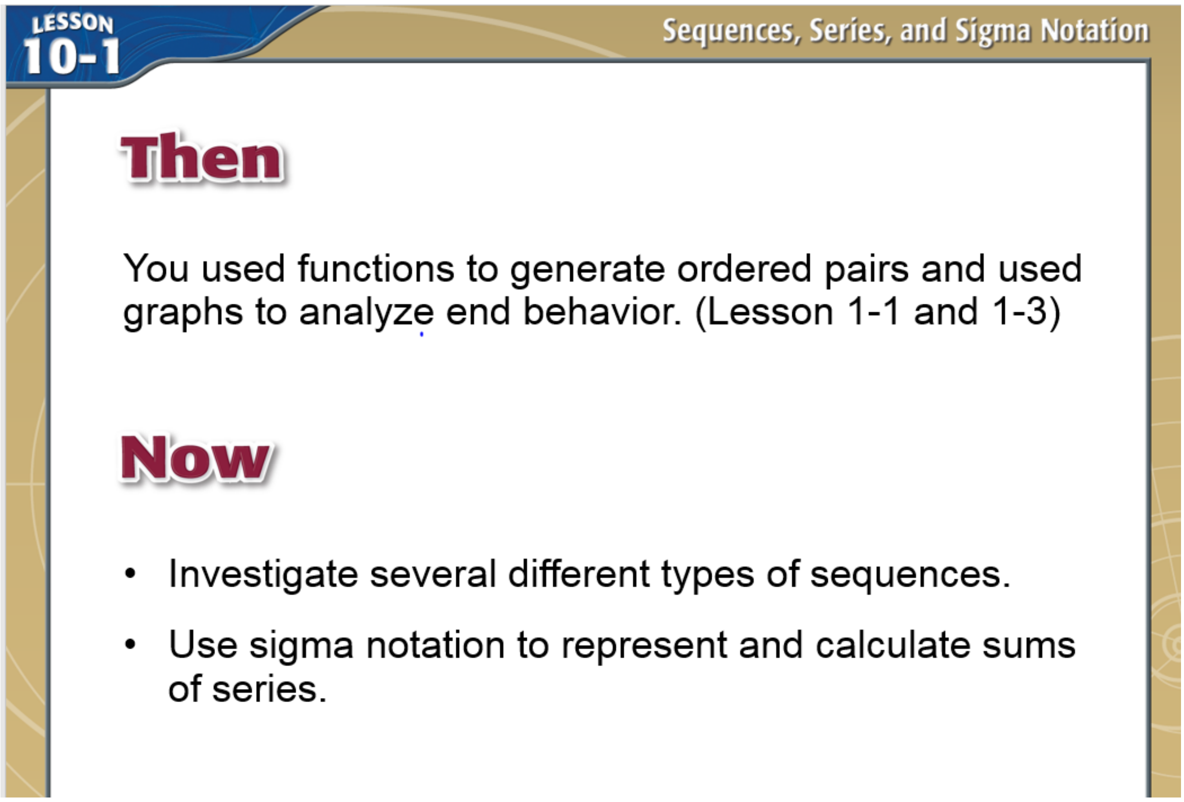
LESSON 10-1
Sequences, Series, and Sigma Notation
Then
You used functions to generate ordered pairs and used graphs to analyze end behavior. (Lesson 1-1 and 1-3)
Now
• Investigate several different types of sequences.
• Use sigma notation to represent and calculate sums of series.

New Vocabulary

- sequence
- term
- finite sequence
- infinite sequence
- recursive sequence
- explicit sequence
- Fibonacci sequence
- converge
- diverge
- series
- finite series
- *n*th partial sum
- infinite series
- sigma notation

10-1 Study Guide and Intervention

Sequences, Series, and Sigma Notation

Sequences A **sequence** is a function with a domain that is the set of natural numbers. The terms of a sequence are the range elements of the function. The *n*th term is written a_n. A term in a **recursive sequence** depends on the previous term. In an **explicit sequence**, any *n*th term can be calculated from the formula. A sequence that approaches a specific value is said to be **convergent**. Otherwise, it is **divergent**.

a_5 = 5th term

Example 1: Find the next four terms of the sequence –7, –3, 4, 14, 27, … .

