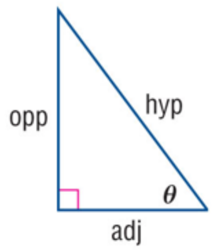
4-1 Study Guide and Intervention

Right Triangle Trigonometry

Values of Trigonometric Ratios The side lengths of a right triangle and a reference angle θ can be used to form six **trigonometric ratios** that define the **trigonometric functions** known as **sine**, **cosine**, and **tangent**. The **cosecant**, **secant**, and **cotangent** ratios are reciprocals of the sine, cosine, and tangent ratios, respectively. Therefore, they are known as **reciprocal functions**.

KeyConcept Trigonometric Functions

Let θ be an acute angle in a right triangle and the abbreviations opp, adj, and hyp refer to the length of the side opposite θ, the length of the side adjacent to θ, and the length of the hypotenuse, respectively.



Then the six trigonometric functions of θ are defined as follows.

sine $(\theta) = \sin\theta = \frac{\text{opp}}{\text{hyp}}$

cosine $(\theta) = \cos\theta = \frac{\text{adj}}{\text{hyp}}$

tangent $(\theta) = \tan\theta = \frac{\text{opp}}{\text{adj}}$

cosecant $(\theta) = \csc\theta = \frac{\text{hyp}}{\text{opp}}$

secant $(\theta) = \sec\theta = \frac{\text{hyp}}{\text{adj}}$

cotangent $(\theta) = \cot\theta = \frac{\text{adj}}{\text{opp}}$

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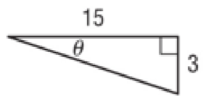
Example: Find the exact values of the six trigonometric functions of θ.



Use the Pythagorean Theorem to determine the length of the hypotenuse.

$15^2 + 3^2 = c^2$	a = 15, b = 3
$234 = c^2$	Simplify.
$c = \sqrt{234}$ or $3\sqrt{26}$	Take the positive square root.

$\sin\theta = \frac{\text{opp}}{\text{hyp}}$ or $\frac{3}{3\sqrt{26}}$ or $\frac{\sqrt{26}}{26}$

$\cos\theta = \frac{\text{adj}}{\text{hyp}}$ or $\frac{15}{3\sqrt{26}}$ or $\frac{\sqrt[5]{26}}{26}$

$\tan\theta = \frac{\text{opp}}{\text{adj}}$ or $\frac{3}{15}$ or $\frac{1}{5}$

$\csc\theta = \frac{\text{hyp}}{\text{opp}}$ or $\frac{3\sqrt{26}}{3}$ or $\frac{26}{\sqrt{26}}$

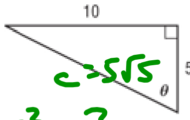
$\sec\theta = \frac{\text{hyp}}{\text{adj}}$ or $\frac{3\sqrt{26}}{15}$ or $\frac{\sqrt{26}}{5}$

$\cot\theta = \frac{\text{adj}}{\text{opp}}$ or $\frac{15}{3}$ or 5

Exercises

Find the exact values of the six trigonometric functions of θ.

1.



$10^2+5^2=c^2$

$\sqrt{125}=\sqrt{c^2}$

$c=5\sqrt5$

$\sin\theta = \frac{\cancel{10}}{\cancel{5}\sqrt5} = \boxed{\frac{2}{\sqrt5}}\cdot\frac{\sqrt5}{\sqrt5} = \frac{2\sqrt5}{5}$

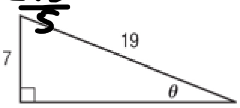
$\cos\theta = \frac{\cancel{5}}{\cancel{5}\sqrt5} = \frac{1}{\sqrt5} = \frac{\sqrt5}{5}$

$\tan\theta = \frac{10}{5} = 2$

$\csc\theta = \frac{\sqrt5}{2}$

$\sec\theta = \sqrt5$

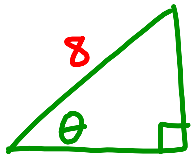
$\cot\theta = \frac{1}{2}$



Use the given trigonometric function value of the acute angle θ to find the exact values of the five remaining trigonometric function values of θ.

3. $\sin\theta = \frac{3}{7}$

4. $\sec\theta = \frac{8}{5} = \frac{hyp}{adj}$



$b=\sqrt{39}$

$5^2+b^2=8^2$

$25+b^2=64$

$-25 \quad -25$

$\sqrt{b^2}=\sqrt{39}$

$\sin\theta = \frac{\sqrt{39}}{8}$

$\cos\theta = \frac{5}{8}$

$\tan\theta = \frac{\sqrt{39}}{5}$

$\csc\theta = \frac{8}{\sqrt{39}}\cdot\frac{\sqrt{39}}{\sqrt{39}} = \frac{8\sqrt{39}}{39}$

$\cot\theta = \frac{5\sqrt{39}}{39}$

Solving Right Triangles To solve a right triangle means to find the measures of all of the angles and sides of the triangle. When the trigonometric value of an acute angle is known, the inverse of the trigonometric function can be used to find the measure of the angle.

Trigonometric Function	Inverse Trigonometric Function
$y = \sin x$	$x = \sin^{-1} y$ or $x = \arcsin y$
$y = \cos x$	$x = \cos^{-1} y$ or $x = \arccos y$
$y = \tan x$	$x = \tan^{-1} y$ or $x = \arctan y$

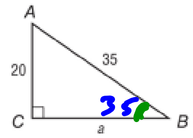
KeyConcept Inverse Trigonometric Functions *angles*

Inverse Sine If θ is an acute angle and the sine of θ is x, then the **inverse sine** of x is the measure of angle θ. That is, if $\sin \theta = x$, then $\sin^{-1} x = \theta$.

