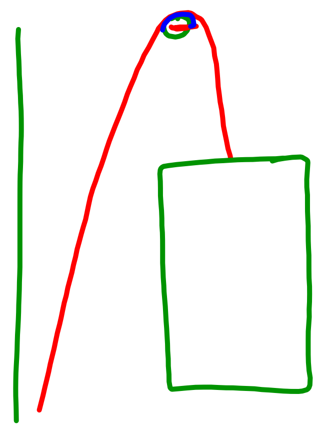
63.



a) $s = r\theta$

$s = (6)(\pi)$

$s = 6\pi$ in

$s = 18.8$ in

$\cancel{180}^\circ \cdot \frac{\pi}{\cancel{180}}$

$\cancel{90}^{.5} \cdot \frac{\pi}{\cancel{180}}$

b) $s = (4)(5\pi)$

$\boxed{s = 20\pi \text{ in} \quad 62.8 \text{ in}}$

69. $\frac{5\pi}{6}$

complement: $\frac{5\pi}{6} + x = \frac{\pi \cdot 3}{2 \cdot 3} \; \frac{3\pi}{6}$ $\quad \cancel{-\frac{2\pi}{6}}$

$-\frac{5\pi}{6} \qquad -\frac{5\pi}{6}$

supplement: $\frac{5\pi}{6} + x = \frac{\pi}{1} \cdot \frac{6}{6} = \frac{6\pi}{6}$ $\quad \frac{\pi}{6}$

$-\frac{5\pi}{6} \qquad -\frac{5\pi}{6}$

0-4 +

5-8 ✓

9+ -

4-3 Study Guide and Intervention

Trigonometric Functions on the Unit Circle

Let θ be any angle in standard position and point $P(x, y)$ be a point on the terminal side of θ.

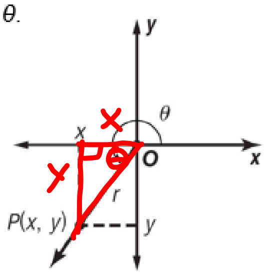
Let r represent the nonzero distance from P to the origin. That is, let $r = \sqrt{x^2 + y^2} \neq 0$.

Then the trigonometric functions of θ are as follows.

$\sin \theta = \frac{y}{r}$ $\quad$ $\csc \theta = \frac{r}{y}$, $y \neq 0$

$\cos \theta = \frac{x}{r}$ $\quad$ $\sec \theta = \frac{r}{x}$, $x \neq 0$

$\tan \theta = \frac{y}{x}$, $x \neq 0$ $\quad$ $\cot \theta = \frac{x}{y}$, $y \neq 0$



You can use the following steps to find the value of a trigonometric function of any angle θ.

1. Find the reference angle θ'.
2. Find the value of the trigonometric function for θ'.
3. Use the quadrant in which the terminal side of θ lies to determine the sign of the trigonometric function value of θ.

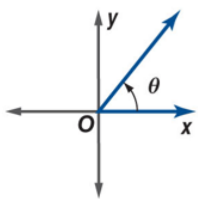
Positive Trig Functions:

Sin	All
Tan	Cos

KeyConcept Reference Angle Rules

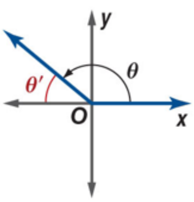
If θ is an angle in standard position, its reference angle θ' is the acute angle formed by the terminal side of θ and the x-axis.