Find the difference between terms to determine a pattern.

$a_2 - a_1 = -3 - (-7) = 4$

$a_3 - a_2 = 4 - (-3) = 7$ The differences are increasing by 3.

$a_4 - a_3 = 14 - 4 = 10$

$a_5 - a_4 = 27 - 14 = 13$

The next four terms are 43, 62, 84, and 109.

Example 2: Find the sixth term of the sequence $a_n = \frac{1}{2n} + 2$.

The sequence is explicit. Substitute 6 for *n*.

$a_6 = \frac{1}{2(6)} + 2 = 2\frac{1}{12}$ or $2.08\overline{3}$

Example 3: Find the third term of the sequence $a_1 = 9$, $a_n = \frac{-a_{n-1}}{3}$.

The sequence is recursive. The first term is given. You need to find the second term before you can find the third term.

$a_2 = \frac{-a_1}{3} = \frac{-9}{3}$ or -3 Substitute 2 for *n*.

$a_3 = \frac{-a_2}{3} = \frac{-(-3)}{3}$ or 1 Substitute 3 for *n*.

The third term is 1.

Exercises

1. Find the next four terms of the sequence 125, 25, 5, 1, … .

$\frac{1}{5}, \frac{1}{25}, \frac{1}{125}, \frac{1}{625}, \ldots\ldots$

Find the specified term of each sequence.

2. tenth term; $a_n = 3n - 7$

$a_{10} = 3(10) - 7$

$a_{10} = 23$

3. third term; $a_1 = 3$, $a_n = \frac{1}{2a_{n-1}}$

$a_2 = \frac{1}{2(a_1)} = \frac{1}{2(3)} = \frac{1}{6}$

$a_3 = \frac{1}{2(a_2)} = \frac{1}{2(\frac{1}{6})} = 3$

4. eighth term; $a_n = \frac{n^3 - 1}{2}$

$a_8 = \frac{(8)^3 - 1}{2} = 255.5$

5. fourth term; $a_1 = 7$, $a_n = 2a_{n-1} + 5$

$a_2 = 2(a_1) + 5 = 2(7) + 5 = 19$

$a_3 = 2(a_2) + 5 = 2(19) + 5 = 43$

$a_4 = 2(a_3) + 5 = 2(43) + 5 = 91$

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21. $a_1 = -64$ $\quad a_n = \frac{3}{4} a_{n-1}$

$a_2 = \frac{3}{4} a_1 = \frac{3}{4}(-64) = -48$

$a_3 = \frac{3}{4} a_2 = \frac{3}{4}(-48) = -36$

$a_4 = \frac{3}{4} a_3 = \frac{3}{4}(-36) = -27$

$a_5 = \frac{3}{4} a_4 = \frac{3}{4}(-27) = -20.25$

$a_6 = \frac{3}{4} a_5 = -15.2$

$a_7 = \frac{3}{4} a_6 = -11.4$

0 - 3 +
4 - 6 ✓
7 ↑ -

KeyConcept Sigma Notation

For any sequence $a_1, a_2, a_3, a_4, \ldots$, the sum of the first k terms is denoted

$$\sum_{n=1}^{k} a_n = a_1 + a_2 + a_3 + \ldots + a_k,$$

where n is the index of summation, k is the upper bound of summation, and 1 is the lower bound of summation.

10-1 Study Guide and Intervention *(continued)*

Sequences, Series, and Sigma Notation

Ex: $\sum_{n=1}^{5} a_n$

Series and Sigma Notation A **series** is the sum of all the terms of a sequence. The ***n*th partial sum** is the sum of the first n terms. A partial sum can be symbolized as S_n. Therefore, S_5 is the sum of the first five terms of a sequence. A series may be written using **sigma notation**, denoted by the Greek letter sigma Σ. A formula is written to the right of sigma. The first number to be substituted for the variable in this formula is given below sigma and the last number to be substituted for the variable is above sigma. The results of each substitution are then added.

$$\sum_{n=1}^{k} a_n = a_1 + a_2 + a_3 + \ldots + a_k$$

Example 1: Find the seventh partial sum of –22, –10, 1, 11, … .