Inverse Cosine If θ is an acute angle and the cosine of θ is x, then the **inverse cosine** of x is the measure of angle θ. That is, if $\cos \theta = x$, then $\cos^{-1} x = \theta$.

Inverse Tangent If θ is an acute angle and the tangent of θ is x, then the **inverse tangent** of x is the measure of angle θ. That is, if $\tan \theta = x$, then $\tan^{-1} x = \theta$.

Example: Solve $\triangle ABC$. Round side measures to the nearest tenth and angle measures to the nearest degree.



28.7

side a

$35^2 - 20^2 = a^2$

$\sqrt{825} = \sqrt{a^2}$

$28.7 = a$

m∠B

$m\angle B = \sin^{-1} \frac{20}{35}$

$= 35^\circ$

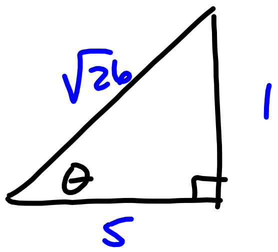
m∠A

$180 - 90 - 35 = 55^\circ$

HW: p. 227

1, 7, 9, 15, 21, 23, 25, 31, 35, 37

15. $\cot\theta = \frac{5}{1} = \frac{adj}{opp}$



$\sin\theta = \frac{1}{\sqrt{26}}\cdot\frac{\sqrt{26}}{\sqrt{26}} = \frac{\sqrt{26}}{26}$

$\cos\theta = \frac{5}{\sqrt{26}}\cdot\frac{\sqrt{26}}{\sqrt{26}} = \frac{5\sqrt{26}}{26}$

$\tan\theta = \frac{1}{5}$

$\csc\theta = \sqrt{26}$

$\sec\theta = \frac{\sqrt{26}}{5}$

$1^2 + 5^2 = c^2$

$\sqrt{26} = \sqrt{c^2}$

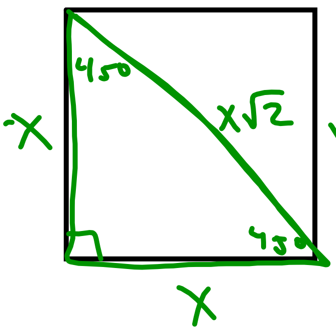
$c = \sqrt{26}$

0·4 +

5-8 ✓

9↑ –

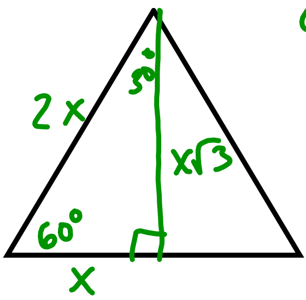
Development of Special Right Triangles



$x^2 + x^2 = c^2$

$\sqrt{2x^2} = \sqrt{c^2}$

$x\sqrt{2} = c$



$a^2 + x^2 = (2x)^2$

$a^2 + x^2 = 4x^2$

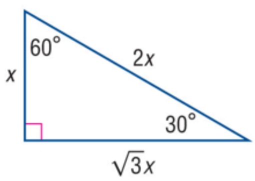
$-x^2 \quad -x^2$

$\sqrt{a^2} = \sqrt{3x^2}$

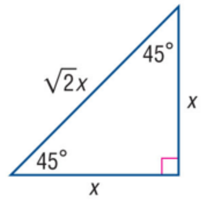
$a = x\sqrt{3}$

KeyConcept Trigonometric Values of Special Angles

30°-60°-90° Triangle



45°-45°-90° Triangle



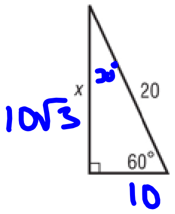
$\sin 30 = \frac{1\cancel{x}}{2\cancel{x}}$

θ	30°	45°	60°
sin θ	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$
cos θ	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$
tan θ	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$

Exercises

Find the value of *x*. Round to the nearest tenth if necessary.

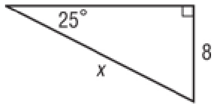
1.



$\frac{2y}{2}=\frac{20}{2}$

$y=10$

2.



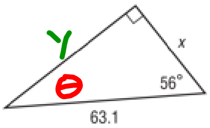
$\cancel{\sin 25 = \frac{8}{x}}$

$\frac{x\sin 25}{\sin 25}=\frac{8}{\sin 25}$

$\boxed{x=18.9}$

Solve each triangle. Round side measures to the nearest tenth and angle measures to the nearest degree.

3.



$m\angle\theta = 180-(90+56)$

$=34°$

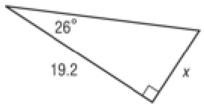
side x

$\cancel{\cos 56 = \frac{x}{63.1}}$

$63.1\cos 56 = x$

$x=35.3$

4.



$35.3^2+y^2=63.1^2$

$1246.1+y^2=3981.6$

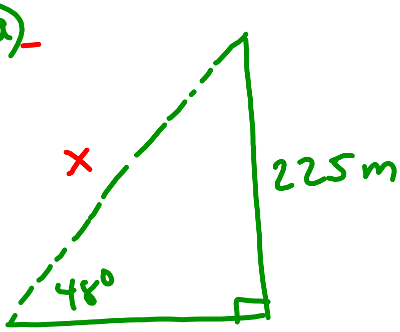
$-1246.1 \qquad -1246.1$

$\sqrt{y^2}=\sqrt{2735.5}$

$y=52.4$

EX: p. 228, #42 - Angle of Elevation and Angle of Depression

a)



b)

$$\sin 48 = \frac{225}{x}$$

$$\frac{x \cancel{\sin 48}}{\cancel{\sin 48}} = \frac{225}{\sin 48}$$

$$x = 302.8 \text{ m}$$

HW: p. 229

39, 41, 45, 49, 51, 57, 61, 63, 65, 75