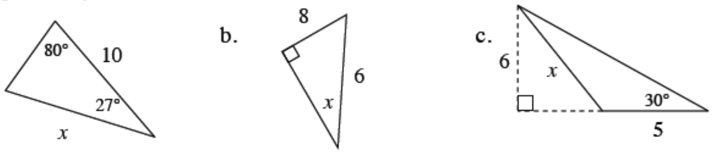
a. What is the surface area of the cylinder? What is the volume?

b. If the cylinder is enlarged with a linear scale factor of 3, what is the volume of the enlarged cylinder? How do you know?

9-47. While Katarina was practicing her figure skating, she wondered how far she had traveled. She was skating a "figure 8," which means she starts between two circles and then travels on the boundary of each circle, completing the shape of a sideways 8. If both circles have a radius of 5 feet, how far does she travel when skating one "figure 8"?



9-48. For each triangle below, solve for x, if possible. If no solution is possible, explain why.



9-49. The mat plan for a three-dimensional solid is shown at right.

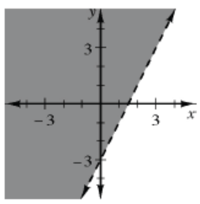
a. On your paper, draw the front, right, and top views of this solid.

b. Find the volume and surface area of the solid.

3	2	0	RIGHT
1	4	1	
0	3	2	

FRONT
Mat Plan

9-50. The graph of the inequality $y > 2x - 3$ is shown at right. On graph paper, graph the inequality $y \le 2x - 3$. Explain what you changed about the graph.



9-51. An international charity builds homes for disaster victims. Often the materials are donated. The charity recently built 45 homes. 20% used granite for the kitchen countertops, while the rest used porcelain tile. 15 of the homes used red oak for the wooden kitchen floor, 20 used white oak, and 10 used maple. If a disaster victim is randomly assigned to a home, what is the probability (in percent) of getting an oak floor with granite countertops?

9-52. **Multiple Choice:** The point $A(-2, 5)$ is rotated 90° counter-clockwise (↺). What are the new coordinates of point A?

a. (2, 5) b. (5, –2) c. (2, –5) d. (–5, –2)

Graph Paper Grid for Problem 9-41

9.1.5 How does the volume change?

Ratios of Similarity

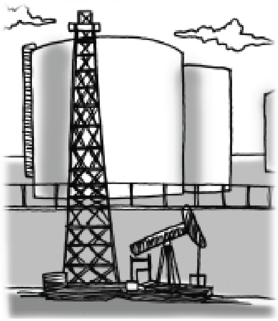
Today, work with your team to analyze the following problems. As you work, think about whether the problem involves volume or area. Also think carefully about how similar solids are related to each other.

9-53. A statue to honor Benjamin Franklin will be placed outside the entrance to the Liberty Bell exhibit hall in Philadelphia. The designers decide that a smaller, similar version will be placed on a table inside the building. The dimensions of the life-sized statue will be four times those of the smaller statue. Planners expect to need 1.5 pints of paint to coat the small statue. They also know that the small statue will weigh 14 pounds.

a. How much paint will be needed to paint the life-sized statue?

b. If the small statue is made of the same material as the enlarged statue, then its weight will change just as the volume changes as the statue is enlarged. How much will the life-size statue weigh?

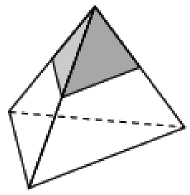
9-54. The Blackbird Oil Company is considering the purchase of 20 new jumbo oil storage tanks. The standard model holds 12,000 gallons. Its dimensions are $\frac{4}{5}$ the size of the similarly shaped jumbo model, that is, the ratio of the dimensions is 4:5.



a. How much more storage capacity would the twenty jumbo models give Blackbird Oil?

b. If jumbo tanks cost 50% more than standard tanks, which tank is a better buy?

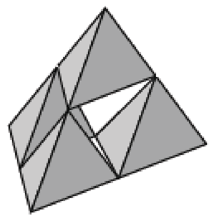
9-55. In problem 7-14 your class constructed a large tetrahedron like the one at right. Assume the dimensions of the shaded tetrahedron at right are half of the dimensions of the similar enlarged tetrahedron.



a. If the volume of the large tetrahedron is 138 in.^{3}, find the volume of the small shaded tetrahedron.

b. Each face of a tetrahedron is an equilateral triangle. If the small shaded tetrahedron has an edge length of 16 cm, find the total surface area for each of the tetrahedra.

c. Your class tried to construct a tetrahedron using four smaller congruent tetrahedra. However, the result left a gap in the center, as shown in the diagram at right. If the volume of each small shaded tetrahedron is 50 in.^{3}, what is the volume of the gap? Explain how you know.

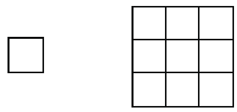


MATH NOTES

METHODS AND MEANINGS

The $r : r^2 : r^3$ Ratios of Similarity

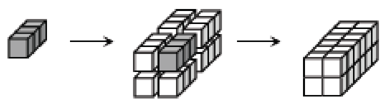
When a two-dimensional figure is enlarged proportionally, its perimeter and area also grow. If the linear scale factor is r, then the perimeter of the figure is enlarged by a factor of r while the area of the figure is enlarged by a factor of r^2. Examine what happens when the square at right is enlarged by a linear scale factor of 3.



$P = 4$ units, $A = 1$ units^2

$P = 4 \cdot \mathbf{3} = 12$ units, $A = 1 \cdot 3^2 = 9$ units^2

When a solid is enlarged proportionally, its surface area and volume also grow. If it is enlarged by a linear scale factor of r, then the surface area grows by a factor of r^2 and the volume grows by a factor of r^3. The example at right shows what happens to a solid when it is enlarged by a linear scale factor of 2.



Original solid SA = 14 units^2 V = 3 units^3

Width, height, and depth are doubled

Result: SA = 56 units^2 V = 24 units^3

Thus, if a solid is enlarged proportionally by a linear scale factor of r, then:

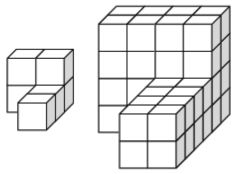
New edge length = $r \cdot$ (corresponding edge length of original solid)

New surface area = $r^2 \cdot$ (original surface area)

New volume = $r^3 \cdot$ (original volume)

Review & Preview

9-56. Consider the two similar solids at right.



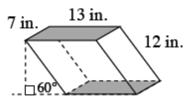
a. Create a mat plan and draw the front, right, and top views for the solid on the left.

b. What is the linear scale factor between the two solids?

c. Find the surface area of each solid. What is the ratio of the surface areas? How is this ratio related to the linear scale factor?

d. Now find the volumes of each solid. How are the volumes related? Compare this to the linear scale factor and record your observations.

9-57. Elliot has a modern fish tank that is in the shape of an oblique prism, shown at right.



a. If the slant of the prism makes a 60° angle with the flat surface on which the prism is placed, find the volume of water the tank can hold. Assume that each base is a rectangle.

b. If Elliot has 25 fish, how crowded are the fish? That is, what is the density of fish, measured in number of fish per cubic inch?

c. What is the density of fish in Elliot's tank in fish per cubic *foot*?

9-58. Decide if the following statements are true or false. If they are true, explain how you know. If they are false, provide a counterexample.

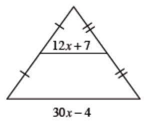
a. If a quadrilateral has two sides that are parallel and two sides that are congruent, then the quadrilateral must be a parallelogram.

b. If the interior angles of a polygon add up to 360°, then the polygon must be a quadrilateral.

c. If a quadrilateral has 3 right angles, then the quadrilateral must be a rectangle.

d. If the diagonals of a quadrilateral bisect each other, then the quadrilateral must be a rhombus.

9-59. Write and solve an equation based on the geometric relationship shown at right.



