69.

b) $y = 3.6 \cdot x^{-1}$

d) $y = 3.6(3.25)^{-1}$

$= 1.12$ atmospheres.

87. $B(r) = 1000(1+r)^3$

$$\frac{y}{1000} = \frac{\cancel{1000}(1+r)^3}{\cancel{1000}}$$

$$\sqrt[3]{\frac{y}{1000}} = \sqrt[3]{(1+r)^{\cancel{3}}}$$

$$\frac{\sqrt[3]{y}}{10} = 1 + r$$

$$-1 \qquad -1$$

$$\frac{\sqrt[3]{B(r)}}{10} - 1 = r$$

0 - 3 +

4 - 6 ✓

7 ↑ -

2-2 Study Guide and Intervention
Polynomial Functions

Graph Polynomial Functions A **polynomial function of degree *n*** is a function of the form

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_2 x^2 + a_1 x + a_0,$$

where n is a nonnegative integer and $a_0, a_1, a_2, \dots, a_{n-1}, a_n$ are real numbers with $a_n \neq 0$. Maxima and minima are located at **turning points**. A polynomial function of degree n has at most n distinct real zeros and at most $n - 1$ turning points.

- All polynomial functions are continuous (no holes, breaks, asymptotes.
- All polynomial functions have smooth, rounded turns (no sharp corners).

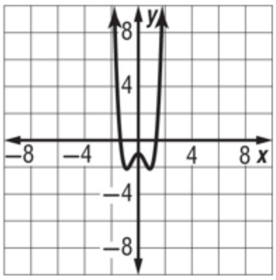
Leading Term Test:

If the leading term has an even degree, the end behavior will be in the same direction. If the leading term has a positive coefficient, both end behaviors approach ∞, if negative, both approach $-\infty$.

If the leading term has an odd degree, the end behavior will be in opposite directions. If the leading term has a positive coefficient, the left end behavior approaches $-\infty$, and the right end behavior approaches ∞. Vice versa if the leading term has a negative coefficient.

Example 1: Graph $f(x) = 2x^4 - 3x^2 - 1$. Describe the end behavior of the graph of the polynomial function using limits. Explain your reasoning using the leading term test.

The degree is 4 and the leading coefficient is 2. Because the degree is even and the leading coefficient is positive, $\lim_{x \to -\infty} f(x) = \infty$ and $\lim_{x \to \infty} f(x) = \infty$.



Example 2: State the number of possible real zeros and turning points of $f(x) = x^3 + 2x^2 - x - 2$. Then determine all of the real zeros by factoring.

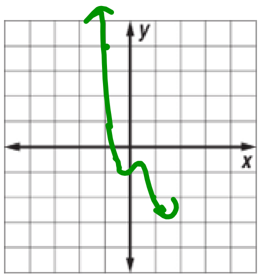
The degree is 3, so f has at most 3 distinct real zeros and at most 3 – 1 or 2 turning points. To find the real zeros, solve the related equation $f(x) = 0$ by factoring.

$x^3 + 2x^2 - x - 2 = 0$	Set $f(x)$ equal to 0.
$x^2(x + 2) - 1(x + 2) = 0$	Group the terms and find the GCF.
$(x^2 - 1)(x + 2) = 0$	Regroup using the Distributive Property.
$(x + 1)(x - 1)(x + 2) = 0$	Factor completely.

So, f has three distinct real zeros, $x = -1$, $x = 1$, and $x = -2$. The graph has two turning points.

Exercises

1. Graph the function $f(x) = -3x^5 + 2x^2 - 1$. Describe the end behavior of the graph of the polynomial function using limits. Explain your reasoning using the leading term test.



$\lim_{x \to -\infty} f(x) = \infty$ $\lim_{x \to \infty} f(x) = -\infty$

2. State the number of possible real zeros and turning points of $f(x) = x^4 - 4x^3 + 5x^2 - 4x + 4$. Then determine all of the real zeros by factoring.

possible zeros $= n = 4$

turning points $= n-1 = 4-1 = 3$

zeros: $(2, 0)$

KeyConcept Repeated Zeros of Polynomial Functions

If $(x - c)^m$ is the highest power of $(x - c)$ that is a factor of polynomial function f, then c is a zero of **multiplicity** m of f, where m is a natural number.

