65. $17 + 21 + 25 + \dots + 61$

$$\sum_{n=4}^{15} 4n+1$$

$$4n+1=61$$

43. $-17 + 1 + 19 + \dots + 649$

$$649 = -17 + (n-1)18$$
$$n = 38$$

$$S_{38} = \frac{38}{2}(-17+649)$$
$$= 12{,}008$$

0-2+
3-4✓
5?-

KeyConcept The *n*th Term of a Geometric Sequence

Words	The nth term of a geometric sequence with first term a_1 and common ratio r is given by $a_n = a_1 r^{n-1}$.
Example	The 9th term of 2, 10, 50, … is $a_9 = 2 \cdot 5^{9-1}$ or 781,250.

Example 1: Find the seventh term of the geometric sequence 8, –24, 72, … .

First, find the common ratio.

$a_2 \div a_1 = -24 \div 8$ or -3

$a_3 \div a_2 = 72 \div (-24)$ or -3

Use the explicit formula $a_n = a_1(r)^{n-1}$ to find a_7. Use $n = 7$, $a_1 = 8$, and $r = -3$.

$a_7 = 8\,(-3)^{7-1}$

$= 5832$

Example 2: Write a sequence that has two geometric means between 6 and 162.

The sequence will resemble 6, __?__, __?__, 162.

This means that $n = 4$, $a_1 = 6$, and $a_4 = 162$. Find r.

$a_n = a_1(r)^{n-1}$	Formula for nth term of a geometric sequence
$162 = 6r^3$	Substitute.
$27 = r^3$	Divide each side by 6.
$3 = r$	Take the cube root of each side.

Determine the geometric means recursively.

$a_2 = 6(3)$ or 18, $a_3 = 18(3)$ or 54

The sequence is 6, 18, 54, 162.

