

## 1-5 Study Guide and Intervention

### Parent Functions and Transformations

**Parent Functions** A parent function is the simplest of the functions in a family.

| Parent Function           | Form                       | Notes   |
|---------------------------|----------------------------|---|
| constant function         | $f(x) = c$                 | graph is a horizontal line  |
| identity function         | $f(x) = x$                 | points on graph have coordinates $(a, a)$   |
| quadratic function        | $f(x) = x^2$               | graph is U-shaped   |
| cubic function            | $f(x) = x^3$               | graph is symmetric about the origin   |
| square root function      | $f(x) = \sqrt{x}$          | graph is in first quadrant  |
| reciprocal function       | $f(x) = \frac{1}{x}$       | graph has two branches  |
| absolute value function   | $f(x) =  x $               | graph is V-shaped   |
| greatest integer function | $f(x) = \lfloor x \rfloor$ | defined as the greatest integer less than or equal to $x$ ; type of step function |

**Example** Describe the following characteristics of the graph of the parent function  $f(x) = x^3$ : domain, range, intercepts, symmetry, continuity, end behavior, and intervals on which the graph is increasing/decreasing.

The graph confirms that  $D = \{x | x \in \mathbb{R}\}$  and  $R = \{y | y \in \mathbb{R}\}$ .

The graph intersects the origin, so the  $x$ -intercept is 0 and the  $y$ -intercept is 0.

It is symmetric about the origin and it is an odd function:

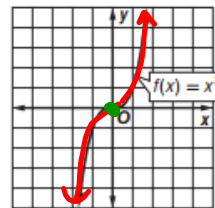
$$f(-x) = -f(x) \quad f(-x) = (-x)^3 = -x^3$$

The graph is continuous because it can be traced without lifting the pencil off the paper.

As  $x$  decreases,  $y$  approaches negative infinity, and as  $x$  increases,  $y$  approaches positive infinity.

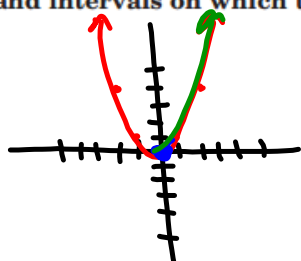
$$\lim_{x \rightarrow -\infty} f(x) = -\infty \quad \text{and} \quad \lim_{x \rightarrow \infty} f(x) = \infty$$

The graph is always increasing, so it is increasing for  $(-\infty, \infty)$ .



**Exercise**

Describe the following characteristics of the graph of the parent function  $f(x) = x^2$ : domain, range, intercepts, symmetry, continuity, end behavior, and intervals on which the graph is increasing/decreasing.



$$D: (-\infty, \infty)$$

$$R: [0, \infty)$$

$$x\text{-int: } (0, 0)$$

$$y\text{-int: } (0, 0)$$

sym. over y-axis

continuous

$$\lim_{x \rightarrow -\infty} f(x) = \infty \quad \lim_{x \rightarrow \infty} f(x) = \infty$$

increasing:  
 $(0, \infty)$

decreasing:  
 $(-\infty, 0)$

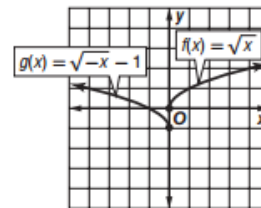
### Parent Functions and Transformations

**Transformations of Parent Functions** Parent functions can be transformed to create other members in a family of graphs.

|                     |  |  |   |
|---------------------|--|--|---|
| <i>moving</i>       | <b>Translations</b>                                    | $g(x) = f(x) + k$ is the graph of $f(x)$ translated... | ... $k$ units up when $k > 0$ .           |
|                     |  |  | ... $k$ units down when $k < 0$ .         |
|                     | $g(x) = f(x - h)$ is the graph of $f(x)$ translated... | ... $h$ units right when $h > 0$ .                     |   |
|                     |  | ... $h$ units left when $h < 0$ .                      |   |
| <b>Reflections</b>  | $g(x) = -f(x)$ is the graph of $f(x)$ ...              | ...reflected in the $x$ -axis.                         |   |
|                     | $g(x) = f(-x)$ is the graph of $f(x)$ ...              | ...reflected in the $y$ -axis.                         |   |
| <i>change shape</i> | <b>Dilations</b>                                       | $g(x) = a \cdot f(x)$ is the graph of $f(x)$ ...       | ...expanded vertically if $a > 1$ .       |
|                     |  |  | ...compressed vertically if $0 < a < 1$ . |
|                     | $g(x) = f(ax)$ is the graph of $f(x)$ ...              | ...compressed horizontally if $a > 1$ .                |   |
|                     |  | ...expanded horizontally if $0 < a < 1$ .              |   |

**Example** Identify the parent function  $f(x)$  of  $g(x) = \sqrt{-x} - 1$ , and describe how the graphs of  $g(x)$  and  $f(x)$  are related. Then graph  $f(x)$  and  $g(x)$  on the same axes.

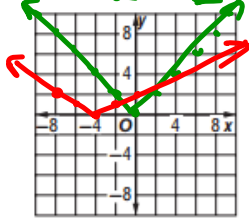
The graph of  $g(x)$  is the graph of the square root function  $f(x) = \sqrt{x}$  reflected in the  $y$ -axis and then translated one unit down.



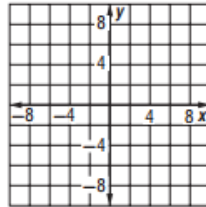
### Exercises

Identify the parent function  $f(x)$  of  $g(x)$ , and describe how the graphs of  $g(x)$  and  $f(x)$  are related. Then graph  $f(x)$  and  $g(x)$  on the same axes.

1.  $g(x) = 0.5|x + 4|$



2.  $g(x) = 2x^2 - 4$



parent function:  $f(x) = |x|$   
 vertical dilation  
 horizontal translation

HW: p. 52, #3, 7, 11, 15, 21, 29, 33