- If a zero c has odd multiplicity, then the graph of f crosses the x-axis at $x = c$ and the value of $f(x)$ changes signs at $x = c$.
- If a zero c has even multiplicity, then the graph of f is tangent to the x-axis at $x = c$ and the value of $f(x)$ does not change signs at $x = c$.

c

c

Ex: Apply the leading term test, determine the zeros and state the multiplicity of any repeated zeros, find a few additional points, and graph:

$f(x) = -2x(x - 4)(3x - 1)^3$

$= -2x(x-4)(3x-1)(3x-1)(3x-1)$

Leading term: $(-2x)(x)(3x)(3x)(3x) = -54x^5$

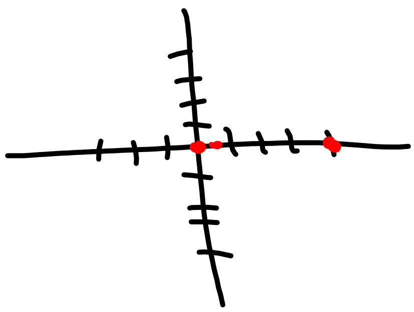
$\lim_{x \to -\infty} f(x) = \infty$ $\lim_{x \to \infty} f(x) = -\infty$

$0 = -2x(x-4)(3x-1)(3x-1)(3x-1)$

$-2x = 0$
$\frac{-2x}{-2} = \frac{0}{-2}$
$x = 0$

$x - 4 = 0$
$+4 \quad +4$
$x = 4$

$3x - 1 = 0$
$+1 \quad +1$
$3x = 1$
$\frac{3x}{3} = \frac{1}{3}$
$x = \frac{1}{3}$, has multiplicity of 3

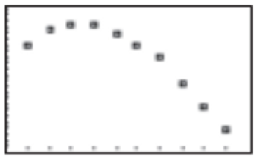


Model Real-World Data with Polynomial Functions You can use a graphing calculator to model data by first creating a scatter plot.

Example: An oil tanker collides with another ship and starts leaking oil at the rate shown in the table. Write a polynomial function to model the set of data.

Time (h)	1	2	3	4	5	6	7	8	9	10
Flow rate (100s of L/h)	18.0	20.5	21.3	21.1	19.9	17.8	15.9	11.3	7.6	3.7

a. Create a scatter plot of the data and determine the type of polynomial function that could be used to represent the data.
Enter the data using the list feature. Let L1 be time and L2 flow rate.
The curve resembles a parabola, so a quadratic function can model the data.



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b. Write a polynomial function to model the data set. Round each coefficient to the nearest thousandth. State the correlation coefficient.
Use the **QUADREG** tool in the **STAT** menu to obtain $f(x) = -0.409x2 + 2.762x + 16.267$.
The correlation coefficient is about 0.99.

c. Use the model to estimate the flow rate in 10.5 hours.
Graph the unrounded regression as **Y1**. Press 2nd **[CALC]**, choose **VALUE** and enter 10.5 for x.
Since $y \approx 0.17$, the flow rate will be about 17 L/h.

d. Use the model to determine the approximate time the flow rate was 1000 liters per hour.
Graph the line $y = 10$. Press 2nd **[CALC]**, choose **intersect**, and choose each graph.
The intersection occurs when $x \approx 8.5$, so the time was about 8.5 hours.

Exercise

The farther a planet is from the Sun, the longer it takes to complete an orbit. Use a graphing calculator to write a polynomial function to model the set of data. Round each coefficient to the nearest thousandth. State the correlation coefficient.
r

Distance (AU)	0.39	0.72	1	1.49	5.19	9.51	19.1	30	39.3
Period (days)	88	225	365	687	4344	10,775	30,681	60,267	90,582

$y = 34.968x^2 + 962.084x - 790.64$

$\sqrt{r^2} = \sqrt{.999719}$

$r = .999859$

27. $\underline{f(x)} = x^{6} - 6x^3 - 16$

possible zeros $= n = 6$

turning points $= n - 1 = 6 - 1 = 5$

$0 = x^6 - 6x^3 - \underline{16}$

$0 = (x^3 - 8)(x^3 + 2)$

$-8x^3$
$+2x^3$
$-6x^3$

$x^3 - 8 = 0$ $\quad x^3 + 2 = 0$

$+8 \quad +8$ $\quad -2 \quad -2$

$\sqrt[3]{x^3} = \sqrt[3]{8}$ $\quad \sqrt[3]{x^3} = \sqrt[3]{-2}$

$\boxed{x = 2 \qquad x = \sqrt[3]{-2}}$

HW:
p.104, 51, 53, 59, 65, 69, 77, 87, 99, 103, 105, 110-113