9-60. Solve each equation below. Check your solution.

a. $20-6(5+2x)=10-2x$

b. $2x^2-9x-5=0$

c. $\frac{3}{5x-1}=\frac{1}{x+1}$

d. $|2x-1|=5$

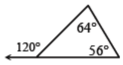
9-61. A new car purchased for $27,000 loses 15% of its value each year.

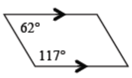
a. What is the multiplier?

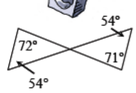
b. Write a function of the form $f(t)=ab^t$ that represents the situation.

c. At the current rate, what will be the value of the car in 5 years?

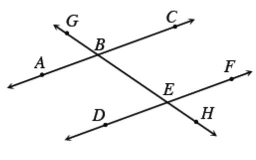
9-62. Examine the information provided in each diagram below. Decide if each figure is possible or not. If the figure is not possible, explain why.

a. 

b. 

c. 

9-63. **Multiple Choice:** For $\angle ABE \cong \angle BEF$ in the diagram below, what must be true?



a. $\angle ABE \cong \angle BED$

b. $\angle ABE \cong \angle GBC$

c. $\overline{AC} \parallel \overline{GH}$

d. $\overline{AC} \parallel \overline{DF}$

e. None of these.

9.2.1 How can I construct it?

Introduction to Constructions

So far in this course, you have used tools such as rulers, tracing paper, protractors, and even computers to draw geometric relationships and shapes. But how did ancient mathematicians accurately construct shapes such as squares or equilateral triangles without these types of tools?

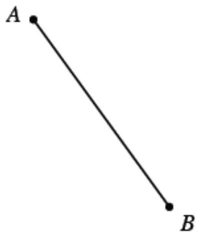
Today you will start by exploring how to construct several geometric relationships and figures with tracing paper. You will then investigate how to construct geometric shapes with tools called a compass and a straightedge, much like the ancient Greeks did more than 2000 years ago. As you study these forms of **construction**, you will not only learn about new geometric tools, but also gain a deeper understanding of some of the special geometric relationships and shapes you have studied so far in this course.

9-64. CONSTRUCTING WITH TRACING PAPER

To start this focus on construction, you will begin with a familiar tool: tracing paper. Obtain several sheets of tracing paper and a straightedge from your teacher. Note: A straightedge is *not* a ruler. It does not have any markings or measurements on it. For example, a 3" × 5" index card makes a good straightedge.

a. Starting with a smooth, square piece of tracing paper, find a way to create parallel lines (or creases). Make sure the lines are *exactly* parallel. Be ready to share with the class how you accomplished this.

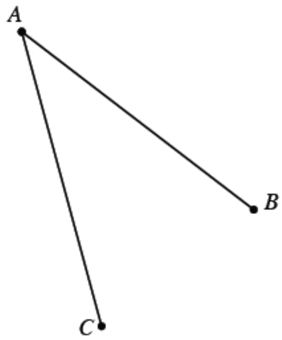
b. With a new piece of tracing paper, trace line segment $\overline{AB}$ at right. Use your straightedge for accuracy. Can you fold the tracing paper so that the resulting crease not only finds the midpoint of $\overline{AB}$ but also is perpendicular to $\overline{AB}$? Remember that this is the **perpendicular bisector** of $\overline{AB}$. Prove that your crease is the perpendicular bisector.



c. On the perpendicular bisector from part (b) above, choose a point C and then connect $\overline{AC}$ and $\overline{BC}$ to form ΔABC. What type of triangle did you construct? Use your geometry knowledge to justify your answer.

d. In part (b), you determined how to use tracing paper and a straightedge to construct a line that bisects another line. How can you construct an angle bisector?

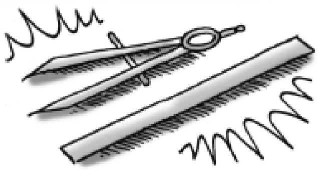
On a piece of tracing paper, trace $\angle BAC$ at right. Construct the angle bisector. That is, find $\overrightarrow{AD}$ such that $\angle BAD \cong \Delta CAD$.



e. Did you know that the angle bisectors of a triangle intersect at a single point? On your tracing paper, connect $\overline{BC}$ to form ΔABC. Then fold to find the angle bisectors of the other two angles, $\angle ABC$ and $\angle BCA$. Mark this special "center" of the triangle, which is sometimes referred to as the **incenter** of the triangle, with a point. Put this tracing paper aside but keep it for problem 9-66.

9-65. CONSTRUCTING WITH A COMPASS AND A STRAIGHTEDGE

Other tools that are often used to construct geometric figures are a compass and a straightedge. Obtain a Lesson 9.2.1 Resource Page from your teacher and explore what types of shapes you can construct using these tools.



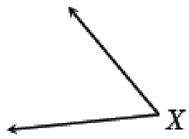
a. Find point C on the resource page. Use your compass to construct two circles with different radii that have a center at point C. Circles that have the same center are called **concentric** circles.

b. With tracing paper, copying a line segment means just putting the tracing paper over the line and tracing it. But how can you copy a line segment using only a compass and a straightedge?

On the resource page, find $\overline{AB}$. Next to $\overline{AB}$, use your straightedge to draw a new line segment. With your team, decide how to use the compass to mark off two points (C and D) so that $\overline{AB} \cong \overline{CD}$. Be ready to share your method with the class.

c. Now construct a new line segment, labeled $\overline{EF}$, that is twice as long as $\overline{AB}$. How can you be sure that $\overline{EF}$ is twice as long as $\overline{AB}$?

d. How can you use these tools to copy an angle? On your resource page, find $\angle X$. With your team, discuss how you can use your compass to construct a new angle ($\angle Y$) on your resource page that is congruent to $\angle X$. Start by drawing a ray with endpoint Y.



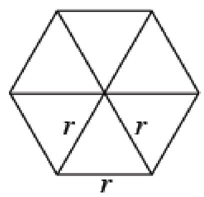
9-66. The incenter of a triangle is special because it is also the center of a circle that lies inside the triangle and intersects each side of the triangle exactly once. This circle is called an **inscribed circle**. To construct this circle, you will need to use both your tracing paper from part (d) and (e) of problem 9-64 and your compass.

a. As Shui thought about constructing a circle that fits inside the triangle, she observed, "*I think ΔABC is an isosceles triangle*." Do you agree? How can you test her conjecture with your tracing paper?

b. Shui thinks that the inscribed circle must pass through the midpoint of $\overline{BC}$. Is this correct? Explain why or why not.

c. With your compass, construct the inscribed circle of ΔABC with the center at the incenter.

9-67. REGULAR HEXAGON



As Shui was completing her homework, she noticed that a regular hexagon has a special quality: when dissected into congruent triangles, the hexagon contains triangles that are all equilateral! *"I bet I can use this fact to help me construct a regular hexagon,"* she told her team.

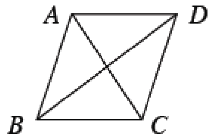
a. On the Lesson 9.2.1 Resource Page, construct a circle with radius r and center H.

b. Mark one point on the circle to be a starting vertex. Since each side of the hexagon has length r, the radius of the circle, carefully use the compass to mark off the other vertices of the hexagon on the circle. Then connect the vertices to create the regular hexagon.

c. When all vertices of a polygon lie on the same circle, the polygon is **inscribed** in the circle. For example, the hexagon you constructed in part (b) is inscribed in $\odot H$. After consulting with your teammates, construct an equilateral triangle that is also inscribed in $\odot H$. You may want to use colored markers or pencils to help distinguish between the hexagon and the triangle.

Review & Preview

9-68. Examine the diagram of $ABCD$ at right.



a. If opposite sides of the quadrilateral are parallel and all sides are congruent, what type of quadrilateral is $ABCD$?

b. List what you know about the diagonals of $ABCD$.

c. Find the area of $ABCD$ if $BC = 8$ and $m\angle ABC = 60°$.