Exercises

$\cdot 2 \quad \cdot 2$

1. Determine the common ratio and find the next three terms of the geometric sequence x, $2x$, $4x$, … .

$r = 2$

$8x, 16x, 32x$

$\cdot -.3 \cdot$

2. Find the seventh term of the geometric sequence 157, –47.1, 14.13, … .

$r = \frac{a_n}{a_{n-1}}$

$a_7 = 157\boxed{(-.3)^{7-1}}$

$a_7 = .114$

$\cdot \frac{1}{2}$
$\div 2$

3. Find the 17th term of the geometric sequence 128, 64, 32, … .

$a_{17} = 128(.5)^{17-1}$

$a_{17} = .002$

4. Find the first term of the geometric sequence for which $a_6 = 0.1$ and $r = 0.2$.

$a_n = a_1 r^{n-1}$

$.1 = a_1 \boxed{(.2)^{6-1}}$

$\boxed{a_1 = 312.5}$

5. Find r of the geometric sequence for which $a_1 = 15$ and $a_{10} = 7680$.

$7680 = 15 r^{10-1}$

$\frac{7680}{15} = \frac{15r^9}{15}$

$\sqrt[9]{512} = \sqrt[9]{r^9}$

$512 \wedge (1 \div 9)$

$r = 2$

6. Write a geometric sequence that has three means between 7 and 567.

$\times 3$

7, <u>21</u>, <u>63</u>, <u>189</u>, 567

$567 = 7r^{5-1}$

$\frac{567}{7} = \frac{7r^4}{7}$

$\sqrt[4]{81} = \sqrt[4]{r^4}$

$r = 3$

HW: p. 615

1, 7, 9, 11, 15, 19, 23, 27, 33, 37

9 a) $C = \pi d$

$C_1 = \pi d_1$

$C_2 = \pi d_2$

$C_3 = \pi d_3$

πd_4

πd_5

$r = \frac{\pi d_2}{\pi d_1} = \frac{d_2}{d_1} = \frac{d_n}{d_{n-1}}$

b) $A = \pi r^2$

$A = \pi \left(\frac{d}{2}\right)^2$

$A_1 = \pi \left(\frac{d_1}{2}\right)^2$

$A_2 = \pi \left(\frac{d_2}{2}\right)^2$

$r = \frac{\pi\left(\frac{d_2}{2}\right)^2}{\pi\left(\frac{d_1}{2}\right)^2}$

$r = \left(\frac{\frac{d_2}{2}}{\frac{d_1}{2}}\right)^2$

$r = \left(\frac{d_2}{d_1}\right)^2$

37. $48, \underline{-120}, \underline{300}, -750$

$-750 = 48r^{4-1}$

$\frac{-750}{48} = \frac{48r^3}{48}$

$\sqrt[3]{-\frac{125}{8}} = \sqrt[3]{r^3}$

$-\frac{5}{2} = r$

19. $b_t = b_0(16)^{t-1}$

$b_4 = 12(16)^{4-1}$

$b_4 =$ ______

$\cdot\frac{1}{2}$

$\div 2$

$16, 8, 4, 2 \ldots$

$a_n = 16\left(\frac{1}{2}\right)^{n-1}$

$2, 4, 8, 16$

0-3+

4-5 ✓

6+-

KeyConcept Sum of a Finite Geometric Series

The sum of a finite geometric series with n terms or the nth partial sum of a geometric series can be found using one of two related formulas.

Formula 1 $S_n = a_1\left(\frac{1 - r^n}{1 - r}\right)$

Formula 2 $S_n = \frac{a_1 - a_n r}{1 - r}$

Example 1: Find the sum of the first 12 terms of the geometric series 6 + 7.5 + 9.375 + … .

The common ratio is 7.5 ÷ 6 or 1.25. Because the first term and number of terms is known, use $S_n = a_1\left(\frac{1-r^n}{1-r}\right)$. Substitute 12 for n, 6 for a_1, and 1.25 for r.

$$S_{12} = 6\left(\frac{1 - 1.25^{12}}{1 - 1.25}\right)$$
$$\approx 325.246$$

Example 2: If possible, find the sum of the geometric series 40 + 8 + 1.6 + … .

The common ratio is 8 ÷ 40 or 0.2. Because $|0.2| < 1$, the series has a sum.

$$S = \frac{a_1}{1 - r}$$
$$= \frac{40}{1 - 0.2} \text{ or } 50$$

Exercises

$\cdot 4 \quad \cdot 4$

1. Find the sum of the first seven terms of –1 + (–4) + (–16) + … .

$$S_7 = -1\left(\frac{1-(4)^7}{1-4}\right)$$

$$r = \frac{a_n}{a_{n-1}}$$

$$\boxed{S_7 = -5461}$$

2. Find the sum of a geometric series if $a_1 = 8$, and $a_n = 0.394$, and $r = \frac{9}{11}$.

$$S_n = \frac{8 - .394\left(\frac{9}{11}\right)}{1-\left(\frac{9}{11}\right)}$$

$$\boxed{S_n = 42.24}$$

3. Find $\sum_{n=1}^{11} 5(1.06)^{n-1}$.

$$a_1 = 5(1.06)^{1-1} = 5$$

$$a_{11} = 5(1.06)^{11-1} = 8.95$$

$$= \frac{5 - 8.95(1.06)}{1 - 1.06}$$

$$= 74.78$$

$$8.95 = 5r^{11-1}$$

$$\frac{8.95}{5} = \frac{5r^{10}}{5}$$

$$\sqrt[10]{\frac{8.95}{5}} \quad \sqrt[10]{r^{10}}$$

$$r = 1.06$$

HW: p. 615,
3, 13, 21, 29, 35, 41, 47, 49, 53, 63, 97

4. Find the sum of the first 16 terms in a geometric series where $a_1 = 1$, and $a_n = -2a_{n-1}$.

KeyConcept The Sum of an Infinite Geometric Series

The sum S of an infinite geometric series for which $|r| < 1$ is given by

$$S = \frac{a_1}{1-r}.$$

If possible, find the sum of each infinite geometric series.

5. $\sum_{n=1}^{\infty} 13\left(\frac{3}{8}\right)^{n-1}$

6. $\sum_{n=1}^{\infty} 3^{n-1}$