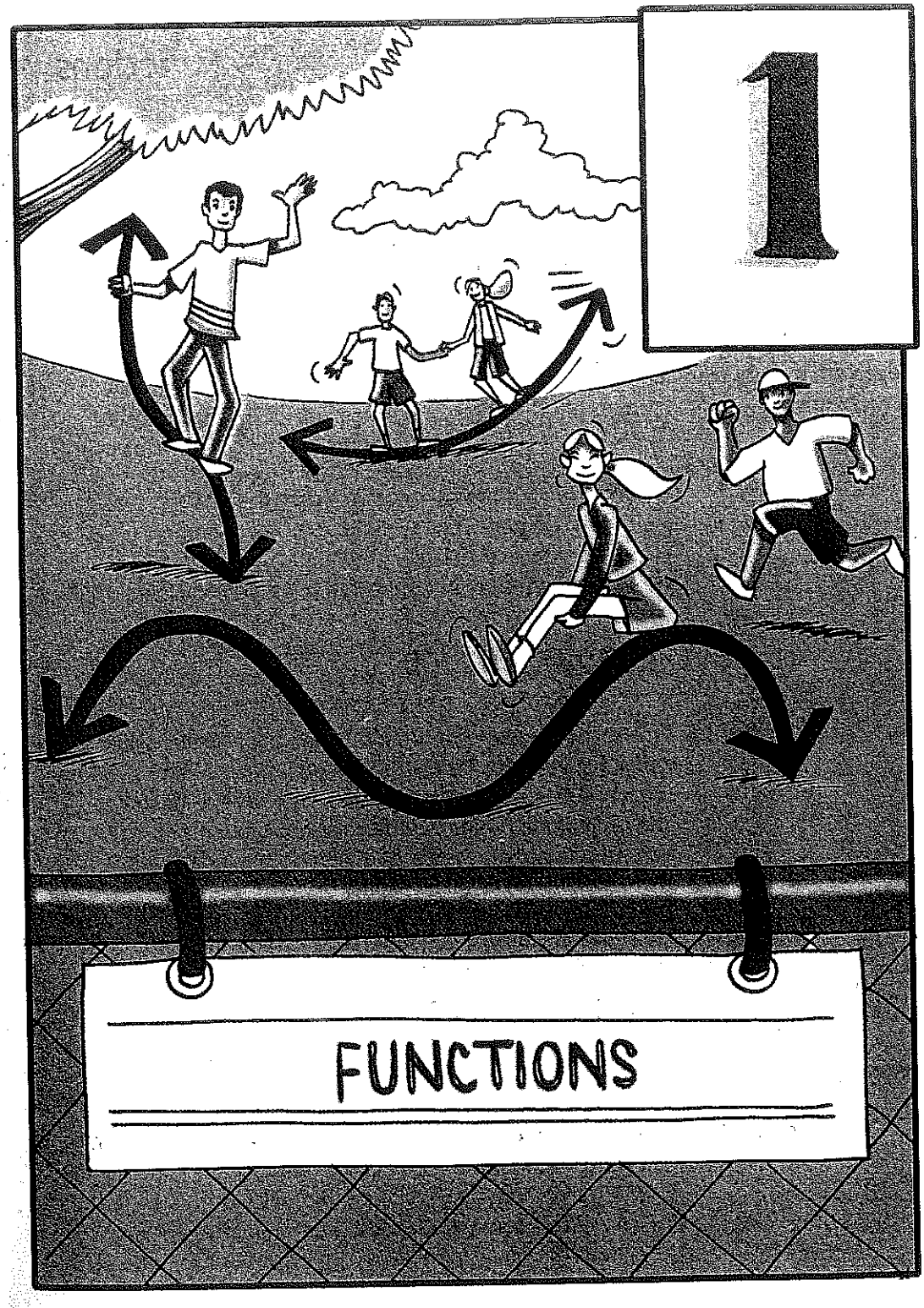

1
FUNCTIONS

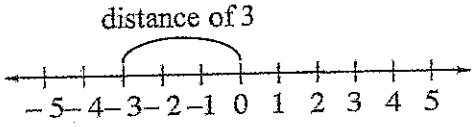
METHODS AND MEANINGS

Definition of Absolute Value

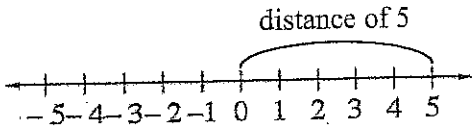
MATH NOTES

Absolute value represents the numerical value of a number without regard to its sign. The symbol for absolute value is two vertical bars, | |. Absolute value can represent the distance on a number line between a number and zero. Since a distance is always positive, the absolute value is *always* either a positive value or zero. The absolute value of a number is *never* negative.

