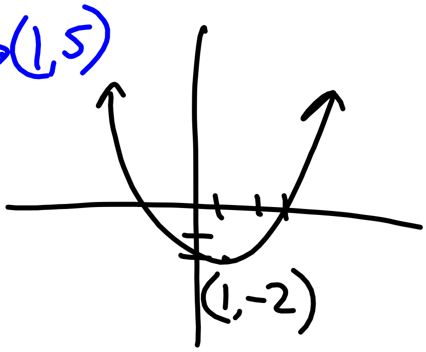
93 vertical dilation → (1, -6)
vertical translation → (1, 5)
reflection → (1, 6)



0 - 3 +
4 - 6 ✓
7 ↑ -

1-6 Study Guide and Intervention

Function Operations and Composition of Functions

Operations with Functions Two functions can be added, subtracted, multiplied, or divided to form a new function. For the new function, the domain consists of the intersection of the domains of the two functions, excluding values that make a denominator equal to zero.

Example 1 **Given $f(x) = x^2 - x - 6$ and $g(x) = x + 2$, find each function and its domain.**

a. $(f + g)(x)$

$$(f + g)x = f(x) + g(x)$$
$$= x^2 - x - 6 + x + 2$$
$$= x^2 - 4$$

The domains of f and g are both $(-\infty, \infty)$, so the domain of $(f + g)$ is $(-\infty, \infty)$.

b. $\left(\frac{f}{g}\right)(x)$

$$\left(\frac{f}{g}\right)x = \frac{f(x)}{g(x)}$$
$$= \frac{x^2 - x - 6}{x + 2}$$
$$= \frac{(x - 3)\cancel{(x + 2)}}{\cancel{x + 2}} = x - 3$$

The domains of f and g are both $(-\infty, \infty)$, but $x = -2$ yields a zero in the denominator of $\left(\frac{f}{g}\right)$. So, the domain is $\{x \mid x \neq -2, x \in \mathbb{R}\}$. $(-\infty, -2) \cup (-2, \infty)$

Example 2 **Given $f(x) = x^2 - 3$ and $g(x) = \frac{1}{x}$, find each function and its domain.**

a. $(f - g)(x)$

$$(f - g)x = f(x) - g(x)$$
$$= x^2 - 3 - \frac{1}{x}$$

The domain of f is $(-\infty, \infty)$ and the domain of g is $(-\infty, 0) \cup (0, \infty)$, so the domain of $(f - g)$ is $(-\infty, 0) \cup (0, \infty)$.

b. $(f \cdot g)(x)$

$$(f \cdot g)x = f(x) \cdot g(x)$$
$$= (x^2 - 3)\frac{1}{x}$$
$$= x - \frac{3}{x}$$

The domain of f is $(-\infty, \infty)$ and the domain of g is $(-\infty, 0) \cup (0, \infty)$, so the domain of $(f - g)$ is $(-\infty, 0) \cup (0, \infty)$.

Exercises

Find $(f+g)(x)$, $(f-g)(x)$, $(f\cdot g)(x)$, and $\left(\frac{f}{g}\right)(x)$ for each $f(x)$ and $g(x)$. State the domain of each new function.

1. $f(x) = x^2 - 1$, $g(x) = \frac{2}{x}$

$(f+g)(x) = f(x) + g(x)$

$= \frac{x^2(x)}{1(x)} - \frac{1(x)}{1(x)} + \frac{2}{x}$

$= \frac{x^3 - x + 2}{x}$

$D: (-\infty, 0) \cup (0, \infty)$

$(f-g)(x) = \frac{(x)x^2}{(x)1} - \frac{1(x)}{1(x)} - \left(\frac{2}{x}\right)$

$= \frac{x^3 - x - 2}{x}$

$D: (-\infty, 0) \cup (0, \infty)$

2. $f(x) = x^2 + 4x - 7$, $g(x) = \sqrt{x}$

$(f\cdot g)(x) = f(x) \cdot g(x)$

$= (x^2 - 1)\left(\frac{2}{x}\right)$

$= \frac{(x)2x}{(x)1} - \frac{2}{x}$

$= \frac{2x^2 - 2}{x}$

$D: (-\infty, 0) \cup (0, \infty)$

$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$

$= \frac{(x^2-1)x}{\left(\frac{2}{x}\right)x}$

$= \frac{x^3 - x}{2}$

$D: (-\infty, 0) \cup (0, \infty)$

$\left(\frac{f}{g}\right)(2) = \frac{(2)^3 - 2}{2}$

$= 3$

or $\frac{f(2)}{g(2)} = \frac{(2)^2 - 1}{\frac{2}{2}} = 3$

Function Operations and Composition of Functions

Compositions of Functions In a function **composition**, the result of one function is used to evaluate a second function.

Given functions f and g, the composite function $f \circ g$ can be described by the equation $[f \circ g](x) = f[g(x)]$. The domain of $f \circ g$ includes all x-values in the domain of g for which $g(x)$ is in the domain of f.

Example **Given $f(x) = 3x^2 + 2x - 1$ and $g(x) = 4x + 2$, find $[f \circ g](x)$ and $[g \circ f](x)$.**

$[f \circ g](x) = f[g(x)]$	Definition of composite functions
$= f(4x + 2)$	Replace $g(x)$ with $4x + 2$.
$= 3(4x + 2)^2 + 2(4x + 2) - 1$	Substitute $4x + 2$ for x in $f(x)$.
$= 3(16x^2 + 16x + 4) + 8x + 4 - 1$	Simplify.
$= 48x^2 + 56x + 15$	
$[g \circ f](x) = g(f(x))$	Definition of composite functions
$= g(3x^2 + 2x - 1)$	Replace f(x) with $3x^2 + 2x - 1$.
$= 4(3x^2 + 2x - 1) + 2$	Substitute $3x^2 + 2x - 1$ for x in $g(x)$.
$= 12x^2 + 8x - 2$	Simplify.

Exercises

For each pair of functions, find $[f \circ g](x)$, $[g \circ f](x)$, and $[f \circ g](4)$.

1. $f(x) = 2x + 1$, $g(x) = x^2 - 2x - 4$

2. $f(x) = 3x^2 - 4$, $g(x) = \frac{1}{x}$

$(f \circ g)(x) = f[g(x)] = 3\left(\frac{1}{x}\right)^2 - 4$

$= 3\left(\frac{1}{x^2}\right) - 4$

$= \frac{3}{x^2} - \frac{4(x^2)}{1(x^2)}$

$= \frac{3 - 4x^2}{x^2}$

5. $f(x) = 3x - 5$, $g(x) = x^2 + 1$

6. $f(x) = \frac{1}{x - 1}$, $g(x) = x^2 - 1$

$(g \circ f)(x) = g[f(x)]$

$= \frac{1}{3x^2 - 4}$

$(f \circ g)(4) = \frac{3 - 4(4)^2}{(4)^2} = \boxed{\frac{-61}{16}}$

HW: p. 61 # 1, 3, 9, 13, 15, 17, 23, 31, 41