9-69. A butterfly house at a local zoo is a rectangular prism with dimensions $20' \times 15' \times 10'$ and contains 625 butterflies.

a. Sketch the prism on your paper.

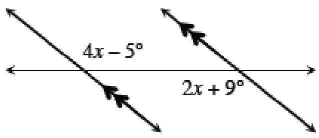
b. What is the volume of the butterfly house? Show your work.

c. How many cubic feet of air is there for each butterfly?

d. **Density** is the quantity of something per unit measure, especially length, area, or volume. For example you might talk about the density of birds on a power wire (maybe a flock lands with 7 birds/meter), population density (the density of Singapore is 7301 people per square meter), or the mass density of an element (iron has density of $7.874\frac{g}{cm^3}$).

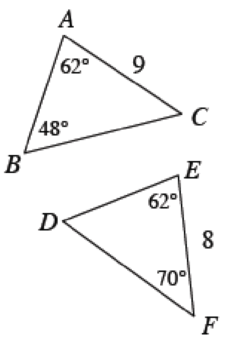
Assuming the butterflies are equally distributed inside the butterfly house, what is the density of butterflies? Explain.

9-70. Use the relationships given in the diagram at right to write and solve an equation for x. Show all work.

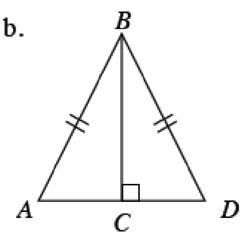


9-71. For each pair of triangles below, determine if the triangles are congruent. If they are congruent, state the congruence property that assures their congruence and write a congruence statement (such as $\Delta ABC \cong \Delta$______).

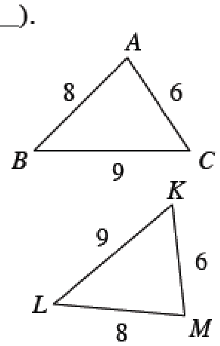
a.



b.



c.



9-72. Write the equation represented by the table below.

IN (x)	–4	–3	–2	–1	0	1	2	3	4
OUT (y)	–26	–20	–14	–8	–2	4	10	16	22

9-73. This problem is a checkpoint for finding probabilities. It will be referred to as Checkpoint 9A.

Because students complained that there were not enough choices in the cafeteria, the student council decided to collect data about the sandwich choices that were available. The cafeteria supervisor indicated that she makes 36 sandwiches each day. Each sandwich consists of bread, a protein, and a condiment. Twelve of the sandwiches were made with white bread, and 24 with whole-grain bread. Half of the sandwiches were made with salami, and the other half were evenly split between turkey and ham. Two-thirds of the sandwiches were made with mayonnaise, and the rest were left plain with no condiment.

a. Organize the possible sandwich combinations of bread, protein, and condiment by making an area model or tree diagram, if possible.

b. Wade likes any sandwich that has salami or mayonnaise on it. Which outcomes are sandwiches that Wade likes? If Wade randomly picks a sandwich, what is the probability he will get a sandwich that he likes? (Hint: You can use W and G to abbreviate the breads. Then use S, T, and H to abbreviate the proteins, and M and P to abbreviate the condiments.)

c. Madison does not like salami or mayonnaise. Which outcomes are sandwiches that Madison likes? If Madison randomly picks a sandwich, what is the probability she will get a sandwich that she likes?

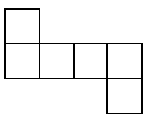
d. If you have not already done so in part (c), show how to use a complement to find the probability Madison gets a sandwich that she likes.

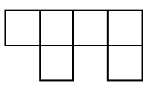
e. Which outcomes are in the event for the intersection of {salami} and {mayonnaise}?

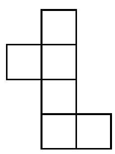
Check your answers by referring to the Checkpoint 9A materials located at the back of your book.

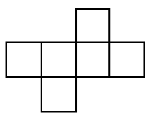
If you needed help solving these problems correctly, then you need more practice. Review the Checkpoint 9A materials and try the practice problems. Also, consider getting help outside of class time. From this point on, you will be expected to do problems like these quickly and easily.

9-74. **Multiple Choice:** Which net below will not produce a closed cube?

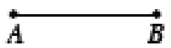
a. 

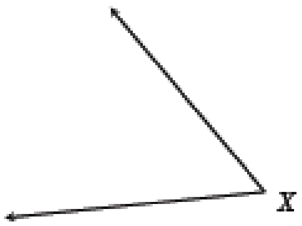
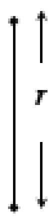
b. 

c. 

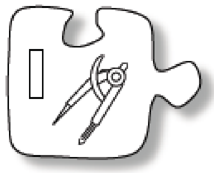
d. 

Construction Toolkit #1

Problem	Construction
9-65. a. Use your compass to construct two circles with different radii that have a center at point C at right.	$\bullet C$
b. In the space at right, use a compass and a straightedge to construct a line segment $\overline{CD}$ that is congruent to $\overline{AB}$ below. 	
c. Now, in the space at right, construct a new line segment, labeled $\overline{EF}$, that is twice as long as $\overline{AB}$ above.	

Problem	Construction
9-66. In the space at right, construct a new angle $\angle Y$ that is congruent to $\angle X$ below.  	
9-67. Construct a circle centered at H with radius r below. Then use what you know about a regular hexagon to construct a regular hexagon.  	

9.2.2 How can I construct it?

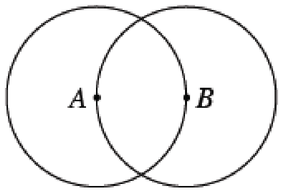


Constructing Bisectors

During Lesson 9.2.1, you studied how to construct geometric relationships such as congruent line segments using tools that include a compass and tracing paper. But what other geometric relationships and shapes can you construct using these tools? Today, as you investigate new ways to construct familiar geometric figures, look for connections to previous course material.

9-75. INTERSECTING CIRCLES

As Ventura was doodling with his compass, he drew the diagram at right. Assume that each circle passes through the center of the other circle.



a. Explain why $\odot A$ and $\odot B$ must have the same radius.

b. On the Lesson 9.2.2 Resource Page provided by your teacher, construct two intersecting circles so that each passes through the other's center. Label the centers *A* and *B*.

c. On your construction, locate the two points where the circles intersect each other. Label these points *C* and *D*. Then construct quadrilateral *ACBD*. What type of quadrilateral is *ACBD*? Justify your answer.

d. Use what you know about the diagonals of *ACBD* to describe the relationship of $\overline{AB}$ and $\overline{CD}$. Make as many statements as you can.

e. What else can this diagram help you construct when given a line segment such as $\overline{AB}$? Share your ideas with the class.

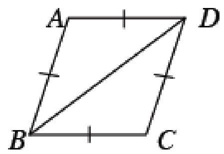
9-76. In problem 9-75, you constructed a rhombus and a perpendicular bisector.

a. In your own words, describe how this process works. That is, given any line segment, how can you find its midpoint? How can you find a line perpendicular to it? Be sure to justify your statements.

b. Test that your directions in part (a) work for line $\overline{KM}$ on the Lesson 9.2.2 Resource Page. In other words, construct a perpendicular bisector of $\overline{KM}$. Label the midpoint of $\overline{KM}$ point N.

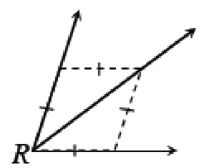
c. Return to your work from part (b) and use it to construct a 45°- 45°- 90° triangle. Prove that your triangle must be isosceles.