For example, the number –3 is 3 units away from 0, as shown on the number line at right. Therefore, the absolute value of –3 is 3. This is written $|-3| = 3$.

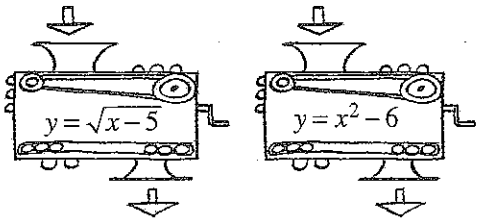


Likewise, the number 5 is 5 units away from 0. The absolute value of 5 is 5, written $|5| = 5$.



Review & Preview

1-4. Angelica is working with function machines. She has the two machines shown at right. She wants to put them in order so that the output of the first machine becomes the input of the second. She wants to use a beginning input of 6.



a. In what order must she put the machines to get a final output of 5?

b. Is it possible for her to find an input that will get a final output of –5? If so, show how she could do that. If not, explain why not.

1-5. Evaluate each absolute value expression. Review the Math Notes box in the lesson for the definition of absolute value.

a. $|54|$ b. $-\left|-7\frac{3}{5}\right|$ c. $|3|-|-1|$ d. $|2.2-5.13|$

1-6. Examine the tile pattern at right.

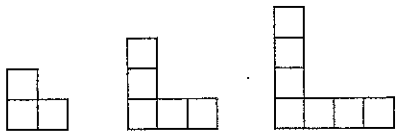


Figure 1 Figure 2 Figure 3

a. On your paper, sketch Figures 4 and 5.

b. How does the pattern grow? Explain how you know.

c. How many tiles will there be in Figure 0 (the figure before Figure 1)? Explain how you know.

1-7. Simplify each expression.

a. $-42+(-17)$ b. $8-(-9)$ c. $8(-9)$

d. $-42 \div (-7)$ e. $-2(-3)(-4)$ f. $-18-7$

g. $(-5)^2$ h. -5^2 i. $\sqrt{49}$

1-8. For each equation below, find y if $x = 2$.

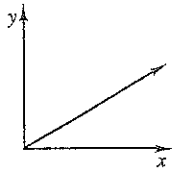
a. $y = 7-|x|$ b. $y = x^2 - 1$ c. $y = \sqrt{x+14}$

METHODS AND MEANINGS

Families of Functions

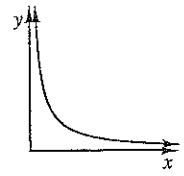
MATH NOTES

There are several "families" of special functions that you will study in this course. One of these is called **direct variation** (also called **direct proportion**) which is a **linear** function. The data you gathered in the "Sign on the Dotted Line" lab (in problem 1-9) is an example of a linear function.



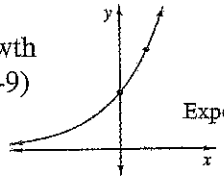
Linear

Another function is **inverse variation** (also called **inverse proportion**). The data collected in the "Hot Tub Design" lab (in problem 1-9) is an example of inverse variation.



Inverse Variation

You also observed an **exponential** function. The growth of infected people in the "Local Crisis" (in problem 1-9) was exponential.



Exponential

Note that we will define and develop these and other functions later in the course, and formally introduce functions in Section 2 of this chapter.

1-13. Consider the situation described below.

a. Meredith lives 24 blocks from her friend's house. If she travels 1 block every minute, how many minutes will it take her to reach her friend's house? What if she travels 2 blocks every minute? Show how you calculated each answer.

b. Copy and complete the table below to represent the amount of time it would take Meredith to get to her friend's house if she traveled at different rates.

