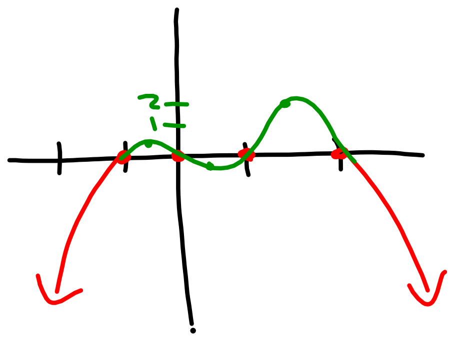
59. zeros: $0, -4$
degree: $5$

$0 = x^4(x+4)$

$f(x) = x^5 + 4x^4$

77. Degree: 6 ✓
4 zeros ✓
$\lim_{x\to\infty} f(x) = -\infty$

$0 = -x^3(x-1)(x+1)(x-2)$




33. $f(x) = x(x+4)(x-1)^2$

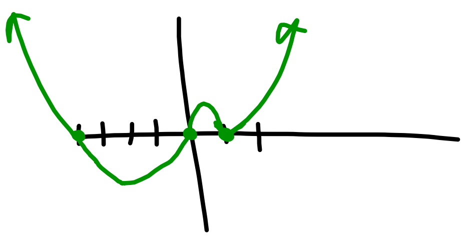
a) leading term: $x^4$
$\lim_{x\to -\infty} f(x) = \infty$ $\lim_{x\to\infty} f(x) = \infty$

b) $0 = x(x+4)(x-1)(x-1)$

$x = 0$

$x+4=0$
$-4\ -4$
$x=-4$

$x-1=0$
$+1\ +1$
$x=1$, multiplicity of 2



0-6 +
7-12 ✓
13 ↑

# 2-3 Study Guide and Intervention

## *The Remainder and Factor Theorems*

**Divide Polynomials** You can divide polynomials by using either long or **synthetic division.** The process of dividing polynomials by using long division is similar to that in arithmetic, with the alternating of dividing, multiplying, and subtracting. To set up long division for polynomials, be sure the dividend is in standard form, with placeholders for missing terms. Synthetic division is a shortcut for long division. It only works when the divisor is in the form *x* - *c*.

Ex: Divide: $(6x^3 + 17x^2 - 104x + 60) \div (2x-5)$

$3x^2 + 16x - 12$

$2x-5 \,\overline{)\, 6x^3 + 17x^2 - 104x + 60}$

$-(6x^3 - 15x^2)$

$32x^2 - 104x$

$-(32x^2 - 80x)$

$-24x + 60$

$-(-24x + 60)$

$0$

$6x^3 + 17x^2 - 104x + 60$

$= (2x-5)(3x^2 + 16x - 12)$

**Example**

**a. Divide $3x^3 - 2x - 4$ by $x + 1$ using long division.**

$$\begin{array}{rl}
 & 3x^2 - 3x + 1 \\
x + 1 & \overline{)\,3x^3 + 0x^2 - 2x - 4} \\
 & (-)\ \underline{3x^3 + 3x^2} \\
 & \quad -3x^2 - 2x \\
 & \quad (-)\ \underline{-3x^2 - 3x} \\
 & \qquad\qquad x - 4 \\
 & \qquad\qquad (-)\ \underline{x + 1} \\
 & \qquad\qquad\qquad -5
\end{array}$$

Insert a placeholder for $x^2$.

Divide $x$ into each term, starting on the left. Write the quotient above the term with the same exponent.

Multiply the quotient by the divisor, and subtract the polynomial from the dividend. Repeat.

The quotient is $3x^2 - 3x + 1 - \frac{5}{x+1}$

**b. Divide $3x^3 - 2x - 4$ by $x + 1$ using synthetic division.**

$$\begin{array}{r|rrr|r}
-1 & 3 & 0 & -2 & -4 \\
 & & -3 & 3 & -1 \\
\hline
 & 3 & -3 & 1 & -5
\end{array}$$

Write the value of $c$ in the box and the coefficients of the dividend in the first row. Insert a placeholder for $x^2$. Bring down the first coefficient. Multiply it by $c$ and write the product under the second coefficient. Add the numbers in this column. Repeat.

