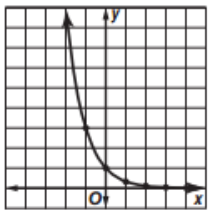
3-1 Study Guide and Intervention

Exponential Functions

Exponential Functions An **exponential function** with base b has the form $f(x) = ab^x$, where x is any real number and a and b are real number constants such that $a \neq 0$, b is positive, and $b \neq 1$. If $b > 1$, then the function is *exponential growth*. If $0 < b < 1$, then the function is *exponential decay*.

Example **Sketch and analyze the graph of $f(x) = \left(\frac{1}{3}\right)^x$. Describe its domain, range, intercepts, asymptotes, end behavior, and where the function is increasing or decreasing.**

x	−3	−2	−1	0	1	2	3
$f(x)$	27	9	3	1	$\frac{1}{3}$	$\frac{1}{9}$	$\frac{1}{27}$



Domain: $(-\infty, \infty)$ Range: $(0, \infty)$

Intercept: $(0, 1)$ Asymptote: x-axis

End behavior: $\lim_{x \to -\infty} f(x) = \infty$ and $\lim_{x \to \infty} f(x) = 0$

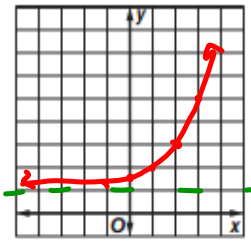
Decreasing: $(-\infty, \infty)$

Exercises

Sketch and analyze the graph of each function. Describe its domain, range, intercepts, asymptotes, end behavior, and where the function is increasing or decreasing.

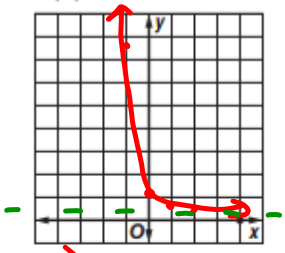
e = natural base
e ≈ 2.718

1. $h(x) = 2^{x-1} + 1$



D: $(-\infty, \infty)$
R: $(1, \infty)$
x-int: none
y-int: $(0, 1.5)$
horizontal asymptote: $y = 1$
end behavior: $\lim_{x \to -\infty} f(x) = 1$ | $\lim_{x \to \infty} f(x) = \infty$
increasing $(-\infty, \infty)$

2. $k(x) = e^{-2x}$



D: $(-\infty, \infty)$
R: $(0, \infty)$
x-int: none
y-int: $(0, 1)$
horizontal asymptote: $y = 0$
$\lim_{x \to -\infty} f(x) = \infty$ $\lim_{x \to \infty} f(x) = 0$
decreasing $(-\infty, \infty)$

Exponential Functions

Exponential Growth and Decay Many real-world situations can be modeled by exponential functions. One of the equations below may apply.

Exponential Growth or Decay	Continuous Exponential Growth or Decay	Compound Interest
$N = N_0(1 + r)^t$	$N = N_0e^{kt}$	$A = P\left[1 + \frac{r}{n}\right]^{nt}$
N is the final amount, N_0 is the initial amount, r is the rate of growth or decay, and t is time.	N is the final amount, N_0 is the initial amount, k is the rate of growth or decay, t is time, and e is a constant.	P is the principal or initial investment, A is the final amount of the investment, r is the annual interest rate, n is the number of times interest is compounded each year, and t is the number of years.

Example 1 **BIOLOGY A researcher estimates that the initial population of a colony of cells is 100. If the cells reproduce at a rate of 25% per week, what is the expected population of the colony in six weeks?**

$N = N_0(1 + r)^t$ Exponential Growth Formula

$= 100(1 + 0.25)^6$ $N_0 = 100$, $r = 0.25$, $t = 6$

≈ 381.4697266 Use a calculator.

There will be about 381 cells in the colony in 6 weeks.

Example 2 **FINANCIAL LITERACY Lance has a bank account that will allow him to invest \$1000 at a 5% interest rate compounded continuously. If there are no other deposits or withdrawals, what will Lance's account balance be after 10 years?**

$A = Pe^{rt}$ Continuous Compound Interest Formula

$= 1000e^{(0.05)(10)}$ $P = 1000$, $r = 0.05$, and $t = 10$

≈ 1648.72 Simplify.

With continuous compounding, Lance's account balance after 10 years will be \$1648.72.

Exercises

1. **FINANCIAL LITERACY** Compare the balance after 10 years of a $5000 investment earning 8.5% interest compounded continuously to the same investment compounded quarterly.

continuously

$N = N_0 e^{kt}$

$= 5000 e^{(.085)(10)}$

$= \$11,698.23$

quarterly

$A = P\left(1 + \frac{r}{n}\right)^{nt}$

$A = 5000\left(1 + \frac{.085}{4}\right)^{4 \cdot 10}$

$A = \$11,594.52$

3. **BIOLOGY** The number of rabbits in a field showed an increase of 10% each month over the last year. If there were 10 rabbits at this time last year, how many rabbits are in the field now?

HW: p. 166, # 5,9,13,15,19,21,25,27,33,35