

$$103 \quad \tan \frac{\pi}{12} = \tan \left(\frac{\pi}{3} - \frac{\pi}{4} \right)$$

$$= \frac{\tan \frac{\pi}{3} - \tan \frac{\pi}{4}}{1 + \tan \frac{\pi}{3} \tan \frac{\pi}{4}}$$

$$= \frac{\sqrt{3} - 1}{1 + (\sqrt{3})(1)}$$

$$= \frac{(\sqrt{3} - 1)(1 - \sqrt{3})}{(1 + \sqrt{3})(1 - \sqrt{3})}$$

$$= \frac{\sqrt{3} - 3 - 1 + \sqrt{3}}{1 - 3}$$

$$= \frac{-2\sqrt{3} - 4}{-2}$$

$$= -\sqrt{3} + 2$$

$$\begin{array}{l} 0-3+ \\ 4-5\checkmark \\ 6\uparrow- \end{array}$$

109. - 5, 1, 7, 13, 19, 25, 31, 37, 43, 49, 55, 61,
67, 73, 79, 85, 91, 97, 103, 109, 115, 121, 127,
133, 139, 145, 151, 157, No solution?

KeyConcept The n th Term of an Arithmetic Sequence**Words**

The n th term of an arithmetic sequence with first term a_1 and common difference d is given by $a_n = a_1 + (n - 1)d$.

+ or -

Example

The 16th term of 2, 5, 8, ... is $a_{16} = 2 + (16 - 1) \cdot 3$ or 47.

Example 1: Find the 38th term of the arithmetic sequence $-7, -5, -3, \dots$

First find the common difference.

$$a_2 - a_1 = -5 - (-7) \text{ or } 2$$

$$a_3 - a_2 = -3 - (-5) \text{ or } 2$$

Use the explicit formula $a_n = a_1 + (n - 1)d$ to find a_{38} . Use $n = 38$, $a_1 = -7$, and $d = 2$.

$$\begin{aligned} a_{38} &= -7 + (38 - 1)2 \\ &= 67 \end{aligned}$$

Example 2: Write an arithmetic sequence that has three arithmetic means between 3.2 and 4.4.

The sequence will have the form 3.2, ? , ? , ? , 4.4. Find d .

$$a_n = a_1 + (n - 1)d \quad \text{Formula for } n\text{th term of arithmetic sequence}$$

$$4.4 = 3.2 + (5 - 1)d \quad \text{Substitute.}$$

$$4.4 = 3.2 + 4d \quad \text{Simplify.}$$

$$d = 0.3$$

Determine the arithmetic means recursively.

$$a_2 = 3.2 + 0.3 = 3.5, a_3 = 3.5 + 0.3 = 3.8, a_4 = 3.8 + 0.3 = 4.1$$

The sequence is 3.2, 3.5, 3.8, 4.1, 4.4.

Exercises

1. Find the 100th term of the arithmetic sequence $1.6, 2.3, 3, \dots$

$$a_n = a_1 + (n-1)d$$

$$a_{100} = 1.6 + (100-1)(.7)$$

$$a_{100} = 70.9$$

$$1.6, 2.3, 3, \dots$$

$\overset{+.7}{\wedge}$
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 $\overset{+.7}{\wedge}$

2. Find the 28th term of the arithmetic sequence $-1, -3, -5, \dots$

$$a_{28} = -1 + (28-1)(-2)$$

$$a_{28} = -55$$

$$\overset{-2}{\wedge}$$

3. Find the first term of the arithmetic sequence for which $a_{15} = 30$ and $d = 1.4$.

$$a_n = a_1 + (n-1)d$$

$$30 = a_1 + (15-1)1.4$$

$$30 = a_1 + 19.6$$

$$a_1 = 10.4$$

4. Find d in the arithmetic sequence for which $a_1 = 6$ and $a_{40} = 142.5$.

$$142.5 = 6 + (40-1)d$$

$$d = 3.5$$

5. Write an arithmetic sequence that has three arithmetic means between 17 and 39.

$$17, \underline{22.5}, \underline{28}, \underline{33.5}, 39$$

$$39 = 17 + (5-1)d$$

$$d = 5.5$$

6. Write an arithmetic sequence that has seven arithmetic means between -2 and 16 .

7. Find the explicit and recursive formula for the n th term in the sequence: $2, 5, 8, \dots$

Recursive

$$a_1 = 2$$

$$a_n = a_{n-1} + 3$$

Explicit

$$a_n = a_1 + (n-1)d$$

$$a_n = 2 + (n-1)3$$

$$a_n = \underline{2} + \underline{3n} - \underline{3}$$

$$a_n = 3n - 1$$

HW: p. 605

1, 7, 9, 11, 13, 19, 23, 27, 31, 33, 37

KeyConcept Sum of a Finite Arithmetic Series

The sum of a finite arithmetic series with n terms or the n th partial sum of an arithmetic series can be found using one of two related formulas.

Formula 1 $S_n = \frac{n}{2}(a_1 + a_n)$

Formula 2 $S_n = \frac{n}{2}[2a_1 + (n - 1)d]$

Example 1: Find the sum of the first 50 terms in the series $11 + 14 + 17 + \dots + 158$.

Because the first and last terms are known, use $S_n = \frac{n}{2}(a_1 + a_n)$.

Substitute 50 for n , 11 for a_1 , and 158 for a_{50} .

$$\begin{aligned} S_{50} &= \frac{50}{2}(11 + 158) \\ &= 4225 \end{aligned}$$

Example 2: Find the 23rd partial sum of the arithmetic series $173 + 162 + 151 + \dots$.

The 23rd term is not known. The first term is known and the common difference can be found by subtracting $162 - 173 = -11$. Use $S_n = \frac{n}{2}[2a_1 + (n - 1)d]$.

$$\begin{aligned} S_{23} &= \frac{23}{2}[2(173) + (23 - 1)(-11)] \\ &= 1196 \end{aligned}$$

Exercises

1. Find the 82nd partial sum of the arithmetic series $-1 + (-4) + (-7) + \dots$

$$S_{82} = \frac{82}{2} [2(-1) + (82-1)(-3)]$$

$$S_{82} = -10,045$$

2. Find the sum of the first 25 terms in the series $7 + 10 + 13 + \dots + 79$.

$$S_{25} = \frac{25}{2} (7 + 79)$$

$$S_{25} = 1,075$$

3. Find the 53rd partial sum of the arithmetic series $12 + 20 + 28 + \dots$

4. Find the sum of the first 42 terms in the series $1.5 + 2 + 2.5 + \dots + 22$.

5. Find $\sum_{n=3}^{13} (3n + 1)$.

$$a_3 = 3(3) + 1 = 10$$

$$a_{13} = 3(13) + 1 = 40$$

$$\frac{11}{2}(10 + 40)$$

$$= 364$$

6. Find $\sum_{n=1}^{42} 2n$.

$$a_1 = 2(1) = 2$$

$$a_{42} = 2(42) = 84$$

$$\frac{42}{2}(2 + 84)$$

$$= 1806$$

Hw: p. 605

3, 13, 21, 29, 35, 39, 41, 47, 49, 53, 63