The coefficients of the terms in the quotient appear in the bottom row, where the powers of the variables decrease by one and the first power is one less than the degree of the dividend. The last number is the remainder.

**Divide using synthetic division.**

**3.** $(x^3 - 28x - 48) \div (x + 4)$

$(x^3 + 0x^2 - 28x - 48) \div (x + 4)$

$$\begin{array}{r|rrrr} -4 & 1 & 0 & -28 & -48 \\ & & -4 & 16 & 48 \\ \hline & 1 & -4 & -12 & 0 \end{array}$$

remainder

$\boxed{x^2 - 4x - 12}$

**4.** $(x^4 - 3x^2 + 12) \div (x - 1)$

$$\begin{array}{r|rrrrr} 1 & 1 & 0 & -3 & 0 & 12 \\ & & 1 & 1 & -2 & -2 \\ \hline & 1 & 1 & -2 & -2 & 10 \end{array}$$

$\boxed{x^3 + x^2 - 2x - 2 + \frac{10}{x-1}}$

**Using the Remainder and Factor Theorems** The Remainder Theorem states that if a polynomial $f(x)$ is divided by $x - c$, the remainder $r$ is $r = f(c)$. This allows you to evaluate $f(x)$ for $x = c$ by dividing $f(x)$ by $x - c$ using synthetic division. This process is called **synthetic substitution**.

The Factor Theorem states that $x - c$ is a factor of $f(x)$ if and only if $f(c) = 0$. This means you can use synthetic substitution to check if linear binomials are factors of polynomials. If the remainder is 0, the binomial is a factor.

**Example**

**a. Find $f(c)$ for $f(x) = x^4 - 5x^2 - 17x - 12$ when $c = -3$.**

Use synthetic substitution.

$$\begin{array}{r|rrrr|r} -3 & 1 & 0 & -5 & -17 & -12 \\ & & -3 & 9 & -12 & 87 \\ \hline & 1 & -3 & 4 & -29 & 75 \end{array}$$

Because the remainder is 75, $f(-3) = 75$.

**b. Determine whether $x - 2$ is a factor of $f(x) = 2x^3 + 5x^2 - 14x - 8$.**

Use synthetic division to test the factor.

$$\begin{array}{r|rrr|r} 2 & 2 & 5 & -14 & -8 \\ & & 4 & 18 & 8 \\ \hline & 2 & 9 & 4 & 0 \end{array}$$

Because the remainder is 0, $x - 2$ is a factor of $f(x)$.

**Exercises**

**Find each $f(c)$ using synthetic substitution.**

**1.** $f(x) = 2x^3 - 3x^2 - 6x + 2$ ; $c = 4$

**2.** $f(x) = x^4 + 5x^2 - 10x - 15$ ; $c = -2$

$$\begin{array}{r|rrrr} 4 & 2 & -3 & -6 & 2 \\ & \downarrow & 8 & 20 & 56 \\ \hline & 2 & 5 & 14 & \boxed{58} \end{array}$$

$f(4) = 58$

Ex: Use the factor theorem to determine if the following are factors of f(x). If possible, write a factored form of f(x).

$f(x) = x^3 - 2x^2 - 29x + 30$; $(x+5)$

$$\begin{array}{r|rrrr} -5 & 1 & -2 & -29 & 30 \\ & \downarrow & -5 & 35 & -30 \\ \hline & 1 & -7 & 6 & 0 \\ & x^2 & -7x & +6 & \end{array}$$

Yes it is since the remainder is 0

$(x+5)(x^2-7x+6)$

$\boxed{(x+5)(x-1)(x-6)}$

HW: p. 115

1, 7, 17, 19, 23, 27, 33, 37, 45, 49