Quadrant I



$\theta' = \theta$

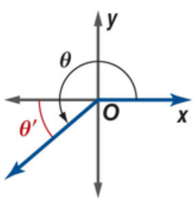
Quadrant II



$\theta' = 180° - \theta$

$\theta' = \pi - \theta$

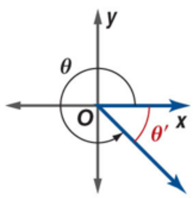
Quadrant III



$\theta' = \theta - 180°$

$\theta' = \theta - \pi$

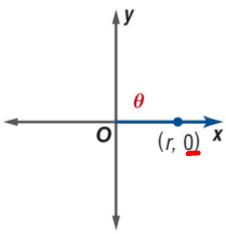
Quadrant IV



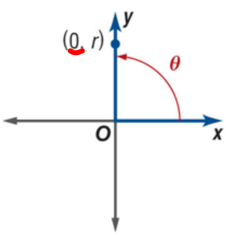
$\theta' = 360° - \theta$

$\theta' = 2\pi - \theta$

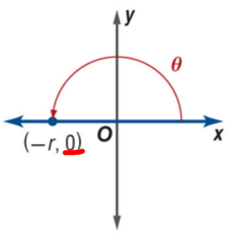
KeyConcept Common Quadrantal Angles



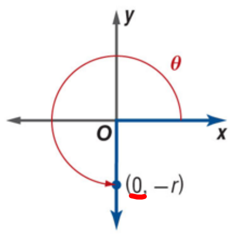
$\theta = 0°$ or 0 radians



$\theta = 90°$ or $\frac{\pi}{2}$ radians



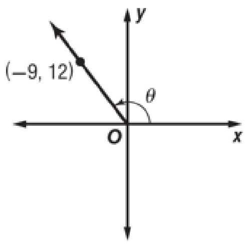
$\theta = 180°$ or π radians



$\theta = 270°$ or $\frac{3\pi}{2}$ radians

Example: Let (–9, 12) be a point on the terminal side of an angle θ in standard position. Find the exact values of the six trigonometric functions of θ.

Use the values of x and y to find r.



$r = \sqrt{x^2 + y^2}$ Pythagorean Theorem

$= \sqrt{(-9)^2 + 12^2}$ $x = -9$ and $y = 12$

$= \sqrt{225}$ or 15 Take the positive square root.

Use $x = -9$, $y = 12$, and $r = 15$ to write the six trigonometric ratios.

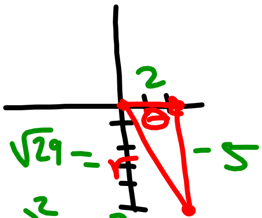
$\sin\theta = \frac{y}{r} = \frac{12}{15}$ or $\frac{4}{5}$ $\cos\theta = \frac{x}{r} = \frac{-9}{15}$ or $-\frac{3}{5}$ $\tan\theta = \frac{y}{x} = \frac{12}{-9}$ or $-\frac{4}{3}$

$\csc\theta = \frac{r}{y} = \frac{15}{12}$ or $\frac{5}{4}$ $\sec\theta = \frac{r}{x} = \frac{15}{-9}$ or $-\frac{5}{3}$ $\cot\theta = \frac{x}{y} = \frac{-9}{12}$ or $-\frac{3}{4}$

Exercises

The given point lies on the terminal side of an angle θ in standard position. Find the values of the six trigonometric functions of θ.

1. (2, –5)



$(2)^2 + (-5)^2 = r^2$

$r = \sqrt{29}$

2. (12, 4)

3. (–3, –8)

$\sin\theta = \frac{-5}{\sqrt{29}} \cdot \frac{\sqrt{29}}{\sqrt{29}} = \frac{-5\sqrt{29}}{29}$

$\cos\theta = \frac{2}{\sqrt{29}} = \frac{2\sqrt{29}}{29}$

$\tan\theta = -\frac{5}{2}$

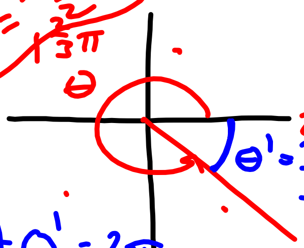
$\csc\theta = \frac{\sqrt{29}}{-5}$

$\sec\theta = \frac{\sqrt{29}}{2}$

$\cot\theta = -\frac{2}{5}$

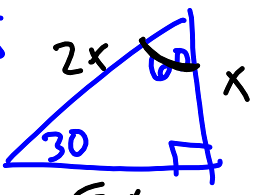
Find the exact value of each expression.