9-77. In problem 9-75, you used the fact that the diagonals of a rhombus are perpendicular bisectors of each other to develop a construction. In fact, most constructions are rooted in the properties of many of the geometric shapes you have studied so far. A rhombus can help you with another important construction.



a. Examine the rhombus $ABCD$ at right. What is the relationship between $\angle ABC$ and $\overline{BD}$?

b. Since the diagonals of a rhombus bisect the angles, use this relationship to construct an angle bisector of $\angle R$ on the resource page. That is, construct a rhombus so that R is one of its vertices. Use only a compass, a straightedge, and a pencil.



9-78. CONSTRUCTION CHALLENGE

On the Lesson 9.2.2 Resource Page, locate $\overline{PQ}$, $\overline{ST}$, and $\angle V$. In the space provided, use the construction strategies you have developed so far to construct a triangle with legs congruent to $\overline{PQ}$ and $\overline{ST}$, with an angle congruent to $\angle V$ in between. Be sure you know how to do this two ways: with a compass and a straightedge and with tracing paper.

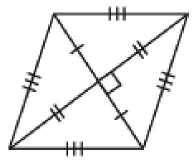
MATH NOTES

METHODS AND MEANINGS

Rhombus Facts

Review what you have previously learned about a rhombus below.

A **rhombus** is a quadrilateral with four equal sides. All rhombi (the plural of rhombus) are parallelograms.



Starting with the definition of a rhombus above, there are several facts about rhombi that can be proved. For example, the diagonals of a rhombus are perpendicular bisectors of each other. That is, they intersect each other at their midpoints and form right angles at that point. Also, all four small triangles are congruent (as well as the larger triangles formed by any two of the smaller triangles). In addition, the diagonals of a rhombus bisect the opposite angles.

Review & Preview

9-79. Unlike a straightedge, a ruler has measurement markings. With a ruler, it is fairly simple to construct a line segment of length 6 cm or a line segment with length 3 inches. But how can you construct a line segment of $\sqrt{2} \approx 1.414213562...$ centimeters? Consider this as you answer the questions below.

a. With a ruler, construct a line segment of 1 cm.

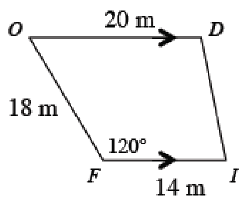
b. What about 1.4 cm? Adjust your line segment from part (a) so that its length is 1.4 cm. Did your line get longer or shorter?

c. Now change the line segment so that its length is 1.41 cm. How did it change?

d. Karen wants to draw a line segment is exactly $\sqrt{2} \approx 1.414213562...$ centimeters long. Is this possible? Why or why not?

9-80. Describe a sequence of steps to construct an equilateral triangle.

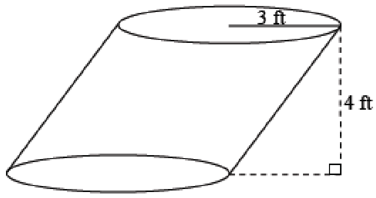
9-81. The floor plan of Marina's local drug store is shown at right. While shopping one day, Marina tied her dog, Mutt, to the building at point F. If Mutt's leash is 4 meters long, what is the area that Mutt can roam? Draw a diagram and show all work.



9-82.

Find the area of the Marina's drugstore (*FIDO*) in problem 9-81 Show all work.

9-83. Compute the volume of the figure at right.



9-84. Which has greater measure: an exterior angle of an equilateral triangle or an interior angle of a regular heptagon (7-gon)? Show all work.

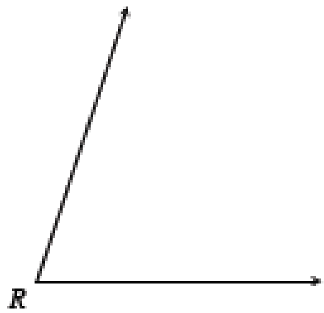
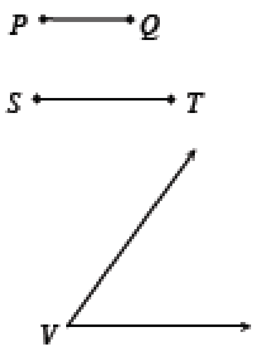
9-85. On graph paper, graph the line $y = -\frac{3}{2}x + 6$. State the x- and y-intercepts.

9-86. **Multiple Choice:** A solid with a volume of 26 in.^{3} was enlarged to create a similar solid with a volume of 702 in.^{3}. What is the linear scale factor between the two solids?

a. 1 b. 2 c. 3 d. 4

Construction Toolkit #2

Problem	Construction
9-75. In the space at right, construct two intersecting circles so that each passes through the other's center. Label the centers A and B. Then follow the directions for parts (c) and (d) in the textbook.	
9-76. Construct a perpendicular bisector of $\overline{KM}$ at right. Then find the midpoint of $\overline{KM}$ and label it N. Finally, construct an isosceles triangle that is *not* equilateral.	

Problem	Construction
9-77. Follow the suggestions in part (b) in the student text to construct the angle bisector of $\angle R$ at right.	
9-78. At right, construct a triangle with legs congruent to $\overline{PQ}$ and $\overline{ST}$, with an angle congruent to $\angle V$ in between. 	

9.2.3 How do I construct it?

More Explorations with Constructions

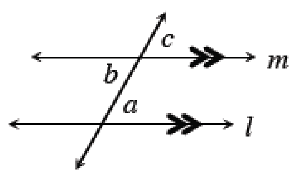
So far, several geometric relationships and properties have helped you develop constructions using a compass and a straightedge. For example, constructing a rhombus helped you construct an angle bisector. Constructing intersecting circles helped you construct a perpendicular bisector. What other relationships can help you develop constructions?

Today you will investigate how to use parallel line theorems to construct a line parallel to a given line through a point not on the line. You will also use the different construction techniques you have learned to construct a square, and will justify that the shape you created meets the definition of a square.

9-87. CONSTRUCTING PARALLEL LINES

In this chapter you have used geometric concepts such as triangle congruence and the special properties of a rhombus to create constructions. How can angle relationships formed by parallel lines help with construction? Consider this question as you answer the questions below.

a. Examine the diagram at right. If $l \parallel m$, what do you know about $\angle a$ and $\angle b$? What about $\angle a$ and $\angle c$? Justify your answer.



b. Neelam thinks that angle relationships can help her construct a line parallel to another line through a given point not on the line. On the Lesson 9.2.3 Resource Page, find line l and point P. Help Neelam construct a line parallel to l through point P by first constructing a transversal through point P that intersects line l.

c. If you have not already done so, complete Neelam's construction by copying an angle formed by the transversal and line l. Explain how you used alternate interior angles or corresponding angles.

9-88. CONSTRUCTING A SQUARE

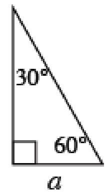
So far, you have developed techniques for constructing congruent angles, congruent line segments, perpendicular and parallel lines, and bisectors of angles and segments. You have constructed a rhombus, a regular hexagon, and an equilateral triangle using just a compass and straightedge. Now you will use what you know to construct a square.

a. What makes a square a square? In other words, what are its unique characteristics? What shapes is it related to?

b. Based on the characteristics you identified in part (a), work with your team to develop a strategy for constructing a square. Keep track of the steps you take on the Lesson 9.2.3 Resource Page, so that you can share them with the class.

c. In order for the shape you constructed in part (c) to be a square, it must have four right angles and four congruent sides. Prove that your shape is a square using the steps of your construction and your geometry knowledge. Be ready to share your justification.

9-89. Consider what you know about all 30°- 60°- 90° triangles.

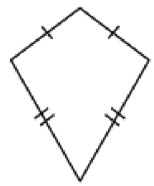


a. Using the information in the triangle at right, how long is the hypotenuse? Explain how you know.

b. Negin (pronounced “Nay-GEEN”) wants to use this relationship to construct a 30°- 60°- 90° triangle. On the Lesson 9.2.3 Resource Page, locate her work so far. She has constructed perpendicular lines and has constructed one side ($\overline{MN}$). Complete her construction so that her triangle has angles 30°, 60°, and 90°.

