93 a) Hearts · Clubs

$= {}_{13}C_3 \cdot {}_{13}C_2$

$= 22,308$

b) Ace · Jacks · Kings

$= {}_4C_1 \cdot {}_4C_2 \cdot {}_4C_2$

$= 144$

c) ${}_{12}C_5 = 792$

95. $A = \begin{bmatrix} -6 & 3 \\ -5 & 11 \end{bmatrix}$ $C = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$

$3C + 2A$

No solution

59. a) 1930 to 2010

D: $[0, 80]$

b) $f(0) = .0001(0)^3 - .001(0)^2 - .825(0) + 12.58 = 12.58\%$

$f(0) = 12.58\%$

In 1930, population growth was 12.58%.

c) $x = 15.4$ $x = 87.7$

In 1945, there was no growth in population.

0-4 +

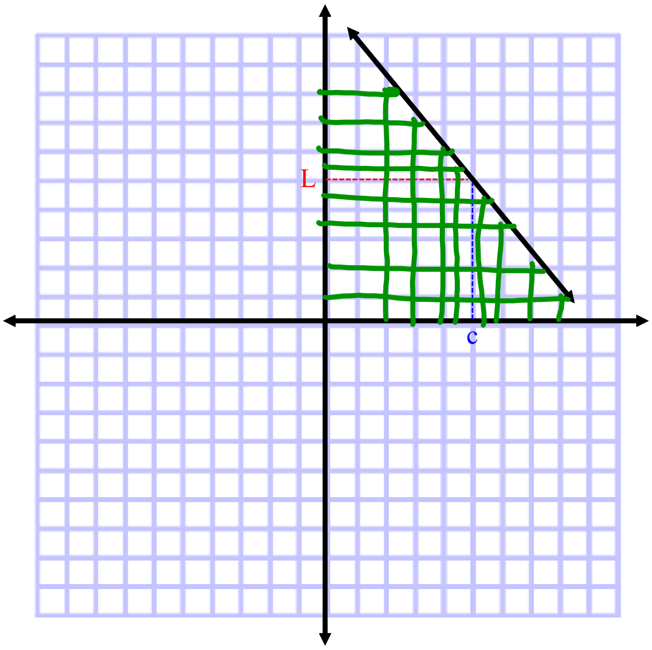
5-8 V

9↑-

d) $f(150) = 203\%$

Limit - If the value of f(x) approaches a unique value, L, as x approaches c from each side, then the limit of f(x) as x approaches c is L.

$$\lim_{x \to c} f(x) = L$$



1-3 Study Guide and Intervention

Continuity, End Behavior, and Limits

Continuity A function $f(x)$ is **continuous** at $x = c$ if it satisfies the following conditions.

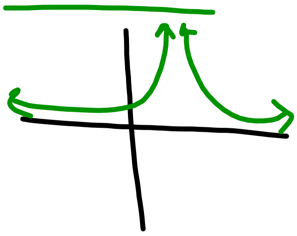
(1) $f(x)$ is defined at c; in other words, $f(c)$ exists.

(2) $f(x)$ approaches the same function value to the left and right of c; in other words, $\lim_{x \to c} f(x)$ exists.

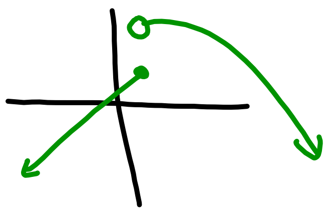
(3) The function value that $f(x)$ approaches from each side of c is $f(c)$; in other words, $\lim_{x \to c} f(x) = f(c)$.

Functions that are not continuous are **discontinuous.** Graphs that are discontinuous can exhibit **infinite discontinuity**, **jump discontinuity**, or **removable discontinuity** (also called **point discontinuity**).