Find the pattern of the sequence to find the fifth, sixth, and seventh terms.

Notice that $a_2 - a_1 = 12$, $a_3 - a_2 = 11$, $a_4 - a_3 = 10$.

Continuing the pattern: $a_5 = 11 + 9 = 20$

$a_6 = 20 + 8 = 28$

$a_7 = 28 + 7 = 35$

The seventh partial sum is $S_7 = -22 + (-10) + 1 + 11 + 20 + 28 + 35$ or 63.

Example 2: Find the sum of the series $n = \sum_{n=1}^{4} \frac{2^n}{4}$.

Find a_1, a_2, a_3, and a_4.

$a_1 = \frac{2^1}{4}$ or $\frac{1}{2}$ $\qquad$ $a_3 = \frac{2^3}{4}$ or 2

$a_2 = \frac{2^2}{4}$ or 1 $\qquad$ $a_4 = \frac{2^4}{4}$ or 4

$$\sum_{n=1}^{4} \frac{2^n}{4} = \frac{1}{2} + 1 + 2 + 4 = \frac{15}{2}$$

Exercises

1. Find the sixth partial sum of $a_n = 4n - 1$.

$a_1 = 4(1) - 1 = 3$

$a_2 = 4(2) - 1 = 7$

$a_3 = 4(3) - 1 = 11$

$a_4 = 4(4) - 1 = 15$

$a_5 = 19$

$a_6 = 23$

$S_6 = 3 + 7 + 11 + 15 + 19 + 23 = 78$

2. Find the fourth partial sum of $a_n = \frac{-2a_{n-1}}{5}$, $a_1 = -1$.

$a_1 = -1$

$a_2 = \frac{-2(a_1)}{5} = \frac{-2(-1)}{5} = \frac{2}{5}$

$a_3 = \frac{-2(\frac{2}{5})}{5} = -\frac{4}{25}$

$a_4 = \frac{-2(-\frac{4}{25})}{5} = \frac{8}{125}$

$\frac{\frac{-2}{1} \cdot \frac{2}{5}}{5} = \frac{\frac{-4}{5}}{5} = \frac{-4}{5} \cdot \frac{1}{5}$

$S_4 = -1 + \frac{2}{5} + -\frac{4}{25} + \frac{8}{125} = -\frac{87}{125}$

$= -1 + .4 + -.16 + .064$

Find each sum.

3. $\sum_{n=3}^{7} n^2 - 2$ $= 7 + 14 + 23 + 34 + 47$

125

$a_3 = (3)^2 - 2 = 7$

$a_4 = (4)^2 - 2 = 14$

$a_5 = (5)^2 - 2 = 23$

$a_6 = (6)^2 - 2 = 34$

$a_7 = (7)^2 - 2 = 47$

4. $\sum_{1}^{6} \quad 3\left(\frac{1}{2}\right)^{n-2}$

$a_1 = 3\left(\frac{1}{2}\right)^{1-2} = 6$

$a_2 = 3\left(\frac{1}{2}\right)^{2-2} = 3$

$a_3 = 3\left(\frac{1}{2}\right)^{3-2} = \frac{3}{2}$

$a_4 = 3\left(\frac{1}{2}\right)^{4-2} = \frac{3}{4}$

$a_5 = 3\left(\frac{1}{2}\right)^{5-2} = \frac{3}{8}$

$a_6 = 3\left(\frac{1}{2}\right)^{6-2} = \frac{3}{16}$

$6 + 3 + \frac{3}{2} + \frac{3}{4} + \frac{3}{8} + \frac{3}{16} = \frac{189}{16}$

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