4. $\sin\frac{5\pi}{3}$ $= -\frac{\sqrt{3}}{2}$



$\frac{5\pi}{3} + \theta' = 2\pi$

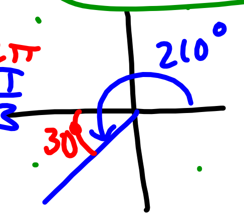
$\frac{5\pi}{3} + \theta' = \frac{6\pi}{3}$



$\sin\frac{\pi}{3} = \frac{\sqrt{3}\cancel{x}}{2\cancel{x}}$

$\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

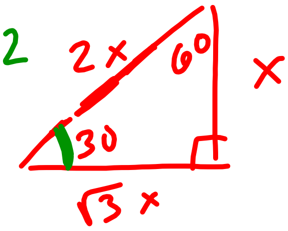
5. csc 210° $= -2$



csc 30° = 2

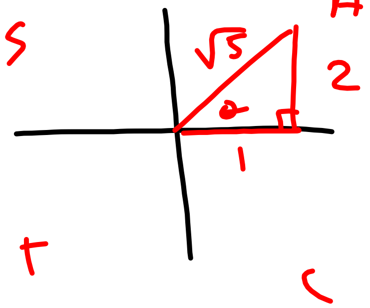
$\frac{2\cancel{x}}{\cancel{x}}$

6. cot (–315°)



HW: p. 251,
1, 7, 11, 15, 17, 19, 21, 23,
25, 29, 33, 37, 39

33. $\tan\theta = \frac{2}{1} = \frac{opp}{adj}$



$1^2 + 2^2 = r^2$

$\sqrt{5} = r$

$\sin\theta = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$

$\cos\theta = \frac{1}{\sqrt{5}} = \frac{\sqrt{5}}{5}$

$\csc\theta = \frac{\sqrt{5}}{2}$

$\sec\theta = \sqrt{5}$

$\cot\theta = \frac{1}{2}$

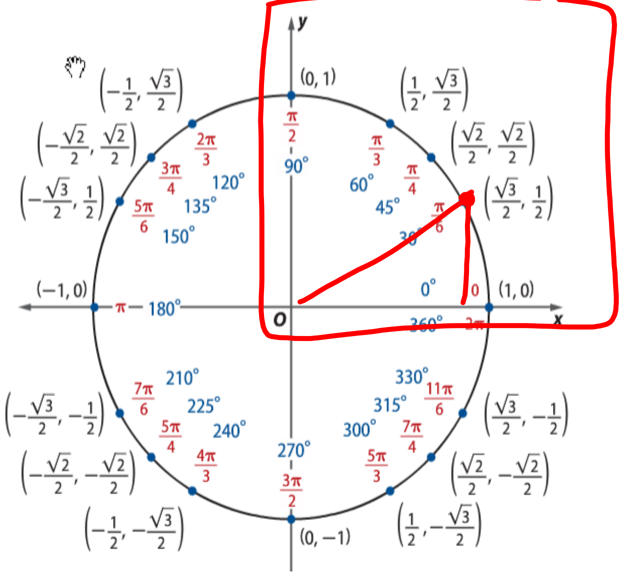
0 - 4 +

5 - 8 ✓

9 r -

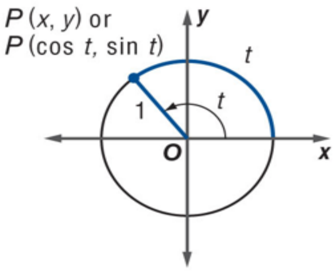
Unit Circle

16-Point Unit Circle



KeyConcept Trigonometric Functions on the Unit Circle

Let t be any real number on a number line and let $P(x, y)$ be the point on t when the number line is wrapped onto the unit circle. Then the trigonometric functions of t are as follows.



$\underline{\sin}\ t = \underline{y}$ $\qquad$ $\underline{\cos}\ t = \underline{x}$ $\qquad$ $\tan t = \frac{y}{x}, x \neq 0$ $\quad \frac{\sin t}{\cos t}$

$\csc t = \frac{1}{y}, y \neq 0$ $\qquad$ $\sec t = \frac{1}{x}, x \neq 0$ $\qquad$ $\cot t = \frac{x}{y}, y \neq 0$ $\quad \frac{\cos t}{\sin t}$

Therefore, the coordinates of P corresponding to the angle t can be written as $P(\cos t, \sin t)$.