Speed (in blocks per minute)	1	2	3	4	6	8	10	12	24
Time to Get to Friend's House (in minutes)									

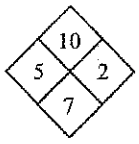
c. What happens to the time it takes to get to her friend's house as Meredith's speed increases? Explain.

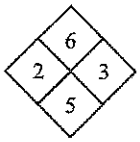
1-14. Evaluate each expression if $r=-3$, $s=4$, and $t=7$.

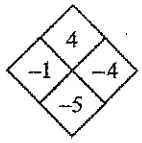
a. $r^2+\sqrt{s}$ b. $\frac{t-r}{s}$ c. $2s^2+r-t$ d. $3(s-t)^2$

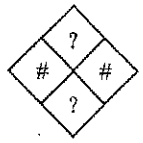
1-15. Finding and using a pattern is an important problem-solving skill you will use in algebra. The patterns in Diamond Problems will be used later in the course to solve other types of algebraic problems.

Look for a pattern in the first three diamonds below. For the fourth diamond, explain how you could find the missing numbers (?) if you know the two numbers (#).

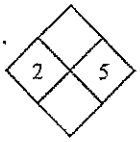


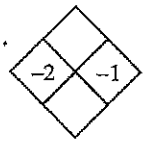


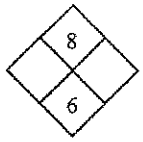


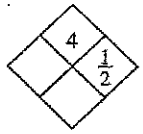


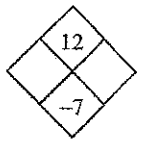
Copy the Diamond Problems below onto your paper. Then use the pattern you discovered to complete each one.

a. 

b. 

c. 

d. 

e. 

1-16. What value(s) of x will make each equation below true?

a. $x+5=5$ b. $2x-6=3x+1-x-7$

c. $3x+1=43$ d. $4x-1=4x+7$

1-17. Simplify each expression.

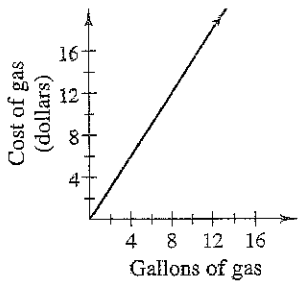
a. $\frac{2}{9}+\left(-\frac{1}{2}\right)$ b. $-\frac{6}{7}-\frac{3}{5}$ c. $\frac{9}{10}\left(-\frac{2}{3}\right)$ d. $\frac{1}{4}\div\frac{2}{7}$

1-18. In December of 2003, the average price for a gallon of regular gas in the United States was \$1.50.

a. At that time, what did it cost to buy 12 gallons of gas?

b. Gerald paid \$12.60 for a tank of gas. How many gallons did he buy?

c. At right is a graph of this situation. Predict how the line would change to represent the average cost of gas in December of 2005, when gas cost \$2.20 per gallon on average.



1-19. Solve each linear equation. Check your solutions.

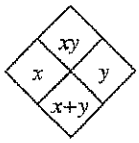
a. $-2x-3=3$ b. $7+2x=4x-3$ c. $6x-10=-8+3x$

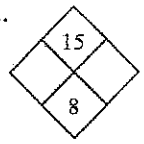
1-20. Evaluate the expressions below for the given values.

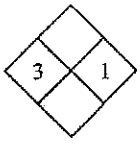
a. $-2x^2-3x+1$ for $x=-3$ b. $8-(3x-2)^2$ for $x=-2$

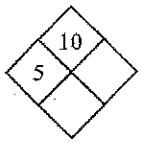
c. $\frac{-3}{k+2}$ for $k=-3$ d. $\frac{15m}{n+1}-m^2+n$ for $m=1$ and $n=2$

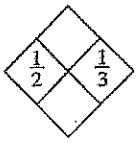
1-21. Copy and complete each of the Diamond Problems below. The pattern used in the Diamond Problems is shown at right.



a. 

b. 

c. 

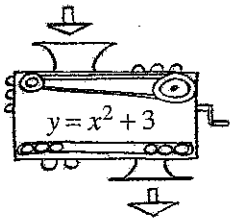
d. 

