$$f(x) = \frac{x^2 - 3}{x - 4}$$

$$0 = \frac{x^2 - 3}{x - 4}$$

$$x^2 - 3 = 0$$

$$+3 \quad +3$$

$$\sqrt{x^2} = \sqrt{3}$$

$$\boxed{x = \pm\sqrt{3}}$$

0 - 5 +

6 - 10 ✓

11 ↑ -

1-7 Study Guide and Intervention

Inverse Relations and Functions

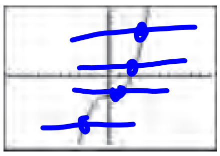
Inverse and One-to-One-Functions Two relations are **inverse relations** if and only if one relation contains the element (b, a) whenever the other relation contains the element (a, b). If the inverse of the function $f(x)$ is also a function, then the inverse is denoted by $f^{-1}(x)$.

A function has an inverse function if and only if each horizontal line intersects the graph of the function in at most one point. This is known as the **horizontal line test**. If a function passes the horizontal line test, it is said to be **one-to-one** because every x-value is matched with exactly one y-value.

Example **Graph each function using a graphing calculator, and apply the horizontal line test to determine whether its inverse function exists. Write *yes* or *no*.**

a. $f(x) = \frac{1}{4}x^3 - 3$

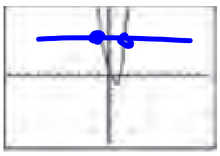
Yes, the inverse function exists. It is not possible to find a horizontal line that intersects the graph of $f(x)$ in more than one point, so you can conclude that $f^{-1}(x)$ exists.



[−10, 10] scl: 1 by [−10, 10] scl: 1

b. $g(x) = x^4 + 2x^2 - 5x + 1$

No, the inverse function does not exist. It is possible to find a horizontal line that intersects the graph in more than point. Therefore, you can conclude that $g^{-1}(x)$ does not exist.



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Exercises

Exercises

Graph each function using a graphing calculator, and apply the horizontal line test to determine whether its inverse function exists. Write *yes* or *no*.

1. $f(x) = \frac{1}{x}$

2. $f(x) = x^2 - 5$

3. $f(x) = x^3 - 8x^2 + 6x - 4$

4. $f(x) = \frac{1}{\sqrt{x - 4}}$

Yes

5. $f(x) = -x^3 + 6$

6. $f(x) = -x^3 + 2x$

No

Inverse Relations and Functions

Find Inverse Functions Follow the steps below to find an inverse function algebraically.

Step 1: Use the horizontal line test to confirm the inverse function exists.

Step 2: Replace $f(x)$ with y and then interchange x and y.

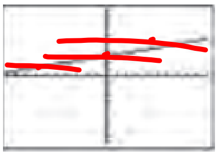
Step 3: Solve for y. Replace y with $f^{-1}(x)$.

Step 4: State any restrictions on the domain.

- You can verify the inverse function by showing that $f[f^{-1}(x)] = x$ and that $f^{-1}[f(x)] = x$

- If given the graph of a function, you can graph its inverse. Locate points on $f(x)$, and reflect them in the line $y = x$. Interchange the x- and y-coordinates, and connect them with a straight line or smooth curve.

Example **Determine whether $f(x) = \frac{1}{4}x + 3$ has an inverse function. If it does, find the inverse function and state any restrictions on the domain.**



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Step 1: The graph shows the function passes the horizontal line test, so the inverse function exists.

Step 2: $y = \frac{1}{4}x + 3$ — Replace $f(x)$ with y.

$x = \frac{1}{4}y + 3$ — Exchange x and y.

Step 3: $4x = y + 12$ — Solve for y.

$y = 4x - 12$

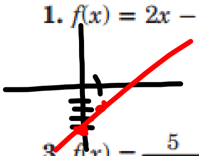
$f^{-1}(x) = 4x - 12$ — Replace y with $f^{-1}(x)$.

Step 4: There are no restrictions on the domain.

Exercises

Determine whether *f* has an inverse function. If it does, find the inverse function and state any restrictions on the domain.

1. $f(x) = 2x - 4$



$y = 2x - 4$

$x = 2y - 4$

$+4 \qquad +4$

$\frac{x+4}{2} = \frac{\cancel{2}y}{\cancel{2}}$

$\frac{x+4}{2} = f^{-1}(x)$

$f(f^{-1}(x)) = \cancel{2}\left(\frac{x+4}{\cancel{2}}\right) - 4$

$= x + \cancel{4} - \cancel{4}$

$= x$

$f^{-1}(f(x)) = \frac{(2x - \cancel{4}) + \cancel{4}}{2}$

$= \frac{\cancel{2}x}{\cancel{2}}$

$= x$

2. $f(x) = (x - 1)^2 + 2$

3. $f(x) = \frac{5}{x-2}$

4. $f(x) = x^3 + 4$

$f(x) = \frac{5}{x-2}$

$y = \frac{5}{x-2}$

$\cancel{x = \frac{5}{y-2}}$

$\frac{\cancel{x}(y-2)}{\cancel{x}} = \frac{5}{x}$

$y \cancel{-2} = \frac{5}{x} + 2$

$\quad +2$

$\boxed{f^{-1}(x) = \frac{5}{x} + 2}$

HW: p. 20

1, 5, 9, 13, 17, 19, 27, 35, 37, 39, 47