Then

You found and graphed the inverses of relations and functions. (Lesson 1-7)

Now

- Evaluate and graph inverse trigonometric functions.
- Find compositions of trigonometric functions.

13. $y = 4\sec\left(x - \frac{3\pi}{4}\right)$

$P = \frac{2\pi}{B}$

$P = \frac{2\pi}{1} = 2\pi$

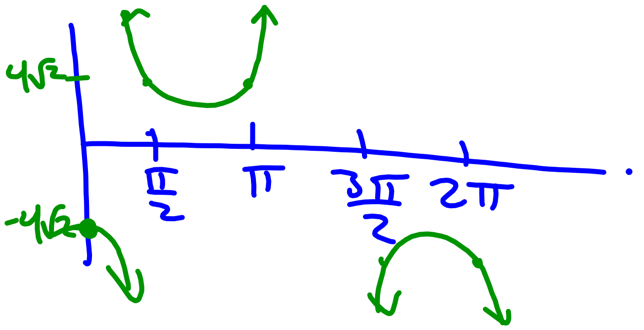
units: $\frac{2\pi}{4} = \frac{\pi}{2}$

$4\sec\left(\pi - \frac{3\pi}{4}\right)$

$4\sec\left(\frac{\pi}{4}\right)$

$4 \cdot \frac{1}{\cos\frac{\pi}{4}} = 4\sqrt{2}$

x	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
y	$-4\sqrt{2}$	$4\sqrt{2}$	$4\sqrt{2}$	$-4\sqrt{2}$	$-4\sqrt{2}$



0 - 3 +
4 - 6 ✓
7 ? -

$-4\sqrt{2}$

$1 \cdot \frac{-}{\sqrt{2}} - \frac{-}{\sqrt{2}}$ & ...

4-6 Study Guide and Intervention

Inverse Trigonometric Functions

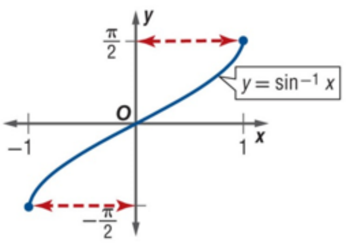
Inverse Trigonometric Functions When restricted to a certain domain, the sine, cosine, and tangent functions have inverse functions known as the arcsine, arccosine, and arctangent functions, respectively.

KeyConcept Inverse Trigonometric Functions

Inverse Sine of x

$\sin^{-1} x = \arcsin x$

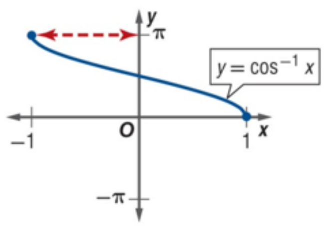
Range: $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$



Inverse Cosine of x

$\cos^{-1} x = \arccos x$

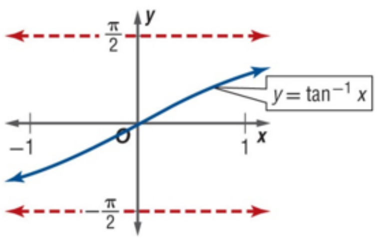
Range: $[0, \pi]$



Inverse Tangent of x

$\tan^{-1} x = \arctan x$

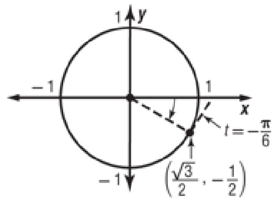
Range: $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$



Examples: Find the exact value of each expression, if it exists.

1. $\sin^{-1}\left(-\frac{1}{2}\right)$

Find a point on the unit circle in the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ with a y-coordinate of $-\frac{1}{2}$.
When $t = -\frac{\pi}{6}$, $\sin t = -\frac{1}{2}$. Therefore, $\sin^{-1}\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$.