1-22. Function Machines

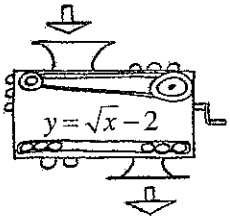
a. If an input of –9 is put into each of the machines at right, what is each output?

b. Eric wants to get an ouput of 0. Can he do this with each machine If so, how? If not, why not?

Relation A



Function B

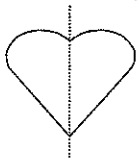


MATH NOTES

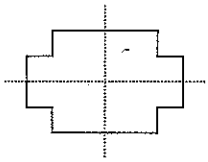
METHODS AND MEANINGS

Lines of Symmetry

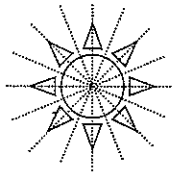
When a graph or picture can be folded so that both sides of the fold will perfectly match, it is said to have **reflective symmetry**. The line where the fold would be is called the **line of symmetry**. Some shapes have more than one line of symmetry. See the examples below.



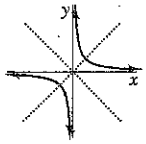
This shape has one line of symmetry.



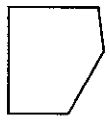
This shape has two lines of symmetry.



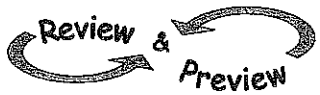
This shape has eight lines of symmetry.



This graph has two lines of symmetry.

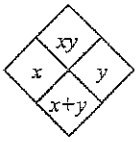


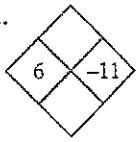
This shape has no lines of symmetry.

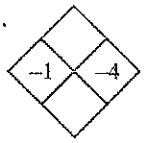


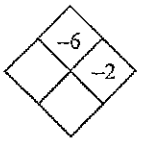
1-25. Freda Function has another quadratic function for you to investigate! Graph the equation $y = x^2 + 3$ and then answer the questions from problem 1-23.

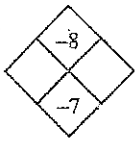
1-26. Copy these Diamond Problems and use the pattern you discovered earlier, shown at right, to complete each of them. Some of these may be challenging!



a. 

b. 

c. 

d. 

1-27. Copy the figure at right onto your paper. Then draw any lines of symmetry.

1-28. Solve the equations below for x and check your solutions.

a. $-3+2x=-x+6$

b. $5-3x=x+1$

c. $-2x=4x+9$

d. $4x+3=x$

1-29. Mr. Guo is thinking of a number. When he takes the absolute value of his number, he gets 15. What could his number be? Is there more than one possible answer?

1-32. LEARNING LOG

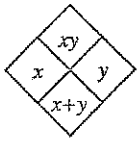
Throughout this course, you will be asked to reflect about your understanding of mathematical concepts in a Learning Log. Writing about your understanding will help you consolidate ideas, develop new ways to describe mathematical ideas, and recognize gaps in your understanding. It is important to write each entry of the Learning Log in your own words so that later you can use your Learning Log as a resource to refresh your memory. Your teacher will tell you where to write your Learning Log entries. Remember to label each entry with a title and a date so that it can be referred to later.

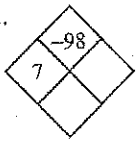
In this first Learning Log entry, as a class, create a list of all the ways to describe a graph from the presentations given by each team. Then, next to each description, create a question that will prompt you to look for this quality in the graphs of other functions you encounter.

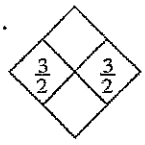
Once your class's list is complete, copy the questions into your first entry in your Learning Log. Title this entry "Graph Investigation Questions" and include today's date.

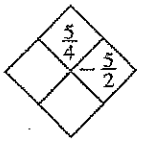
Review & Preview

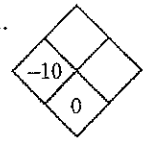
1-33. Copy these Diamond Problems and use the pattern you discovered earlier, shown at right, to complete each of them. Some of these may be challenging!



a. 

b. 

c. 

d. 