9-90. CONSTRUCTING OTHER GEOMETRIC SHAPES

What about constructing a kite? On the Lesson 9.2.3 Resource Page, use a compass and a straightedge to construct a kite. Remember that a kite is defined as a quadrilateral with two pairs of adjacent, congruent sides. Be prepared to explain to the class how you constructed your kite.



Methods and Meanings

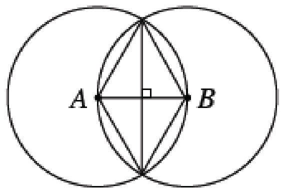
Math Notes

Constructing a Perpendicular Bisector

A perpendicular bisector of a given segment can be constructed using tracing paper or using a compass and a straightedge.

With tracing paper: To construct a perpendicular bisector with tracing paper, first copy the line segment onto the tracing paper. Then fold the tracing paper so that the endpoints coincide (so that they lie on top of each other). When the paper is unfolded, the resulting crease is the perpendicular bisector of the line segment.

With a compass and a straightedge: One way to construct a perpendicular bisector with a compass and a straightedge is to construct a circle at each endpoint of the line segment with a radius equal to the length of the line segment. Then use the straightedge to draw a line through the two points where the circles intersect. This line will be the perpendicular bisector of the line segment.



Review & Preview

9-91. Examine the mat plan of a three-dimensional solid at right.

6	0	0
3	0	1
2	6	6

RIGHT

FRONT

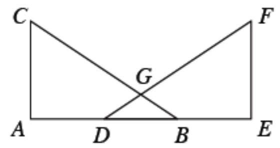
Mat Plan

a. On your paper, draw the front, right, and top views of this solid.

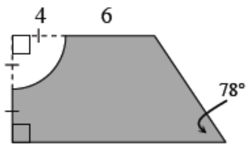
b. Find the volume of the solid.

c. If the length of each edge of the solid is divided by 2, what will the new volume be? Show how you got your answer.

9-92. Examine the diagram at right. Given that $\Delta ABC \cong \Delta EDF$, prove that ΔDBG is isosceles. Use any format of proof that you prefer.



9-93. The Portland Zoo is building a new children's petting zoo pen that will contain 6 goats. One of the designs being considered is shown at right (the shaded portion). Assume the measurements are in meters.



a. What is the area of the petting zoo?

b. How many meters of fence are needed to enclose the petting zoo area.

c. What will be the density of goats in the pen? Show how you got your answer.

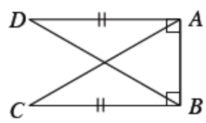
9-94. Sylvia has 14 coins, all nickels and quarters. If the value of the coins is \$2.90, how many of each type of coin does she have? Explain your method.

9-95. West High School has a math building in the shape of a regular polygon. When Mrs. Woods measured an interior angle of the polygon (which was inside her classroom), she got 135°.

a. How many sides does the math building have? Show how you got your answer.

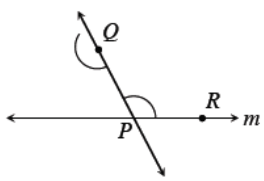
b. If Mrs. Wood's ceiling is 10 feet high and the length of one side of the building is 25 feet, find the volume of West High School's math building.

9-96. Given the information in the diagram at right, prove that $\angle C \cong \angle D$. Write your proof using any format studied so far.



9-97. Write the equation of an exponential function that passes through the points (2, 48) and (5, 750).

9-98. **Multiple Choice:** Jamila has started to construct a line parallel to line m through point Q at right. Which of the possible strategies below make the most sense to help her find the line parallel to m through point Q?



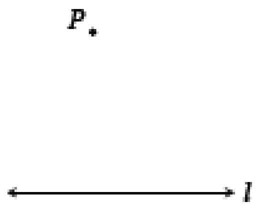
a. Measure $\angle QPR$ with a protractor.

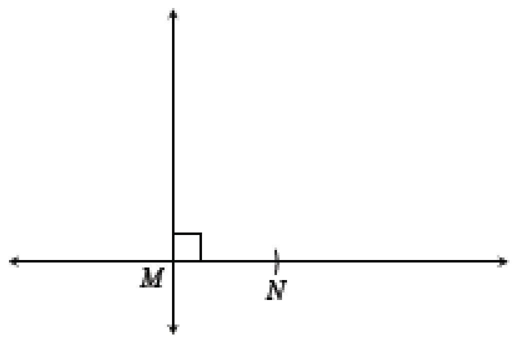
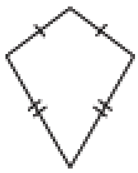
b. Use the compass to measure the arc centered at P, then place the point of the compass where the arc centered at Q meets $\overline{QP}$, and mark that measure off on the arc.

c. Construct $\overline{QR}$.

d. Measure PR with a ruler.

Construction Toolkit #3

Problem	Construction
9-87. Construct a line parallel to *l* through point *P* by first constructing a transversal through point *P* that intersects line *l*. Then use your compass to copy an angle formed by the transversal and draw a line parallel to *l*.	
9-88. Develop a strategy for constructing a square.	

Problem	Construction
9-89. Negin's construction is started at right. Complete her construction so that her triangle has angles 30°, 60°, and 90°.	
9-90. Construct a kite. 	

9.2.4 What more can I construct?

Other Constructions

So far in this section, you have developed a basic library of constructions that can help create many of the geometric shapes and relationships you have studied in Chapters 1 through 8. For example, you can construct a rhombus, an isosceles triangle, a right triangle, a regular hexagon, and an equilateral triangle.

As you continue your investigation of geometric constructions today, keep in mind the following focus questions:

What geometric principles or properties can I use?

Why does it work?

Is there another way?

9-99. TEAM CHALLENGE

Albert has a neat trick. Given any triangle, he can place it on the tip of his pencil and it balances on his first try! The whole class wonders, "*How does he do it?*"

Your Task: Construct a triangle and find its point of balance. This point, called a **centroid**, is special not only because it is the center of balance, but also because it is where the **medians** of the triangle meet. Read more about medians of a triangle in the Math Notes box for this lesson and then follow the directions below.

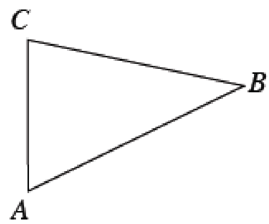
a. After reading about medians and centroids in the Math Notes box for this lesson, draw a large triangle on a piece of unlined paper provided by your teacher. (Note: Your team will work together on one triangle.)

b. Working together, carefully construct the three medians and locate the centroid of the triangle.

c. Once your team is convinced that your centroid is accurate, glue the paper to a piece of cardstock or cardboard provided by your teacher. Carefully cut out the triangle and demonstrate that your centroid is, in fact, the center of balance of your triangle! Good luck!

9-100. In problem 9-99, you constructed a centroid of a triangle, which is special because it is a center of balance in a triangle. However, there are other important points in a triangle. For example, visualize a point inside of ΔABC below that is the same distance from each vertex.

a. Find ΔABC on the Lesson 9.2.4 Resource Page provided by your teacher. Using a compass and a straightedge, find a line that represents all the points that are equidistant (the same distance) from point A and point B. Justify your answer.

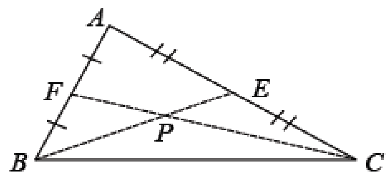


b. Joanna asks, "*How can we find one point that is the same distance from C, too?*" Talk about this with your team and test out your ideas until you find one point that is equidistant from A, B, and C.

c. Joanna points out that the intersection of the perpendicular bisectors of the sides of the triangle is equidistant from the vertices. "*That means there is a circle that passes through all three vertices!*" she noted. Use your compass to draw this circle. Where is its center?

