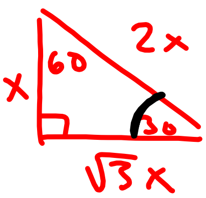
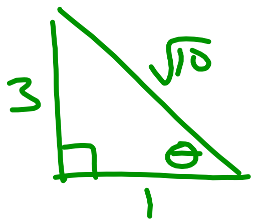
59. $\sec 30 = \frac{hyp}{adj} = \frac{2\cancel{x}}{\sqrt{3}\cancel{x}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$

$= \frac{2\sqrt{3}}{3}$



11.

11. $\tan\theta = \frac{3}{1} = \frac{opp}{adj}$



$1^2 + 3^2 = c^2$

$\sqrt{10} = \sqrt{c^2}$

$\sin\theta = \frac{3}{\sqrt{10}} = \frac{3\sqrt{10}}{10}$

$\cos\theta = \frac{1}{\sqrt{10}} = \frac{\sqrt{10}}{10}$

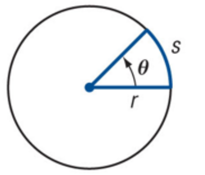
$\csc\theta = \frac{\sqrt{10}}{3}$

$\sec\theta = \sqrt{10}$

$\cot\theta = \frac{1}{3}$

KeyConcept Radian Measure

Words	The measure θ in radians of a central angle of a circle is equal to the ratio of the length of the intercepted arc s to the radius r of the circle.	
Symbols	$\theta = \frac{s}{r}$, where θ is measured in radians (rad)	
Example	A central angle has a measure of 1 radian if it intercepts an arc with the same length as the radius of the circle.	



$\theta = 1$ radian when $s = r$.

4-2 Study Guide and Intervention

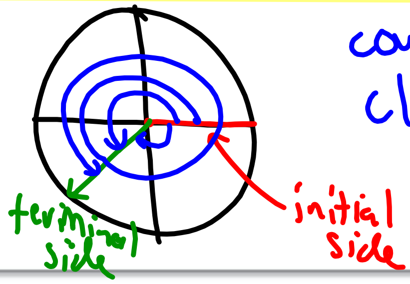
Degrees and Radians

Angles and Their Measures One complete rotation can be represented by 360° or 2π radians. Thus, the following formulas can be used to relate **degree** and **radian** measures.

KeyConcept Degree/Radian Conversion Rules

1. To convert a degree measure to radians, multiply by $\frac{\pi \text{ radians}}{180°}$.
2. To convert a radian measure to degrees, multiply by $\frac{180°}{\pi \text{ radians}}$.

If two angles have the same initial and terminal sides, but different measures, they are called **coterminal angles**.



counterclockwise rotation = positive ∠'s
clockwise rotation = negative ∠'s

KeyConcept Coterminal Angles

Degrees	Radians
If α is the degree measure of an angle, then all angles measuring $\alpha + 360n°$, where n is an integer, are coterminal with α.	If α is the radian measure of an angle, then all angles measuring $\alpha + 2n\pi$, where n is an integer, are coterminal with α.

Example: Write each degree measure in radians as a multiple of π and each radian measure in degrees.

a. 36°

$36° = 36°\left(\frac{\pi \text{ radians}}{180°}\right)$ Multiply by $\frac{\pi \text{ radians}}{180°}$

$= \frac{\pi}{5}$ radians or $\frac{\pi}{5}$ Simplify

b. $-\frac{17\pi}{3}$

$-\frac{17\pi}{3} = -\frac{17\pi}{3}$ radians Multiply by $\frac{180°}{\pi \text{ radians}}$

$= -\frac{17\pi}{3} \text{ radians}\left(\frac{180°}{\pi \text{ radians}}\right) = -1020°$ Simplify

Exercises

Write each degree measure in radians as a multiple of π and each radian measure in degrees.

1. $-250°$ $\frac{-250}{1}\left(\frac{\pi}{180}\right) = -\frac{25\pi}{18}$

2. $6°$

3. $-145°$

7. 4π $\frac{4\pi}{1}\left(\frac{180}{\pi}\right) = 720°$

8. $\frac{13\pi}{30}$

9. -1

Identify all angles that are coterminal with the given angle.

13. $-\frac{\pi}{2}$ $-\frac{\pi}{2} + 2n\pi$

14. $135°$ $135° + 360°n$

15. $\frac{5\pi}{3}$

Applications with Angle Measure The rate at which an object moves along a circular path is called its **linear speed**. The rate at which the object rotates about a fixed point is called its **angular speed**.

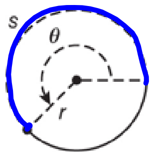
Suppose an object moves at a constant speed along a circular path of radius r.

If s is the arc length traveled by the object during time t, then the object's *linear speed* v is given by

$$V = \frac{s}{t},$$

If θ is the angle of rotation (in radians) through which the object moves during time t, then the *angular speed* ω of the object is given by

$$\omega = \frac{\theta}{t}.$$

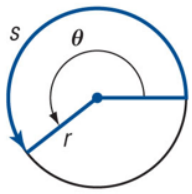


KeyConcept Arc Length

If θ is a central angle in a circle of radius r, then the length of the intercepted arc s is given by

$$s = r\theta,$$

where θ is measured in radians.

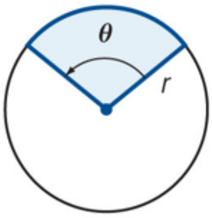


KeyConcept Area of a Sector

The area A of a sector of a circle with radius r and central angle θ is

$$A = \frac{1}{2}r^2\theta,$$

where θ is measured in radians.



Example: Determine the angular speed and linear speed if 8.2 revolutions are completed in 3 seconds and the distance from the center of rotation is 7 centimeters. Round to the nearest tenth.

The angle of rotation is $8.2 \times 2\pi$ or 16.4π radians.

$\omega = \frac{\theta}{t}$	Angular speed
$= \frac{16.4\pi}{3}$	θ = 16.4π radians and t = 3 seconds
≈ 17.17403984	Use a calculator.

Therefore, the ar 'ar speed is about 17.2 radians per second.

The linear speed is $\frac{r\theta}{t}$.

$V = \frac{s}{t}$,	Linear speed
$= \frac{r\theta}{t}$	$s = r\theta$
$= \frac{7(16.4\pi)}{3}$	r = 7 centimeters, θ = 16.4π radians, and t = 3 seconds
$= 120.218278877$	Use a calculator.

Therefore, the linear speed is about 120.2 centimeters per second.

Exercises

Find the rotation in revolutions per minute given the angular speed and the radius given the linear speed and the rate of rotation.

1. $\omega = 2.7$ rad/s

$\omega = \frac{\theta}{t}$

$60(2.7) = \left(\frac{\theta}{60}\right)60$

$\theta = \frac{162 \text{ radians}}{(2\pi)}$

$\boxed{25.78 \text{ rev./min}}$

2. $\omega = \frac{4}{3}\pi$ rad/hr

720°

5. $V = 118$ ft/min , 3.6 rev/s

6. $V = 256$ in./h, 0.5 rev/min

Ex: Change 61.325° to degrees, minutes, + seconds

61° 19' 30"

HW: p.238
1-19 odd, 25, 27, 31, 35, 37, 43, 45