1-34. Evaluate the following absolute value expressions.

a. $|-100|-98$

b. $5|2-8|$

c. $|-13|+|0|$

d. $14-|-10+3|$

1-35. The solution to the equation $x^3 = 64$ is called the **cube root** of 64. The idea is similar to the idea of a square root, except that the value must be cubed (multiplied by itself three times) to become 64. One way to write the cube root of 64 is using the notation $\sqrt[3]{64}$. Use this information to evaluate each of the following expressions.

a. $\sqrt[3]{64}$ b. $\sqrt[4]{16}$ c. $\sqrt[3]{-8}$ d. $\sqrt[3]{125}$

1-36. Solve the following linear equations.

a. $8x+1=-x-1$ b. $-4x-3=3x-4-7x$

c. $4-5x=1+6x$ d. $7-x+3=9x+10$

1-37. Examine the tile pattern shown at right.

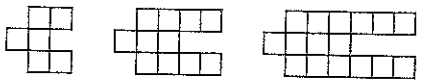


Figure 1 Figure 2 Figure 3

a. On graph paper, draw Figure 0 and Figure 4.

b. How many tiles will Figure 10 have? How do you know?

1-38. Chari performed a series of jumps on a trampoline. Her coach measured the height of each jump. The coach's data was recorded in the table at right.

Jump Number	Height (feet)
1	0.5
2	0.9
3	1.6
4	2.9
5	5.2

a. Make a graph of the data.

b. Fully describe the graph.

c. If this pattern continues, what are a reasonable maximum and minimum for the graph?

d. Which family of functions could model this data? Review the Lesson 1.1.2 Math Note if you need help.

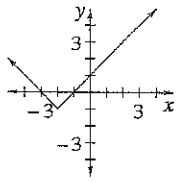
1-39. Use the idea of cube root from problem 1-35 to evaluate the following expressions.

a. $\sqrt[3]{1}$ b. $\sqrt[3]{0}$ c. $\sqrt[3]{2^3}$ d. $\sqrt[3]{7^3}$

Review & Preview

1-47. Use your list of graph investigation questions from your Learning Log to answer questions about the graph shown at right.

$y = |x+2|-1$



1-48. Calculate the value of each expression below.

a. $|-4|-3$ b. $|6-11+3|$ c. $-9-|-2|$ d. $5|6|-2$

1-49. Throughout this book, key problems have been selected as "checkpoints." Each checkpoint problem is marked with an icon like the one at left. These checkpoint problems are provided so that you can check to be sure you are building skills at the expected level. When you have trouble with checkpoint problems, refer to the review materials and practice problems that are available in the "Checkpoint Materials" section at the back of your book.

This problem is a checkpoint for solving linear equations without parentheses. It will be referred to as Checkpoint 1.

Solve each equation.

a. $3x+7=-x-1$ b. $1-2x+5=4x-3$

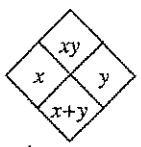
c. $4x-2+x=-2+2x$ d. $3x-4+1=-2x-5+5x$

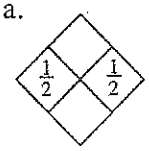
Check your answers by referring to the Checkpoint 1 materials located at the back of your book.

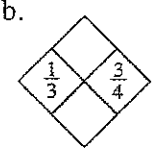
If you needed help solving these problems correctly, then you need more practice. Review the Checkpoint 1 materials and try the practice problems. Also consider getting help outside of class time. From this point on, you will be expected to do problems like this one quickly and easily.

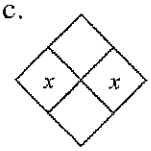
1-50. Graph the points (–3, 4) and (1, 1). If you drew a line through the points, name 3 other points that would be on the line. How did you find them?

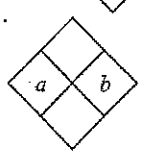
1-51. Copy and complete each of the Diamond Problems below. The pattern used in the Diamond Problems is shown at right.



a. 

b. 

c. 

d. 