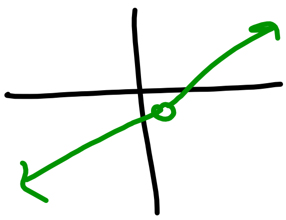
infinite



jump



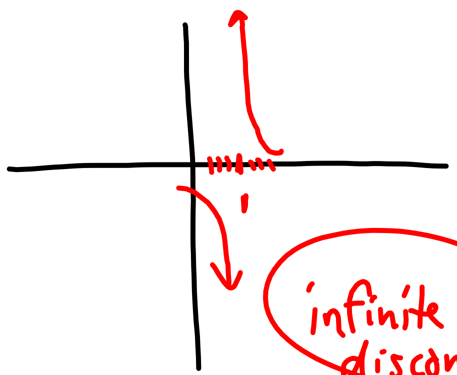
removable



Example Determine whether each function is continuous at the given x-value. Justify using the continuity test. If discontinuous, identify the type of discontinuity as *infinite*, *jump*, or *removable*.

a. $f(x) = 2|x| + 3$; $x = 2$

b. $f(x) = \frac{2x}{(x^2 - 1)}$; $x = 1$



infinite discontuity

1. $f(1) = \frac{2(1)}{(1)^2 - 1} \rightarrow$ undefined

2.

x	.9	.99	.999	1	1.001	1.01	1.1
y	-9.49	-99.49	-999.49	und.	1000.49	100.49	10.49

Exercises

Determine whether each function is continuous at the given x-value. Justify your answer using the continuity test. If discontinuous, identify the type of discontinuity as *infinite*, *jump*, or *removable*.

1. $f(x) = \begin{cases} 2x + 1 \text{ if } x > 2 \\ x - 1 \text{ if } x \leq 2 \end{cases}; x = 2$

Approximating zeros

Ex: Determine between which consecutive integers the real zeros of each function are located on the given interval.

$f(x) = x^3 - 4x + 2$; [-4,4]

* A change of signs signifies a zero.

x	-4	-3	-2	-1	0	1	2	3	4
$f(x)$	-46	-13	2	5	2	-1	2	17	50

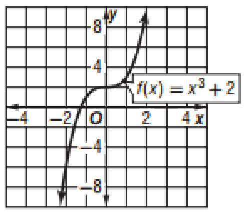
Between $x=0$ & $x=1$, $x=-3$ & $x=-2$, $x=1$ & $x=2$

Continuity, End Behavior, and Limits

End Behavior The **end behavior** of a function describes how the function behaves at either end of the graph, or what happens to the value of $f(x)$ as x increases or decreases without bound. You can use the concept of a limit to describe end behavior.

> Left-End Behavior (as x becomes more and more negative): $\lim\limits_{x \to -\infty} f(x)$
>
> Right-End Behavior (as x becomes more and more positive): $\lim\limits_{x \to \infty} f(x)$
>
> The $f(x)$ values may approach negative infinity, positive infinity, or a specific value.

Example **Use the graph of $f(x) = x^3 + 2$ to describe its end behavior. Support the conjecture numerically.**



As x decreases without bound, the y-values also decrease without bound. It appears the limit is negative infinity: $\lim\limits_{x \to -\infty} f(x) = -\infty$.

As x increases without bound, the y-values increase without bound. It appears the limit is positive infinity: $\lim\limits_{x \to \infty} f(x) = \infty$.

Construct a table of values to investigate function values as $|x|$ increases.

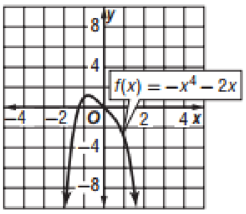
x	−1000	−100	−10	0	10	100	1000
$f(x)$	−999,999,998	−999,998	−998	2	1002	1,000,002	1,000,000,002

As $x \longrightarrow -\infty$, $f(x) \longrightarrow -\infty$. As $x \longrightarrow \infty$, $f(x) \longrightarrow \infty$. This supports the conjecture.

Exercises

Use the graph of each function to describe its end behavior. Support the conjecture numerically.

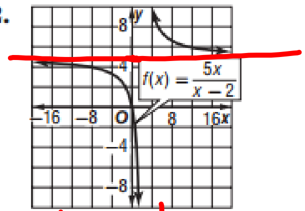
1.



$$\lim_{x \to \infty} f(x) = 5$$

$$\lim_{x \to -\infty} f(x) = 5$$

2.



x	-1000	-100	-10	0	10	100	1000
Y	4.99	4.90	4.16	0	6.25	5.10	5.01

HW: p. 30, # 1, 5, 9, 11, 23, 25, 31, 35, 51