2-4 Study Guide and Intervention

Zeros of Polynomial Functions

Finding Real Zeros of Polynomial Functions The Rational Zero Theorem describes how the leading coefficient and constant term of a polynomial function with integer coefficients determine a list of all possible rational zeros.

Possible Rational Zeros = Factors of the constant
Factors of the leading coefficient

Descartes' Rule of Signs states that the number of positive real zeros is equal to the number of sign changes in f(x) or less than that by an even number. The number of negative real zeros is the same as the number of sign changes in f(-x) or less than that by an even number.

Example

a. List all possible rational zeros of $h(x) = x^3 - 5x^2 - 17x - 6$. Then determine which, if any, are zeros.

The leading coefficient is 1 and the constant term is -6. Possible rational zeros: $\frac{\text{Factors of }-6}{\text{Factors of 1}} = \frac{\pm 1, \pm 2, \pm 3, \pm 6}{\pm 1} \text{ or } \pm 1, \pm 2, \pm 3, \pm 6$

By synthetic substitution, you can determine that x = -2 is a rational zero.

The depressed polynomial is $x^2 - 7x - 3$. You can use the quadratic formula to find the two irrational roots.

b. Describe the possible real zeros of $g(x) = 4x^4 - 13x^3 - 21x^2 + 38x - 8$.

Examine the variations in sign for g(x) and for g(-x).

$$g(x) = 4x^{4} - 13x^{3} - 21x^{2} + 38x - 8$$

$$g(-x) = 4(-x)^{4} - 13(-x)^{3} - 21(-x)^{2} + 38(-x) - 8$$

$$= 4x^{4} + 13x^{3} - 21x^{2} - 38x - 8$$

By Descartes' Rule of Signs, g(x) has either 3 or 1 positive real zeros and 1 negative real zero. The graph shows it has 1 negative real zero and 3 positive real zeros.

Exercises

1. List all possible rational zeros of $h(x) = x^3 - 5x^2 - 4x + 20$. Then determine which, if any, are zeros.

2. Describe the possible real zeros of $f(x) = x^5 - 10x^4 + 7x^3 - x^2 + 4x + 20$.

posior mg.

positive Zens

4 sign changes means 4 positive real zeros or 2 positive real zeros.

2.4 notes - zeros of a	polynomial	function.notebook
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Zeros of Polynomial Functions

Finding Complex Zeros of Polynomial Functions Polynomials can have zeros in the complex number system. The Fundamental Theorem of Algebra states that a polynomial function of degree n has at least one zero in the complex number system. In fact, it has exactly n zeros, including repeated zeros. The Conjugate Root Theorem states that if a + bi is a zero, then its complex conjugate a - bi is also a zero.

Example 1: Write a polynomial function of least degree with real coefficients in standard form that has 0 and $\sqrt{2}i$ as zeros.

Because $\sqrt{2}i$ is a zero, you know that $-\sqrt{2}i$ is also a zero.

$$f(x) = a(x-0)(x-\sqrt{2}i) \left[x-(-\sqrt{2}i)\right]$$
 Write the function as factors.
$$f(x) = 1x(x-\sqrt{2}i)(x+\sqrt{2}i)$$
 Let $a=1$. Simplify.
$$= x(x^2-2i^2)$$
 Multiply.
$$= x(x^2+2)$$
 Simplify. $i^2=-1$ Distribute.

A function of least degree that has 0, $\sqrt{2}i$, and $-\sqrt{2}i$ as zeros is $f(x) = x^3 + 2x$.

Example 2: Find all complex zeros of $p(x) = x^4 - 5x^3 + 3x^2 + 19x - 30$ given that x = 2 + i is a zero of p. Then write the linear factorization of p(x).

Because $x = 2 + \mathbf{i}$ is a zero, you know that $x = 2 - \mathbf{i}$ is also a zero of p. First divide by $2 + \mathbf{i}$ to get the depressed polynomial. Then divide it by $2 - \mathbf{i}$.

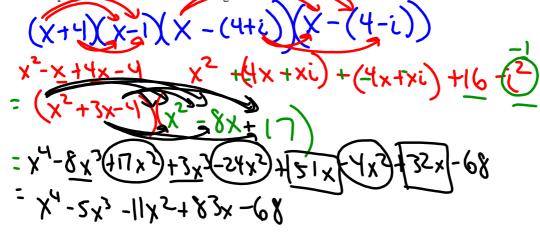
Using the known zeros and the depressed polynomial from the last division, you can write $p(x) = [x - (2 + i)][x - (2 - i)](x^2 - x - 6)$.

Factor the remaining depressed polynomial, $x^2 - x - 6 = (x - 3)(x + 2)$.

The four zeros of p are $2 + \mathbf{i}$, $2 - \mathbf{i}$, 3, and -2. The linear factorization of p is $p(x) = [x - (2 + \mathbf{i})] [x - (2 - \mathbf{i})] (x - 3)(x + 2)$.

Exercises

1. Write a polynomial function of least degree with real coefficients in standard form that has the zeros -4, 1, and 4 + i.



2. Find all complex zeros of $p(x) = x^4 + 8x^3 + 16x^2 + 200x - 225$ given that x = 5i is a zero of p. Then write the linear factorization of p(x).

HW: p.127, # 1,7,11,15,19,27,29,33,39,43,49