Counting Techniques

Multiplication rule of counting:

If there are n possible outcomes for event A, and m possible outcomes for event B, then there are nm possible outcomes for event A followed by event B.

Ex: rolling 2 fair dice

Dice 1: 6 outcomes
Dice 2: 6 outcomes

$6 \cdot 6 =$ 36 outcomes

Ex: flipping a coin + choosing a card from a deck

coin: 2 outcomes
card: 52 outcomes

$2 \cdot 52 =$ 104 outcomes

$n! = n$ factorial

$n! = n(n-1)(n-2) \ldots\ldots \cdot 1$

$5! = 5(4)(3)(2)(1) = 120$

$8! = 8(7)(6)(5)(4)(3)(2)(1)$

Counting Rule for Permutations

The number of ways to arrange in order n distinct objects, taking them r at a time is

$$P_{n,r} = {}_nP_r = \frac{n!}{(n-r)!}$$

Ex: You are at a horse race. Assuming all horses have equal chances of winning how many different combinations are there of winning the trifecta if there are 8 horses competing?

$${}_nP_r = \frac{n!}{(n-r)!} = \frac{8!}{(8-3)!} = \frac{8!}{5!} = \frac{8\cdot7\cdot6\cdot\cancel{5\cdot4\cdot3\cdot2\cdot1}}{\cancel{5\cdot4\cdot3\cdot2\cdot1}}$$

$= 336$

1 2 3 4 5 6 7 8

Counting Rule for Combinations

The number of combinations of n objects taken r at a time is:

$$C_{n,r} = {}_nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

Ex: You are at a horse race. You are betting on the top 3 horses. Find the number of combinations given 8 horses competing.

$$\binom{8}{3} = \frac{8!}{3!(8-3)!} = \frac{8!}{3!\,5!} = 56$$

HW: p. 235

7-30 odds need to show work
evens do on calc.

$\binom{8}{3}$ $_8C_3$ $\frac{8!}{3!(8-3)!}$

probability extension → skip