Example: Find the exact value of tan $\frac{5\pi}{3}$. If undefined, write *undefined*.

$\frac{5\pi}{3}$ corresponds to the point $(x, y) = \left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$ on the unit circle.

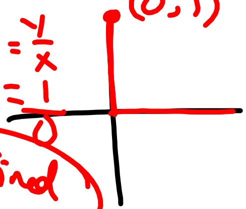
$\tan t = \frac{y}{x}$	Definition of tan t
$\tan \frac{5\pi}{3} = \frac{-\frac{\sqrt{3}}{2}}{\frac{1}{2}}$	$x = \frac{1}{2}$ and $y = -\frac{1}{2}$ when $t = \frac{5\pi}{3}$
$\tan \frac{5\pi}{3} = -\sqrt{3}$	Simplify

Exercises

Find the exact value of each expression. If undefined, write *undefined*.

1. $\tan \frac{\pi}{2} = \frac{y}{x} = \frac{1}{0}$

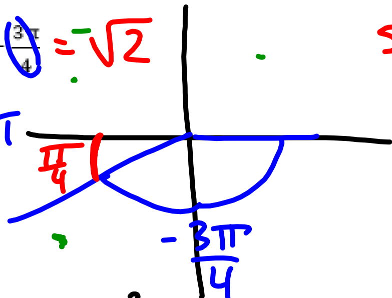
$(0,1)$



undefined

2. $\sec -\frac{3\pi}{4} = -\sqrt{2}$

$\frac{4\pi}{4} = \pi$



$\frac{\pi}{4}$

$-\frac{3\pi}{4}$

$\sec \frac{\pi}{4} = \frac{1}{\cos \frac{\pi}{4}}$

$= \frac{1}{\frac{\sqrt{2}}{2}}$

$= \frac{1}{1} \cdot \frac{2}{\sqrt{2}}$

$= \frac{2}{\sqrt{2}}$

$= \frac{\cancel{2}\sqrt{2}}{\cancel{2}}$

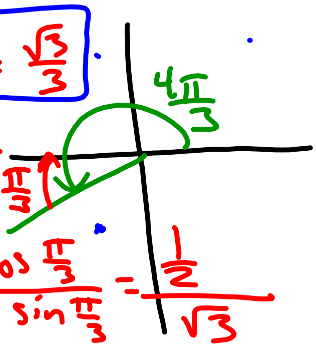
$= \sqrt{2}$

5. $\boxed{\cot \frac{4\pi}{3} = \frac{\sqrt{3}}{3}}$

$1\frac{1}{3}\pi$

$\frac{3\pi}{3} = \pi$

$\frac{4\pi}{3}$



$\frac{\pi}{3}$

$\cot \frac{\pi}{3} = \frac{\cos \frac{\pi}{3}}{\sin \frac{\pi}{3}} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}}$

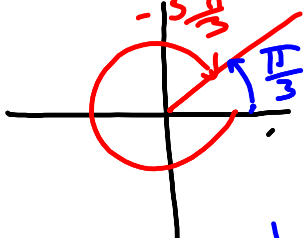
$= \frac{1}{\cancel{2}} \cdot \frac{\cancel{2}}{\sqrt{3}}$

$= \frac{1}{\sqrt{3}}$

$= \frac{\sqrt{3}}{3}$

6. $\csc -\frac{5\pi}{3} = -1\frac{2}{3}\pi$

$-\frac{5\pi}{3}$



$\frac{\pi}{3}$

$\csc \frac{\pi}{3} = \frac{1}{\sin \frac{\pi}{3}} = \frac{1}{\frac{\sqrt{3}}{2}}$

$= \frac{1}{1} \cdot \frac{2}{\sqrt{3}}$

$= \frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$

$\boxed{\csc -\frac{5\pi}{3} = \frac{2\sqrt{3}}{3}}$

HW: p.251, 43, 49, 53, 55, 61, 63, 67, 73

KeyConcept Periodic Functions

A function $y = f(t)$ is periodic if there exists a positive real number c such that $f(t + c) = f(t)$ for all values of t in the domain of f.

The smallest number c for which f is periodic is called the **period** of f.