13. $f(x) = x^3 - x^2 - 3$ $[-2, 4]$

x	-2	-1	0	1	2	3	4
f(x)	-15	-5	-3	-3	1	15	45

$[1, 2]$

27.

x	-1000	-100	-10	0	10	100	1000
f(x)	-.008	-.08	-.8	0	.8	.08	.008

$f(x) = \frac{16x^2}{2x^3+5x+2}$

$\lim_{x \to -\infty} f(x) = 0$ $\lim_{x \to \infty} f(x) = 0$

0 - 4 +
5 - 8 ✓
9 ↑ -

1.4 Study Guide and Intervention

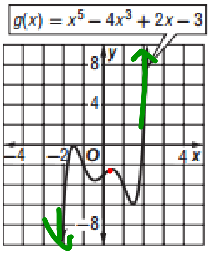
Extrema and Average Rates of Change

Increasing and Decreasing Behavior Functions can increase, decrease, or remain constant over a given interval. The points at which a function changes its increasing or decreasing behavior are called critical points. A critical point can be a **relative minimum**, **absolute minimum**, **relative maximum**, or **absolute maximum**. The general term for minimum or maximum is extremum or **extrema**.

· inflection point

Example **Estimate to the nearest 0.5 unit and classify the extrema for the graph of $f(x)$. Support the answers numerically.**

Graphically:



relative maximum: (-1.5, 0), (.5, -3)

relative minimum: (-.5, -3) (1.5, -6)

Numerically

x	-1000	-2	-1.5	-1	-.5	0	.5	1	1.5	2	1000
f(x)	-9000000000	-7	-.09	-2	-3.5	-3	-2.5	-4	-5.9	1	9000000000

Exercises

Use a graphing calculator to approximate to the nearest hundredth the relative or absolute extrema of each function. State the x-value(s) where they occur.

1. $f(x) = 2x^6 + 2x^4 - 9x^2$

relative max: $(\underline{0}, 0)$

absolute: $(-\underline{1}, -5)$ & $(\underline{1}, -5)$

x	-1000	-1.5	-1	-.5	0	.5	1	1.5	1000
f(x)	2000.....	12.6	$\boxed{-5}$	-2.09	$\boxed{0}$	-2.09	$\boxed{-5}$	12.6	2000000000000000000

Extrema and Average Rates of Change

Average Rate of Change The **average rate of change** between any two points on the graph of f is the slope of the line through those points. The line through any two points on a curve is called a **secant line**.

The average rate of change on the interval $[x_1, x_2]$ is the slope of the secant line, m_{sec}.

$$m_{sec} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

Example **Find the average rate of change of $f(x) = 0.5x^3 + 2x$ on each interval.**

x_1 x_2

a. [−3, −1]

$$\frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{f(-1) - f(-3)}{-1 - (-3)}$$ Substitute −3 for x_1 and −1 for x_2.

$$= \frac{[0.5(-1)^3 + 2(-1)] - [0.5(-3)^3 + 2(-3)]}{-1 - (-3)}$$ Evaluate $f(-1)$ and $f(-3)$.

$$= \frac{-2.5 - (-19.5)}{-1 - (-3)} \text{ or } \frac{17}{2}$$ Simplify.

b. [−1, 1]

$$\frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{f(1) - f(-1)}{1 - (-1)}$$ Substitute −1 for x_1 and 1 for x_2.

$$= \frac{2.5 - (-2.5)}{1 - (-1)} \text{ or } \frac{5}{2}$$ Evaluate and simplify.

Exercises

Find the average rate of change of each function on the given interval.

x_1 x_2

1. $f(x) = x^4 + 2x^3 - x - 1$; $[-3, -2]$

$$M_{sec} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

$$= \frac{(-2)^4 + 2(-2)^3 - (-2) - 1 - \left[(-3)^4 + 2(-3)^3 - (-3) - 1\right]}{-2 - -3} = \frac{1 - 29}{1} = \frac{-28}{1} = -28$$

2. $f(x) = x^4 + 2x^3 - x - 1$; $[-1, 0]$

$f(x_1) =$

$f(x_2) =$

5. $f(x) = x^4 + 8x - 3$; $[-4, 0]$

6. $f(x) = -x^4 + 8x - 3$; $[0, 1]$

HW: p. ~~32~~ 40, #3, 7, 13, 17, 25, 29, 35, 39, 47