Review & Preview

1-57. If $f(x) = x^2$, then $f(4) = 4^2 = 16$. Find:

a. $f(1)$ b. $f(-3)$ c. $f(t)$

1-58. Evaluate each expression.

a. $\sqrt[3]{27}$ b. $\sqrt{144}$ c. $\sqrt{3^2}$ d. $\sqrt[4]{2^4}$

1-59. Graph and fully describe the function $y = \sqrt[3]{x} - 2$.

1-60. A line passes through the points A(–3, –2) and B(2, 1). Does it also pass through the point C(5, 3)? Justify your conclusion.

1-61. Find the following absolute values.

a. $|0.75|$ b. $|-99|$

c. $|4 - 2 \cdot 3|$ d. $|\pi|$

1-66. If $g(x)=\sqrt{x-7}$, find $g(8)$, $g(32)$, and $g(80)$.

1-67. Solve each equation below. Check each solution.

a. $6-x-3=10$

b. $100x+300=200$

c. $\frac{1}{3}x+4=x-2$

d. $36-2x=-x+2$

1-68. Find $f(-4)$ for each function below.

a. $f(x)=|x-3|$

b. $f(x)=-5|x|$

c. $f(x)=|x+1|$

d. $f(x)=|x+3|-6$

1-69. Graph and fully describe the function $f(x)=-x^2+3$. Graph values of x from -3 to 3.

1-70. Find the corresponding inputs or outputs for the following functions. If there is no solution, explain why not. Be careful: In some cases, there may be no solution or more than one possible solution.

a. $x=8 \rightarrow f(x)=|x| \rightarrow f(8)=?$

b. $x=? \rightarrow f(x)=3-\sqrt{x} \rightarrow f(x)=2$

c. $k=-6 \rightarrow f(k)=\frac{k}{2}+1 \rightarrow f(-6)=?$

d. $x=3 \rightarrow f(x)=\sqrt{x-5} \rightarrow f(3)=?$

METHODS AND MEANINGS

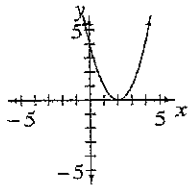
MATH NOTES

Functions

A relationship between inputs and outputs is a **function** if there is no more than one output for each input. We often write a function as y = some expression involving x, where x is the input and y is the output. The following is an example of a function.

$$y=(x-2)^2$$

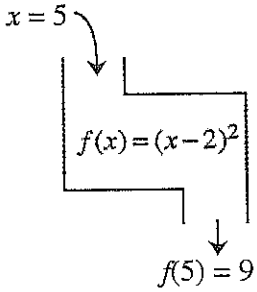
x	−2	−1	0	1	2	3	4	5
y	16	9	4	1	0	1	4	9



In the example above the value of y depends on x, so y is also called the **dependent variable** and x is called the **independent variable**.

Another way to write a function is with the notation "$f(x)$ =" instead of "y =". The function named "f" has output $f(x)$. The input is x.

In the example at right, $f(5) = 9$. The input is 5 and the output is 9. You read this as, "f of 5 equals 9."

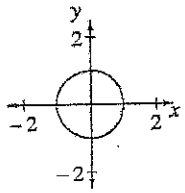


The set of all inputs for which there is an output is called the **domain**. The set of all possible outputs is called the **range**. In the example above, notice that you can input any x-value into the equation and get an output. The domain of this function is "all real numbers" because any number can be an input. But the outputs are all greater than or equal to zero. The range is $y \geq 0$.

$x^2+y^2=1$ is not a function because there are two y-values (outputs) for some x-values, as shown below.

$$x^2+y^2=1$$

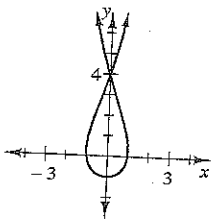
x	−1	0	0	1
y	0	−1	1	0



Review & Preview

1-78. Which of the relationships below are functions? If a relationship is not a function, give a reason to support your conclusion.

a.



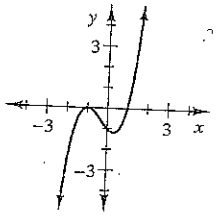
b.

x	y
–3	19
5	19
19	0
0	–3

c.

x	7	–2	0	7	4
y	10	0	10	3	0

d.



