LESSON 5-1

Trigonometric Identities

Then

You found trigonometric values using the unit circle. (Lesson 4-3)

Now

- Identify and use basic trigonometric identities to find trigonometric values.
- Use basic trigonometric identities to simplify and rewrite trigonometric expressions.

5-1 Study Guide and Intervention

Trigonometric Identities

Basic Trigonometric Identities An equation is an **identity** if the left side is equal to the right side for all values of the variable for which both sides are defined. Trigonometric identities are identities that involve trigonometric functions.

KeyConcept Reciprocal and Quotient Identities

Reciprocal Identities			Quotient Identities
$\sin\theta = \frac{1}{\csc\theta}$	$\cos\theta = \frac{1}{\sec\theta}$	$\tan\theta = \frac{1}{\cot\theta}$	$\tan\theta = \frac{\sin\theta}{\cos\theta}$
$\csc\theta = \frac{1}{\sin\theta}$	$\sec\theta = \frac{1}{\cos\theta}$	$\cot\theta = \frac{1}{\tan\theta}$	$\cot\theta = \frac{\cos\theta}{\sin\theta}$

KeyConcept Pythagorean Identities

$\sin^2\theta + \cos^2\theta = 1$ $\tan^2\theta + 1 = \sec^2\theta$ $\cot^2\theta + 1 = \csc^2\theta$

$y^2 + x^2 = r^2$

KeyConcept Cofunction Identities

$\sin\theta = \cos\left(\frac{\pi}{2} - \theta\right)$	$\tan\theta = \cot\left(\frac{\pi}{2} - \theta\right)$	$\sec\theta = \csc\left(\frac{\pi}{2} - \theta\right)$
$\cos\theta = \sin\left(\frac{\pi}{2} - \theta\right)$	$\cot\theta = \tan\left(\frac{\pi}{2} - \theta\right)$	$\csc\theta = \sec\left(\frac{\pi}{2} - \theta\right)$

$f(-x) = f(x)$ even

$f(-x) = -f(x)$ odd

KeyConcept Odd-Even Identities

$\sin\left(-\frac{\pi}{3}\right) = -\sin\frac{\pi}{3}$

$\sin(-\theta) = -\sin\theta$	$\cos(-\theta) = \cos\theta$	$\tan(-\theta) = -\tan\theta$
$\csc(-\theta) = -\csc\theta$	$\sec(-\theta) = \sec\theta$	$\cot(-\theta) = -\cot\theta$

Example **If $\sin\theta = \frac{3}{5}$ and $0° < \theta < 90°$, find $\tan\theta$.**

Use two identities to relate $\sin\theta$ and $\tan\theta$.

$\sin^2\theta + \cos^2\theta = 1$ Pythagorean Identity

$\left(\frac{3}{5}\right)^2 + \cos^2\theta = 1$ $\sin\theta = \frac{3}{5}$

$\cos^2\theta = \frac{16}{25}$ Simplify.

$\cos\theta = \pm\sqrt{\frac{16}{25}}$ or $\pm\frac{4}{5}$ Take the square root of each side.

Since $0° < \theta < 90°$, $\cos\theta$ is positive.

Thus, $\cos\theta = \frac{4}{5}$.

Now find $\tan\theta$.

$\tan\theta = \frac{\sin\theta}{\cos\theta}$ Quotient identity

$\tan\theta = \frac{\frac{3}{5}}{\frac{4}{5}}$ $\sin\theta = \frac{3}{5}$, $\cos\theta = \frac{4}{5}$

$\tan\theta = \frac{3}{4}$ Simplify.

Exercises

Find the value of each expression using the given information.