9-101. As the Math Notes box for this lesson states, the point at which the medians of a triangle meet is called the **centroid**. However, how can you be certain that the medians of a triangle will always meet at a single point? In this problem, you will provide some of the reasoning for a proof that the medians will always meet at a single point.

a. ΔABC has midpoints E and F on sides $\overline{AC}$ and $\overline{AB}$ as shown in the diagram at right. $\overline{BE}$ and $\overline{CF}$ intersect at P. Why are $\overline{BE}$ and $\overline{CF}$ medians?



b. Draw segment $\overline{FE}$ and explain how you know that $\overline{FE}$ is parallel to $\overline{BC}$ and half its length.

c. Prove that $\Delta FPE \sim \Delta CPB$.

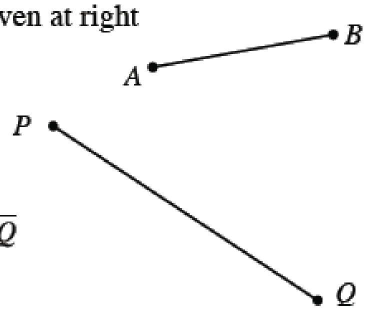
d. Use the relationship of the triangles to identify the ratios $\frac{EP}{PB}$ and $\frac{FE}{BC}$. How does this help explain that $\frac{BP}{PE} = \frac{2}{3}$?

e. What if you had started with a different pair of medians for this triangle? Let point D be the midpoint of $\overline{BC}$. Can the same logic be used to show that $\overline{AD}$ and $\overline{BE}$ will intersect at a point with the same ratio of lengths as in part (d)? Explain.

f. Explain how this proves that the medians will intersect at a single point.

9-102. What other specific triangles can you construct? Choose at least three of the different triangles below and, if possible, construct them using compass and straightedge or tracing paper and the techniques you have developed so far. Be prepared to share your strategies, and to justify your steps for creating each shape or to justify why the shape could not be constructed.

- An isosceles triangle with side length $\overline{AB}$ given at right
- A triangle with side lengths 3, 4, and 5 units
- A 30°-60°-90° triangle
- A triangle with side lengths 2, 3 and 6 units
- An isosceles right triangle with leg length $\overline{PQ}$
- A scalene triangle



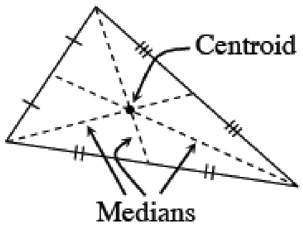
MATH NOTES

METHODS AND MEANINGS

Centroid and Medians of a Triangle

A line segment connecting a vertex of a triangle to the midpoint of the side opposite the vertex is called a **median**.

Since a triangle has three vertices, it has three medians. An example of a triangle with its three medians is provided at right.



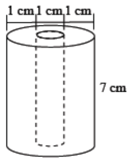
The point at which the three medians intersect is called a **centroid**. The centroid is also the center of balance of a triangle.

Since the three medians intersect at a single point, this point is called a **point of concurrency**. You will learn about other points of concurrency in a later chapter.

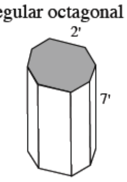
Review & Preview

9-103. Find the volumes of the solids below.

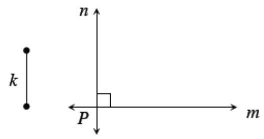
a. cylinder with a hole



b. regular octagonal prism

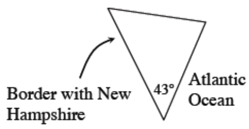


9-104. Jillian is trying to construct a square. She has started by constructing two perpendicular lines, as shown at right. If she wants each side of the square to have length k, as shown at right, describe how she should finish her construction.



9-105. Without using a calculator, find the sum of the interior angles of a 1002-gon. Show all work.

9-106. York County, Maine, is roughly triangular in shape. To help calculate its area, Sergio has decided to use a triangle to model the region, as shown at right. According to his map, the border with New Hampshire is 165 miles long, while the coastline along the Atlantic Ocean is approximately 100 miles long. If the angle at the tip of Maine is 43°, as shown in the diagram, what is the approximate area of York County?



9-107. This problem is a checkpoint for exponential function problems. It will be referred to as Checkpoint 9B.

a. Graph $y = 2(0.75)^x$.

b. Write the equation for the exponential function based on the table at right.

x	$f(x)$
1	23
2	52.9
3	121.67
4	

c. The population of Flood River City is now 42,000. Experts predict the population will decrease 25% each year for the next five years. What will be the population in five years?

d. A share of Orange stock that was worth \$25 in 2000 was worth \$60 in 2010. What is the annual multiplier and percent increase?

Check your answers by referring to the Checkpoint 9B materials located at the back of your book.

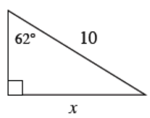
If you needed help solving these problems correctly, then you need more practice. Review the Checkpoint 9B materials and try the practice problems. Also, consider getting help outside of class time. From this point on, you will be expected to do problems like these quickly and easily.

9-108. Copy the following words and their lines of reflection onto your paper. Then use your visualization skills to help draw the reflected images.

a. **REFLECT**

b. **PRISM**

9-109. **Multiple Choice:** Solve this problem without a calculator: Examine the triangle at right. Find the approximate value of x. Use the values in the trigonometric table below as needed.



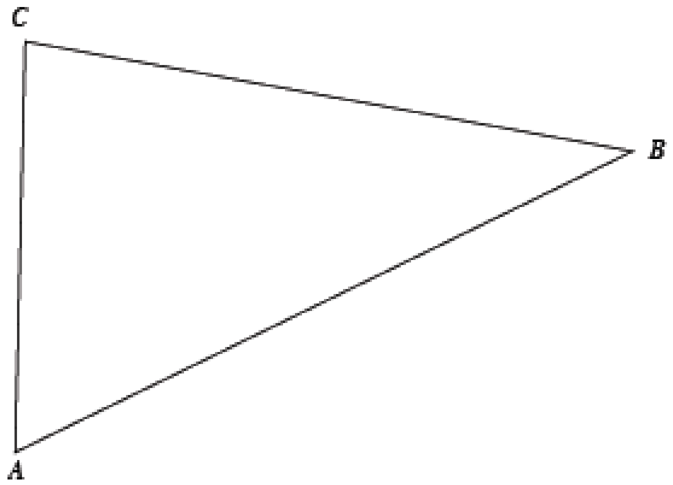
a. 4.69 b. 5.32

c. 8.83 d. 18.81

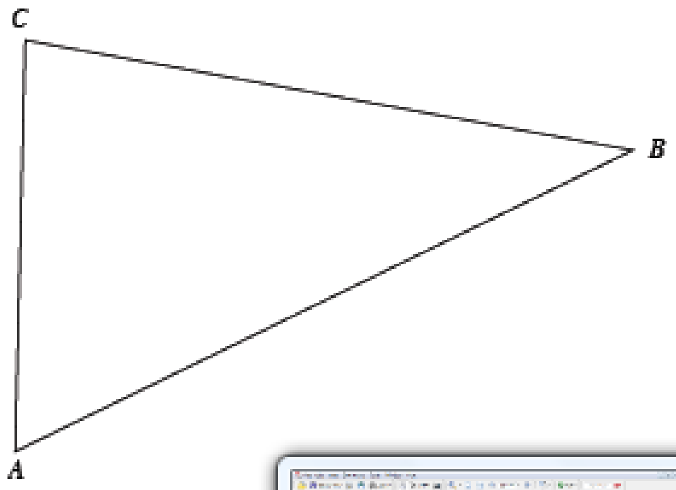
e. None of these

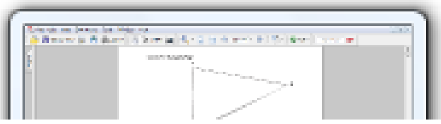
θ	$\cos\theta$	$\sin\theta$	$\tan\theta$
28°	0.883	0.469	0.532
62°	0.469	0.883	1.881

Lesson 9.2.4 Resource Page



Lesson 9.2.4 Resource Page





Chapter 9 Closure What have I learned?

