53. $x^5-3x^4-4x^3+12x^2-32x+96$ $\quad -2i,\ 2i$

$$\begin{array}{r|rrrrrr} -2i & 1 & -3 & -4 & 12 & -32 & 96 \\ & \downarrow & -2i & -4+6i & 12+16i & 32-48i & -96 \\ \hline & 1 & -3-2i & -8+6i & 24+16i & -48i & 0 \end{array}$$

$$\begin{array}{r|rrrrr} 2i & 1 & -3-2i & -8+6i & 24+16i & -48i \\ & & 2i & -6i & -16i & 48i \\ \hline & 1 & -3 & -8 & 24 & 0 \end{array}$$

$(x-2i)(x+2i)(x^3-3x^2-8x+24)$

$x^2(x-3)-8(x-3)$

$(x-3)(x^2-8)$

$(x-2i)(x+2i)(x-3)(x^2-8)$

$$\boxed{(x-2i)(x+2i)(x-3)(x+2\sqrt{2})(x-2\sqrt{2})}$$

$x^2-8=0$
$\quad +8\ \ +8$
$\sqrt{x^2}=\sqrt{8}$
$x=\pm 2\sqrt{2}$

98. $(t^2+3t-9)(5-t)^{-1}$

$$\frac{t^2+3t-9}{5-t}$$

$$\frac{(t^2+3t-9)-1}{(5-t)-1}$$

$$\frac{-t^2-3t+9}{t-5}$$

$$\begin{array}{r|rrr} 5 & -1 & -3 & 9 \\ & \downarrow & -5 & -40 \\ \hline & -1 & -8 & -31 \end{array}$$

$\frac{-x}{-1}\quad \frac{-8}{-1}\quad \cdot\ \frac{\frac{31}{x-5}}{-1}$

$x+8-\frac{31}{5-x}$

0-3+
4-6v
71-

2-5 Study Guide and Intervention

Rational Functions

Graphs of Rational Functions **Asymptotes** are lines that a graph approaches.

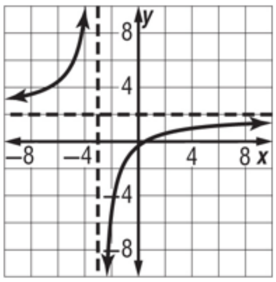
For $f(x) = \frac{a(x)}{b(x)}$, where $a(x)$ and $b(x)$ have no common factors other than one, and n is the degree of $a(x)$ and m is the degree of $b(x)$:

- **Vertical asymptotes:** Occur where the denominator is equal to 0
- **Horizontal asymptotes:** end behavior
  - If $n < m$, the asymptote is $y = 0$.
  - If $n = m$, the asymptote is $y = c$, where c is the ratio of the leading coefficients of the numerator and denominator.
  - If $n > m$, there is no horizontal asymptote.
- **Oblique asymptotes:** occurs if the degree of the numerator is one bigger than the denominator. The equation will be the quotient of the rational function without the remainder.

  Slant
- x-intercepts, if any, occur at the real zeros of $a(x)$. The y-intercept, if it exists, is the value of f when $x = 0$.

Example: Determine any asymptotes and intercepts for $f(x) = \frac{2x-1}{x+3}$. Then graph the function and state its domain.

Vertical asymptotes: The zero of the denominator is –3. The vertical asymptote is $x = -3$.



Horizontal asymptotes: The degree of the numerator equals the degree of the denominator, so the horizontal asymptote is $y = 2$.

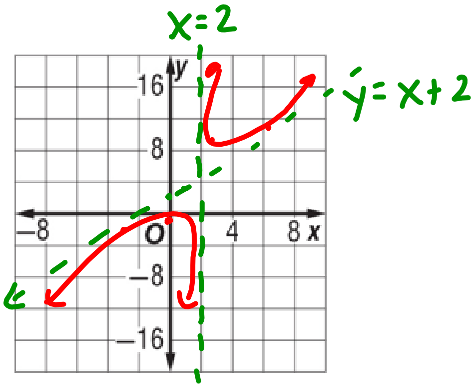
Intercepts: To find x-intercepts, find the zeros of the numerator by solving $2x - 1 = 0$. So, the x-intercept is $\frac{1}{2}$. To find the y-intercept, substitute 0 for x. The y-intercept is $-\frac{1}{3}$.