1. If $\cot\theta = \frac{12}{5}$, find $\tan\theta$.

$\frac{5}{12}$

2. If $\sin\theta = -\frac{1}{4}$, find $\csc\theta$.

-4

5. If $\cot\alpha = -\frac{4}{3}$ and $\boxed{\sin\alpha < 0,}$ find $\cos\alpha$ and $\csc\alpha$.

adj
opp

$\cos\alpha = \frac{4}{5}$

$\csc\alpha = \frac{5}{-3}$

$\frac{1}{x-3} + \frac{2}{3-x}$

Ex: If sin x = -0.37, find cos $\left(x - \frac{\pi}{2}\right)$

$= \cos\left(-\left(\frac{\pi}{2} - x\right)\right)$

$= \cos\left(\frac{\pi}{2} - x\right)$

$= \sin x$

$= -.37$

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1-21 odd

17. $\csc\theta = -1.24$

$\sec\left(\theta - \frac{\pi}{2}\right)$

$\sec -\left(\frac{\pi}{2} - \theta\right)$

$\sec\left(\frac{\pi}{2} - \theta\right)$

$\csc\theta$

-1.24

19. $\tan\theta = -1.52$

$\cot\left(\theta - \frac{\pi}{2}\right)$

$\cot -\left(\frac{\pi}{2} - \theta\right)$

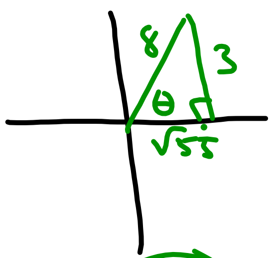
$-\underline{\cot\left(\frac{\pi}{2} - \theta\right)}$

$-\tan\theta$

$-(-1.52)$

1.52

13. $\csc\theta = \frac{8}{3}$ $\frac{hyp}{opp}$



$3^2 + x^2 = 8^2$

$9 + x^2 = 64$

$-9 \qquad -9$

$\sqrt{x^2} = \sqrt{55}$

$x = \sqrt{55}$

$\cos\theta = \frac{\sqrt{55}}{8}$

$\tan\theta = \frac{3}{\sqrt{55}} \cdot \frac{\sqrt{55}}{\sqrt{55}} = \frac{3\sqrt{55}}{55}$

Trigonometric Identities

Simplify and Rewrite Trigonometric Expressions You can apply trigonometric identities and algebraic techniques such as substitution, factoring, and simplifying fractions to simplify and rewrite trigonometric expressions.

Example **Simplify each expression.**

a. $\sec x - \cos x$

$\sec x - \cos x = \frac{1}{\cos x} - \cos x$	Reciprocal Identity
$= \frac{1 - \cos^2 x}{\cos x}$	Add.
$= \frac{\sin^2 x}{\cos x}$	Pythagorean Identity
$= \sin x \left(\frac{\sin x}{\cos x}\right)$	Factor.
$= \sin x \tan x$	Quotient Identity

b. $\csc x \cot^2 x + \frac{1}{\sin x}$

$\csc x \cot^2 x + \frac{1}{\sin x} = \csc x \cot^2 x + \csc x$	Reciprocal Identity
$= \csc x\,(\csc^2 x - 1) + \csc x$	Pythagorean Identity
$= \csc^3 x - \csc x + \csc x$	Distributive Property
$= \csc^3 x$	Simplify.

Exercises

Simplify each expression.

1. $\cos x\,(\tan x + \cot x)$

$\cos x\left(\frac{\sin x}{\cos x}\frac{(\sin x)}{(\sin x)} + \frac{\cos x}{\sin x}\frac{(\cos x)}{(\cos x)}\right)$

$\cos x\left(\frac{\boxed{\sin^2 x + \cos^2 x}}{\sin x \cos x}\right)$

$\frac{\cancel{\cos x}}{1}\left(\frac{1}{\sin x\cancel{\cos x}}\right) = \frac{1}{\sin x} = \boxed{\csc x}$

4. $(\sec x - \tan x)(\csc x + 1)$

3. $\frac{\csc^2 x}{\boxed{1 + \tan^2 x}}$

$= \frac{\csc^2 x}{\sec^2 x}$

$= \frac{\frac{1}{\sin^2 x}}{\frac{1}{\cos^2 x}} = \frac{1}{\sin^2 x} \cdot \frac{\cos^2 x}{1}$

$= \frac{\cos^2 x}{\sin^2 x}$

$= \cot^2 x$

7. $\csc x \sin x + \cot^2 x$

$\frac{1}{\cancel{\sin x}} \cdot \cancel{\sin x} + \cot^2 x$

$\boxed{1 + \cot^2 x}$

$\csc^2 x$

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23-35 odd, 51, 53, 69, 70