2. $\cos^{-1} 4$

Because the domain of the inverse cosine function is $[-1, 1]$ and $4 > 1$, there is no angle with a cosine of 4. Therefore, the value of $\cos^{-1} 4$ does not exist.

Exercises

Find the exact value of each expression, if it exists.

2. $\arcsin \frac{\sqrt{3}}{2}$ $= \frac{\pi}{3}$ and $\cancel{\frac{2\pi}{3}}$

3. $\cos^{-1} \frac{\sqrt{2}}{2}$ $= \frac{\pi}{4}$ and $\cancel{\frac{7\pi}{4}}$

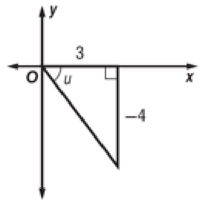
Compositions of Trigonometric Functions Because the domains of the trigonometric functions are restricted to obtain the inverse trigonometric functions, the composition of a trigonometric function and its inverse does not follow the rules that you learned in Lesson 1-7. The properties that apply to trigonometric functions and their inverses are summarized below.

Inverse Properties of Trigonometric Functions	
$f(f^{-1}(x)) = x$	$f^{-1}(f(x)) = x$
If $-1 \le x \le 1$, then $\sin(\sin^{-1} x) = x$.	If $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$, then $\sin^{-1}(\sin x) = x$.
If $-1 \le x \le 1$, then $\cos(\cos^{-1} x) = x$.	If $0 \le x \le \pi$, then $\cos^{-1}(\cos x) = x$.
If $-\infty < x < \infty$, then $\tan(\tan^{-1} x) = x$.	If $-\frac{\pi}{2} < x < \frac{\pi}{2}$, then $\tan^{-1}(\tan x) = x$.

Example: Find the exact value of cos $\left[\tan^{-1}\ -\frac{4}{3}\right]$.

To simplify the expression, let $u = \tan -1\left(-\frac{4}{3}\right)$, so $\tan u = -\frac{4}{3}$.

Because the tangent function is negative in Quadrants II and IV and the domain of the inverse tangent function is restricted to Quadrants I and IV, u must lie in Quadrant IV.



Using the Pythagorean Theorem, you can find that the length of the hypotenuse is 5. Now solve for cos u.

$\cos u = \frac{\text{adj}}{\text{hyp}}$ Cosine function

$= \frac{3}{5}$ adj = 3 and hyp = 5

So, $\cos\left[\tan^{-1}\ -\frac{4}{3}\right] = \frac{3}{5}$.

Exercises

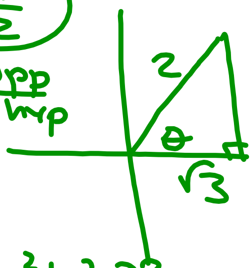
Find the exact value of each expression, if it exists.

1. $\sin\left(\sin^{-1} -\frac{3}{4}\right) = -\frac{3}{4}$

2. $\cos^{-1}\left(\cos \frac{\pi}{2}\right) = \frac{\pi}{2}$

5. $\cos\left(\underbrace{\arcsin \frac{1}{2}}_{\theta}\right) = \frac{\sqrt{3}}{2}$

$\sin\theta = \frac{1}{2} = \frac{opp}{hyp}$

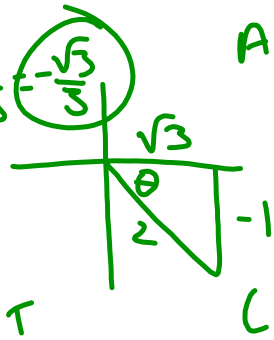


$1^2 + x^2 = 2^2$

$x = \sqrt{3}$

6. $\tan\left(\underbrace{\arcsin -\frac{1}{2}}_{\theta}\right) = -\frac{1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$

$\sin\theta = -\frac{1}{2}$



HW: p. 288, 1, 5, 7, 13, 15, 21, 23, 31, 35, 37,
41, 45