Graph the asymptotes and intercepts. Find and plot points in each interval.

$D = \{x \mid x \neq -3, x \in \mathbb{R}\}$

Exercise

Determine any asymptotes and intercepts for $f(x) = \frac{x^2 + 1}{x - 2}$. Then graph the function and state its domain.



vertical asymptotes

$x - 2 = 0$
$\underline{+2 \quad +2}$
$x = 2$

horizontal asymptote
none

oblique asymptote

$$\begin{array}{r|rrr} 2 & 1 & 0 & 1 \\ & & 2 & 4 \\ \hline & 1 & 2 & 5 \end{array}$$

$x + 2 + \frac{5}{x-2}$

D: $(-\infty, 2) \cup (2, \infty)$

$y = x + 2$

x-int (zero)

$0 = \frac{x^2+1}{x-2}$

$x^2 + 1 = 0$
$\underline{-1 \quad -1}$
$\sqrt{x^2} = \sqrt{-1}$

none

y-int

$y = \frac{(0)^2 + 1}{(0) - 2}$

$y = -\frac{1}{2}$

Rational Equations Rational equations involving fractions can be solved by multiplying each term in the equation by the least common denominator (LCD) of all the terms of the equation. Always check your solutions to a rational equation as some may be extraneous.

Example: Solve each equation.

a. $\mathbf{x - \frac{8}{x-4} = 11}$

$x - \frac{8}{x-4} = 11$	Original equation
$x(x-4) - \frac{8}{x-4}(x-4) = 11(x-4)$	Multiply each term by the LCD, $x - 4$.
$x^2 - 4x - 8 = 11x - 44$	Distributive Property
$x^2 - 15x + 36 = 0$	Simplify.
$(x-12)(x-3) = 0$	Factor.
$x = 12$ or 3	Zero Product Property

b. $\mathbf{\frac{x+1}{3(x-2)} = \frac{5x}{6} + \frac{1}{x-2}}$

$\frac{x+1}{3(x-2)} = \frac{5x}{6} + \frac{1}{x-2}$	Original equation
$6(x-2)\left[\frac{x+1}{3(x-2)}\right] = 6(x-2)\left(\frac{5x}{6} + \frac{1}{x-2}\right)$	Multiply each side by the LCD, $6(x - 2)$.
$2(x+1) = (x-2)(5x) + 6(1)$	Multiply.
$2x + 2 = 5x^2 - 10x + 6$	Simplify.
$5x^2 - 12x + 4 = 0$	Write in standard form.
$(5x-2)(x-2) = 0$	Factor.
$5x - 2 = 0 \qquad x - 2 = 0$	Zero Product Property
$x = \frac{2}{5} \qquad x = 2$	Simplify.

Since x cannot equal 2 because a zero denominator results, the only solution is $\frac{2}{5}$.

Exercises

Solve each equation.

1. $x\,(x) + \left(\frac{10}{x}\right)x = (7)\,x$

3. $\frac{4}{b-3} + \frac{3}{b} = \frac{-2b}{b-3}$

$$x^2 + 10 = 7x$$
$$-7x \quad -7x$$
$$x^2 - 7x + 10 = 0$$
$$(x-5)(x-2) = 0$$
$$x - 5 = 0 \qquad x - 2 = 0$$
$$+5 \; +5 \qquad +2 \; +2$$
$$\boxed{x = 5 \qquad x = 2}$$

5. $4x(x+3)\,\frac{x+3}{x} + \frac{7x}{x+3} = \frac{23}{4}\;4x(x+3)$

$$4(x+3)(x+3) + 7x(4x) = 23x(x+3)$$
$$4(x^2+6x+9) + 28x^2 = 23x^2 + 69x$$
$$4x^2 + 24x + 36 + 28x^2 = 23x^2 + 69x$$
$$-69x \quad -23x^2 \quad -23x^2 \quad -69x$$
$$\frac{9x^2}{9} - \frac{45x}{9} + \frac{36}{9} = \frac{0}{9}$$
$$x^2 - 5x + 4 = 0$$
$$(x-4)(x-1) = 0$$
$$x - 4 = 0 \qquad x - 1 = 0$$
$$+4 \; +4 \qquad +1 \; +1$$
$$\boxed{x = 4 \qquad x = 1}$$

HW: p. 138, # 1, 7, 13, 15, 25, 27, 33, 35, 39