Reflection and Synthesis

The activities below offer you a chance to reflect about what you have learned during this chapter. As you work, look for concepts that you feel very comfortable with, ideas that you would like to learn more about, and topics you need more help with. Look for connections between ideas as well as connections with material you learned previously.

① TEAM BRAINSTORM

What have you studied in this chapter? What ideas were important in what you learned? With your team, brainstorm a list. Be as detailed as you can. To help get you started, lists of Learning Log entries, Toolkit entries, and Math Notes boxes are given below.

What topics, ideas, and words that you learned *before* this chapter are connected to the new ideas in this chapter? Again, be as detailed as you can.

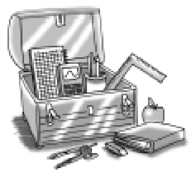
How long can you make your list? Challenge yourselves. Be prepared to share your team's ideas with the class.

Learning Log Entries

- Lesson 9.1.1 – Volume of a Three-Dimensional Shape
- Lesson 9.1.2 – Finding Volume
- Lesson 9.1.4 – Volumes of Similar Solids

Toolkit Entries

- Construction Toolkit (Lesson 9.2,1. 9.2.2, and 9.2.3 Resource Pages)



Math Notes

- Lesson 9.1.2 – Polyhedra and Prisms
- Lesson 9.1.3 – Volume and Total Surface Area of a Solid
- Lesson 9.1.5 – The $r : r^2 : r^3$ Ratios of Similarity
- Lesson 9.2.2 – Rhombus Facts
- Lesson 9.2.3 – Constructing a Perpendicular Bisector
- Lesson 9.2.4 – Centroid and Medians of a Triangle

② MAKING CONNECTIONS

Below is a list of the vocabulary used in this chapter. Make sure that you are familiar with all of these words and know what they mean. Refer to the glossary or index for any words that you do not yet understand.

base	bisect	centroid
circle	compass	concentric circles
construction	cylinder	density
incenter	inscribed	lateral face
line segment	linear scale factor	mat plan
median	net	oblique
perimeter	perpendicular bisector	polygon
polyhedron	point of concurrency	prism
ratio	rhombus	similar
solid	straightedge	surface area
three-dimensional	volume	

Make a concept map showing all of the connections you can find among the key words and ideas listed above. To show a connection between two words, draw a line between them and explain the connection. A word can be connected to any other word as long as you can justify the connection. For each key word or idea, provide an example or sketch that shows the idea.

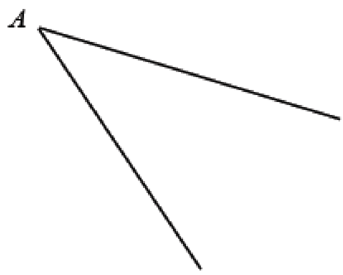
While you are making your map, your team may think of related words or ideas that are not listed here. Be sure to include these ideas on your concept map.

③ PORTFOLIO: EVIDENCE OF MATHEMATICAL PROFICIENCY

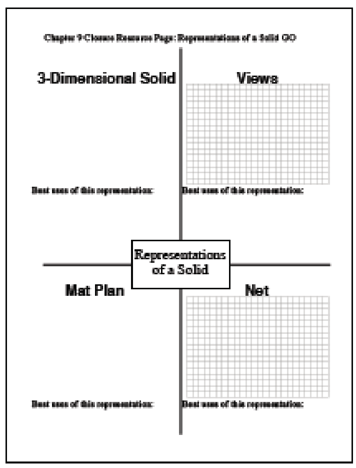
Showcase your understanding of the different representations of a three-dimensional solid by describing the advantages and disadvantages of each representation, including how each representation makes it easier or more difficult to find the surface area and volume of the solid.

Showcase your constructions ability by using only a compass and straightedge to construct a line segment that is $\sqrt{5}$ units long. Assume the segment below is one unit long and use it to make your line segment. Show your construction marks, and explain each step clearly and in detail to a student that does not know how to complete this construction.

Copy the angle at right. Explain each step clearly and in detail in constructing its angle bisector.



Your teacher may give you the Chapter 9 Closure Resource Page: Representations of a Solid Graphics Organizer, and/or the Chapter 9 Closure Resource Page: Constructions Graphics Organizer to work on (or you can download these pages from www.cpm.org). A Graphic Organizer is a tool you can use to organize your thoughts, showcase your knowledge, and communicate your ideas clearly.



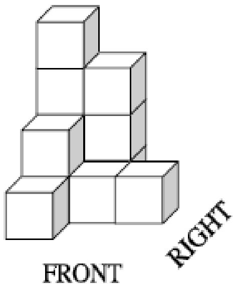
④ WHAT HAVE I LEARNED?

Most of the problems in this section represent typical problems found in this chapter. They serve as a gauge for you. You can use them to determine which types of problems you can do well and which types of problems require further study and practice. Even if your teacher does not assign this section, it is a good idea to try these problems and find out for yourself what you know and what you still need to work on.

Solve each problem as completely as you can. The table at the end of the closure section has answers to these problems. It also tells you where you can find additional help and practice with problems like these.

CL 9-110. On her paper, Kaye has a line with points A and B on it. Explain how she can use a compass to find a point C so that B is a midpoint of $\overline{AC}$. If you have access to a compass, try this yourself.

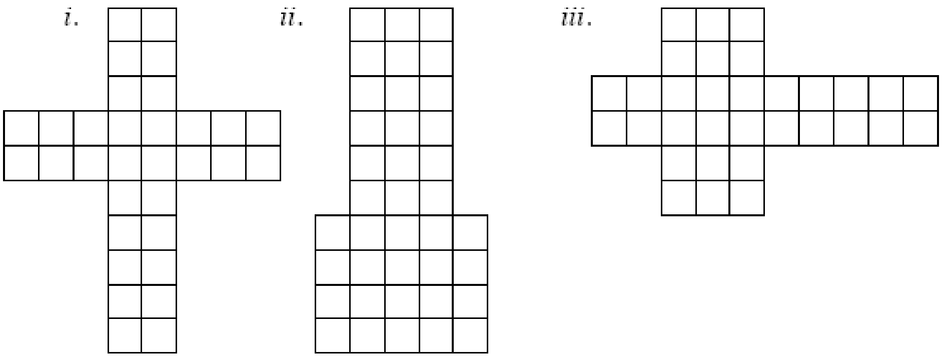
CL 9-111. Assume that the solid at right has no hidden cubes.



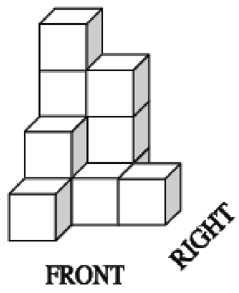
a. On graph paper, draw the front, right, and top views of this solid.

b. Find the volume and surface area of the cube.

c. Which net(s) below would have the same volume as the solid at right when it is folded to create a box?



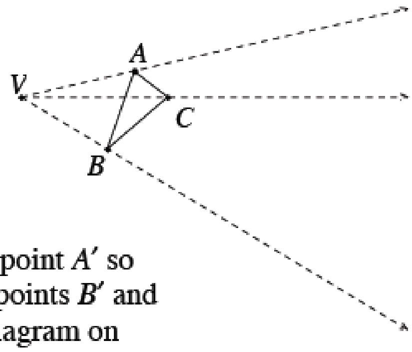
CL 9-112. The solid from problem CL 9-111 is redrawn at right.



a. If this solid were enlarged by a linear scale factor of 4, what would the volume and surface area of the new solid be?

b. Enrique enlarged the solid at right so that its volume was 1500 cubic units. What was his linear scale factor? Justify your answer.

