2-1 Study Guide and Intervention

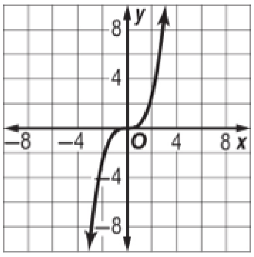
Power and Radical Functions

Power Functions A **power function** is any function of the form $f(x) = ax^n$ where a and n are nonzero constant real numbers. For example, $f(x) = 2x^2$, $f(x) = x^{\frac{1}{2}}$, or $f(x) = \sqrt{x}$ are power functions.

Example: Graph and analyze $f(x) = \frac{1}{3}x^3$. Describe the domain, range, intercepts, end behavior, continuity, and where the function is increasing or decreasing.

Evaluate the function for several x-values in its domain. Then use a smooth curve to connect each of these points to complete the graph.

x	-3	-2	-1	0	1	2	3
$f(x)$	-9	$-\frac{8}{3}$	$-\frac{1}{3}$	0	$\frac{1}{3}$	$\frac{8}{3}$	9



Domain: $(-\infty, \infty)$ Range: $(-\infty, \infty)$

Intercept: 0

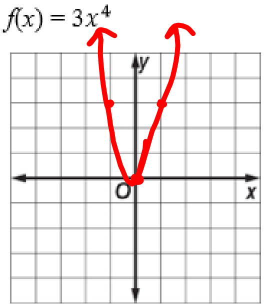
End behavior: $\lim_{x \to -\infty} f(x) = -\infty$ and $\lim_{x \to \infty} f(x) = \infty$

Continuity: continuous for all real numbers Increasing: $(-\infty, \infty)$

Exercises

Graph and analyze each function. Describe the domain, range, intercepts, end behavior, continuity, and where the function is increasing or decreasing.

1. $f(x) = 3x^4$



x	-3	-2	-1	0	1	2	3
f(x)	243	48	3	0	3	48	243

D: $(-\infty, \infty)$

R: $[0, \infty)$

x-int: $(0,0)$

y-int: $(0,0)$

$\lim_{x \to -\infty} f(x) = \infty$

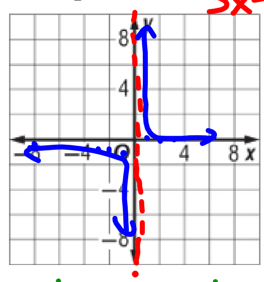
$\lim_{x \to \infty} f(x) = \infty$

continuous? yes

increasing: $(0, \infty)$

decreasing: $(-\infty, 0)$

2. $f(x) = \frac{1}{5}x^{-3} = \frac{1}{5x^3}$



x	-3	-2	-1	0	1	2	3
f(x)	-.007	-.025	-.2	und	.2	.025	.007

D: $(-\infty, 0) \cup (0, \infty)$

R: $(-\infty, 0) \cup (0, \infty)$

x-int: NONE

y-int: NONE

$\lim_{x \to -\infty} f(x) = 0$

$\lim_{x \to \infty} f(x) = 0$

continuous? no

increasing: none

decreasing: $(-\infty, 0) \cup (0, \infty)$

Radical Functions and Equations A **radical function** is a function that has at least one radical expression containing the independent variable. For example, $f(x) = 2\sqrt[4]{x^3}$ is a radical function.

A **radical equation** is an equation in which the variable is in the radicand. To solve a radical equation, isolate the radical expression. Then raise each side of the equation to the power of the index of the radical. Then check for **extraneous solutions**. These are solutions that do not satisfy the original equation.

Example: Solve $3 = \boxed{\sqrt[3]{x^2 - 2x + 1}} - 1$.

Step 1

$3 = \sqrt[3]{x^2 - 2x + 1} - 1.$	Original equation
$(4)^3 = \left(\sqrt[3]{x^2 - 2x + 1}\right)^3$	Isolate the radical.
$64 = x^2 - 2x + 1$	Cube each side.
$0 = x^2 - 2x - 63$	Subtract 64 from each side.
$0 = (x - 9)(x + 7)$	Factor.
$x - 9 = 0$ or $x + 7 = 0$	Zero Product Property
$\underline{x = 9}$ $\quad$ $\underline{x = -7}$	Solve.

Step 2

Check both solutions.

$3 = \sqrt[3]{x^2 - 2x + 1} - 1.$	$3 = \sqrt[3]{x^2 - 2x + 1} - 1.$
$3 \stackrel{?}{=} \sqrt[3]{(9)^2 - 2(9) + 1} - 1$	$3 \stackrel{?}{=} \sqrt[3]{(-7)^2 - 2(-7) + 1} - 1$
$3 \stackrel{?}{=} \sqrt[3]{64} - 1$	$3 \stackrel{?}{=} \sqrt[3]{64} - 1$
$3 \stackrel{?}{=} 4 - 1$	$3 \stackrel{?}{=} 4 - 1$
$3 = 3$ ✓	$3 = 3$ ✓

Both solutions check, so the solutions are -7 and 9.

Exercises

Solve each equation.

1. $\sqrt[3]{x^2 - 1} - 6 = -4$

2. $\sqrt{6n - 3} = \sqrt{-15 + 7n}$

3. $4x = 21 + \sqrt{56 - x}$

$-21 \quad -21$

$(4x-21)^2 = (\sqrt{56-x})^2$

$(4x-21)(4x-21) = 56 - x$

$16x^2 - 84x - 84x + 441 = 56 - x$

$+x \quad -56 \quad -56 + x$

$16x^2 - 167x + 385 = 0$

a b c

$x = \dfrac{-(-167) \pm \sqrt{(-167)^2 - 4(16)(385)}}{2(16)}$

$x = 7$ ~~3.4375~~

check: $4(7) = 21 + \sqrt{56-7}$

$28 = 28$

$4(3.4375) = 21 + \sqrt{56 - 3.435}$

$\cancel{13.75 = 28.25}$

4. $\sqrt[5]{40 - 4x} + 15 = 17$

$(\sqrt{6n-3})^2 = (\sqrt{-15+7n})^2$

$6n - 3 = -15 + 7n$

$-6n \qquad -6n$

$-3 = -15 + n$

$+15 \quad +15$

$n = 12$

Check: $\sqrt{6(12) - 3} = \sqrt{-15 + 7(12)}$

: $\sqrt{69} = \sqrt{69}$ ✓

HW: p. 92

3, 15, 19, 23, 33, 37, 47, 49, 53