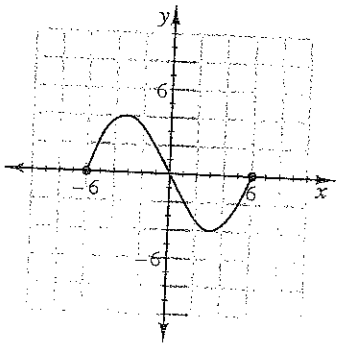
1-79. Find the x- and y-intercepts for the graphs of the relationships in problem 1-78.

1-80. Find the inputs for the following functions with the given outputs. If there is no possible input for the given output, explain why not.

a. $x = ?$ → $f(x) = 3x - 7$ → $f(x) = -1$

b. $x = ?$ → $f(x) = \sqrt{2x - 6}$ → $f(x) = 10$

1-81. Use the relationship graphed at right to answer the questions below.



a. Is the relationship a function?

b. What is the domain?

c. What is the range?

1-82. What value(s) of x will make each equation true?

a. $\sqrt[3]{x} = -2$

b. $\sqrt{x} = 12$

c. $|x + 1| = 4$

④ WHAT HAVE I LEARNED?

Most of the problems in this section represent typical problems found in this chapter. They serve as a gauge for you. You can use them to determine which types of problems you can do well and which types of problems require further study and practice. Even if your teacher does not assign this section, it is a good idea to try these problems and find out for yourself what you know and what you still need to work on.

Solve each problem as completely as you can. The table at the end of the closure section has answers to these problems. It also tells you where you can find additional help and practice with problems like these.

CL 1-83. Use the Order of Operations to simplify the following expressions.

a. $5-2\cdot 3^2$

b. $(-2)^2$

c. $18\div 3\cdot 6$

d. -2^2

e. $(5-3)(5+3)$

f. $24\cdot\frac{1}{4}\div -2$

g. Why are your answers for parts (b) and (d) different?

CL 1-84. Copy the pattern below onto graph paper. Draw the 1st and 5th figures on your paper.

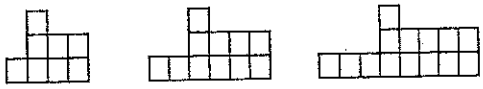


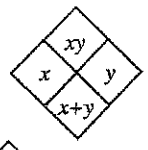
Figure 2 Figure 3 Figure 4

a. How many tiles are in each figure?

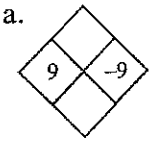
b. Describe how the pattern is changing.

c. How many tiles would the 6th figure have? The 10th figure?

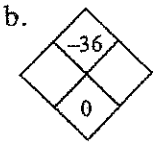
CL 1-85. Copy and complete each of the Diamond Problems below. The pattern used in the Diamond Problems is shown at right.



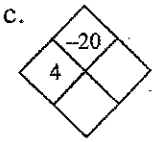
a.



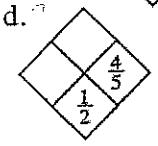
b.



c.



d.



CL 1-86. Graph and fully describe the function $y=2\sqrt{x-1}+3$.

CL 1-87. Solve each equation. Check your solution.

a. $3x-1=4x+8-x$

b. $-10+5x=7x-4$

c. $28-6x+4=30-3x$

d. $4x-1=9x-1-5x$

CL 1-88. Find $f(4)$ for each function below.

a. $f(x)=-|x-7|+3$

b. $f(x)=\frac{\sqrt{x+12}}{4}$

c. $f(x)=2-\sqrt[3]{x+23}$

CL 1-89. Evaluate each expression.

a. $2\div|3-4|$

b. $11|-6|+15$

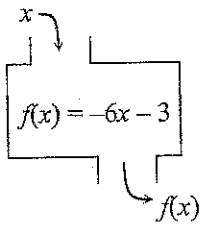
c. $-19+\sqrt[3]{-8}$

d. $-11-\sqrt{16}$

CL 1-90. Use the function machine shown at right to answer the following questions.

a. If the input is –8, what is the output?

b. If the output was 21, what was the input?



CL 1-91. Check your answers using the table at the end of the closure section. Which problems do you feel confident about? Which problems were hard? Use the table to make a list of topics you need help with and a list of topics you need to practice more.