CL 9-113. After constructing a ΔABC, Pricilla decided to try a little experiment. She chose a point V outside of ΔABC and then constructed rays $\overrightarrow{VA}$, $\overrightarrow{VB}$, and $\overrightarrow{VC}$. Her result is shown at right. Copy this diagram onto your paper.

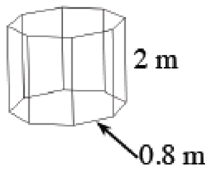


a. Pricilla then used a compass to mark point A' so that $VA = AA'$. She also constructed points B' and C' using the same method. For the diagram on your paper, locate A', B', and C'.

b. Now connect the new points to form $\Delta A'B'C'$. What is the relationship between ΔABC and $\Delta A'B'C'$? Explain what happened.

c. If the area of ΔABC is 19 cm^2 and its perimeter is 15 cm, find the area and perimeter of $\Delta A'B'C'$.

CL 9-114. A restaurant has a giant fish tank, shown at right, in the shape of an octagonal prism.



a. Find the volume and surface area of the fish tank if the base is a regular octagon with side length 0.8 m and the height of the prism is 2 m.

b. What is the density of fish if there are 208 fish in the tank?

CL 9-115. Answer the questions about the angles of polygons below, if possible. If it is not possible, explain how you know it is not possible.

a. Find the sum of the interior angles of a 28-gon.

b. If the exterior angle of a regular polygon is 42°, how many sides does the polygon have?

c. Find the measure of each interior angle of a pentagon.

d. Find the measure of each interior angle of a regular decagon.

CL 9-116. Fill in the blanks in each statement below with one of the quadrilaterals listed at right so that the statement is *true*. Use each quadrilateral name only once.

List:
Kite
Rectangle
Rhombus
Trapezoid

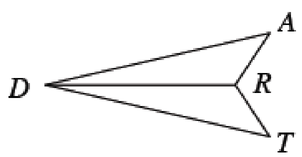
a. If a shape is a square, then it must also be a ______.

b. The diagonals of a ____________ must be perpendicular to each other.

c. If the quadrilateral has only one line of symmetry, then it could be a _____________.

d. If a quadrilateral has only two sides that are congruent, then the shape could be a ______.

CL 9-117. Copy quadrilateral *DART*, shown at right, onto your paper. If $\overline{DR}$ bisects $\angle ADT$ and if $\angle A \cong \angle T$, prove that $\overline{DA} \cong \overline{DT}$.

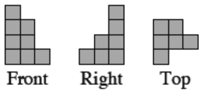
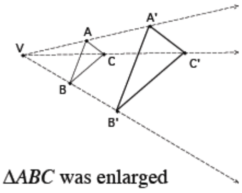
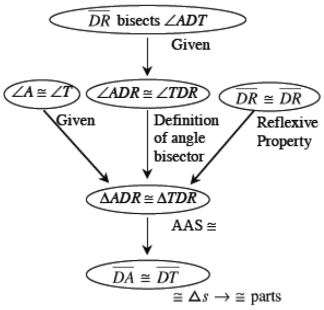


CL 9-118. After Myong's cylindrical birthday cake was sliced, she received the slice at right. If her birthday cake originally had a diameter of 14 inches and a height of 6 inches, find the volume of her slice of cake.

CL 9-119. Check your answers using the table at the end of the closure section. Which problems do you feel confident about? Which problems were hard? Use the table to make a list of topics you need help on and a list of topics you need to practice more.

Answers and Support for Closure Activity #4
What Have I Learned?

Note: MN = Math Note, LL = Learning Log

Problem	Solution	Need Help?	More Practice
CL 9-110.	She should match the length of $\overline{AB}$ with her compass. Then, with the point of the compass at point B, she should mark a point on the line on the side of point B opposite point A. Then she should label that point C.	Lesson 9.2.1 Construction Toolkit	Problems 9-79, 9-80, 9-98, and 9-104
CL 9-111.	a.  b. $V = 12$ un^3, SA = 42 units^2 c. All three nets will form a box with volume 12 units^3.	Lessons 9.1.1 and 9.1.2 MN: 9.1.3 LL: 9.1.1	Problems 9-7, 9-21, 9-49, 9-56, 9-74, and 9-91
CL 9-112.	a. $V = 12(4)^3 = 768$ units^3, $SA = 42(4)^2 = 672$ units^2 b. Linear scale factor = 5	Lessons 9.1.4 and 9.1.5 MN: 9.1.5	Problems 9-45, 9-46, 9-56, 9-86, and 9-91
CL 9-113.	a.  b. ΔABC was enlarged (or dilated) to create a similar triangle with a linear scale factor of 2. c. $A = 19(2)^2 = 76$ units^2; $P = 15(2) = 30$ units	Lessons 8.2.1, 8.2.2, and 9.2.1 MN: 3.1.1, 8.2.1, and 9.1.5 Construction Toolkit	Problems 8-71, 8-83, 9-8, 9-20, and 9-33
CL 9-114.	a. Area of base ≈ 3.09 m^2 Volume ≈ 6.18 m^3 Surface Area ≈ 18.98 m^2 b. density ≈ 33.66 fish/m^3	Lessons 9.1.2 and 9.1.3 MN: 8.3.1 and 9.1.3 LL: 8.1.5 and 9.1.2	Problems 9-26, 9-34, 9-38, 9-40, 9-46, 9-57, 9-69, 9-83, 9-95, and 9-103
CL 9-115.	a. 4680° b. Not possible because 42° does not divide evenly into 360°. c. Not possible because it is not stated that the pentagon is regular. d. 144°	Lessons 8.1.2, 8.1.3, and 8.1.4 MN: 7.1.4 and 8.1.5 LL: 8.1.2, 8.1.3, and 8.1.4	Problems CL 8-138, 9-22, 9-84, 9-95, and 9-105
CL 9-116.	a. Rectangle b. Rhombus c. Kite d. Trapezoid	Lessons 7.3.1 and 7.3.3 MN: 7.2.3, 8.1.2, and 9.2.2	Problems CL 7-156, 8-12, 8-61, and 9-58
CL 9-117.		Section 3.2 and Lessons 6.1.1 through 6.1.4 MN: 3.2.2, 3.2.4, 6.1.4, 7.1.3, and 7.2.1 LL: 3.2.2	Problems CL 3-123, CL 4-123, CL 5-140, CL 6-101, CL 7-155, CL 8-134, 9-35, 9-92, and 9-96
CL 9-118.	$V \approx 97.49$ cubic inches	Lesson 9.1.3 MN: 8.3.2, 8.3.3, and 9.1.3 LL: 9.1.2	Problems 9-38, 9-40, 9-46, and 9-103

Base	Bisect
Centroid	Circle
Compass	Concentric circles
Construction	Cylinder
Density	Incenter

Inscribed	Lateral face
Line segment	Linear scale factor
Mat plan	Median
Net	Oblique
Perimeter	Perpendicular bisector

Polygon	Polyhedron
Point of concurrency	Prism
Ratio	Rhombus
Similar	Solid
Straightedge	Surface area

Three-dimensional	Volume

Chapter 9 Closure Resource Page

Representations of a Solid Graphic Organizer

3-Dimensional Solid

Best uses and qualities of this representation:

Views

Best uses and qualities of this representation:

Representations of a Solid

Mat Plan

Best uses and qualities of this representation:

Net

Best uses and qualities of this representation:

Chapter 9 Closure Resource Page

Construction Graphic Organizer

Name of geometric shape: ______________________________

Steps you used to create this construction:

Justify that your construction created the shape you wanted.

Construction using **Tracing Paper**

Steps you used to create this construction:

Justify that your construction created the shape you wanted.

Same construction using **Compass and Straightedge**