Then

You simplified trigonometric expressions. (Lesson 5-1)

Now

- Verify trigonometric identities.
- Determine whether equations are identities.

79. $\tan(\arctan 3)$

$\tan^{-1}$

3

81. $\cos\left(\frac{\pi}{2} - \cos^{-1}\frac{\sqrt{2}}{2}\right)$

$\cos\left(\frac{\pi}{2} - \frac{\pi}{4}\right)$

$\cos\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$

0-3+
4-6✓
7↑-

ConceptSummary Strategies for Verifying Trigonometric Identities

- Start with the more complicated side of the identity and work to transform it into the simpler side, keeping the other side of the identity in mind as your goal.
- Use reciprocal, quotient, Pythagorean, and other basic trigonometric identities.
- Use algebraic operations such as combining fractions, rewriting fractions as sums or differences, multiplying expressions, or factoring expressions.
- If no other strategy presents itself, try converting the entire expression to one involving only sines and cosines.

5-2 Study Guide and Intervention

Verifying Trigonometric Identities

Verify Trigonometric Identities To **verify an identity** means to *prove* that both sides of the equation are equal for all values of the variable for which both sides are defined.

Example **Verify that $\frac{\sec^2 x - 1}{\sec^2 x} = \sin^2 x$.**

The left-hand side of this identity is more complicated, so start with that expression first.

$$\frac{\sec^2 x - 1}{\sec^2 x} = \frac{(\tan^2 x + 1) - 1}{\sec^2 x}$$ Pythagorean Identity

$$= \frac{\tan^2 x}{\sec^2 x}$$ Simplify.

$$= \frac{\left(\frac{\sin^2 x}{\cos^2 x}\right)}{\frac{1}{\cos^2 x}}$$ Quotient Identity and Reciprocal Identity

$$= \frac{\sin^2 x}{\cos^2 x} \cdot \cos^2 x$$ Simplify.

$$= \sin^2 x \checkmark$$ Multiply.

Notice that the verification ends with the expression on the other side of the identity.

Verify each identity.

1. $\sec\theta - \cos\theta = \sin\theta\tan\theta$

$\frac{1}{\cos\theta} - \frac{\cos\theta}{1}\left(\frac{\cos\theta}{\cos\theta}\right) =$

$\frac{1}{\cos\theta} - \frac{\cos^2\theta}{\cos\theta} =$

$\frac{1-\cos^2\theta}{\cos\theta} =$

$\frac{\sin^2\theta}{\cos\theta} =$

$= \frac{\sin\theta\cdot\sin\theta}{\cos\theta}$

$= \tan\theta\sin\theta$ ✓

2. $\sec\theta = \sin\theta(\tan\theta + \cot\theta)$

3. $\tan\theta\csc\theta\cos\theta = 1$

$\tan\theta\cdot\frac{1}{\sin\theta}\cdot\cos\theta =$

$\frac{\sin\theta}{\cos\theta}\cdot\frac{1}{\sin\theta}\cdot\cos\theta =$

$\frac{\cos\theta}{1}$

$1 =$ ✓

4. $\frac{\csc^2\theta - \cot^2\theta}{1-\sin^2\theta} = \sec^2\theta$

$\frac{\frac{1}{\sin^2\theta} - \frac{\cos^2\theta}{\sin^2\theta}}{1-\sin^2\theta} =$

$\frac{\frac{1-\cos^2\theta}{\sin^2\theta}}{1-\sin^2\theta} =$

$\frac{\frac{1-\cos^2\theta}{\sin^2\theta}}{\cos^2\theta} =$

$\frac{\frac{\sin^2\theta}{\sin^2\theta}}{\cos^2\theta} =$

$\frac{1}{\cos^2\theta} =$

$\sec^2\theta =$ ✓

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1-17 odd

5-2 Study Guide and Intervention *(continued)*

Verifying Trigonometric Identities

Identifying Identities and Nonidentities You can use a graphing calculator to test whether an equation might be an identity by graphing the functions related to each side of the equation. If the graphs of the related functions do not coincide for all values of x for which both functions are defined, the equation is not an identity. If the graphs appear to coincide, you can verify that the equation is an identity by using trigonometric properties and algebraic techniques.

Example **Use a graphing calculator to test whether $\csc \theta - \sin \theta = \cot \theta \cos \theta$ is an identity. If it appears to be an identity, verify it. If not, find an x-value for which both sides are defined but not equal.**

The equation *appears* to be an identity because the graphs of the related functions coincide. Verify this algebraically.

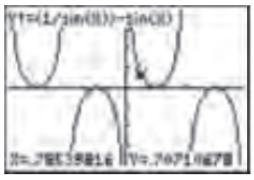
$\csc \theta - \sin \theta = \frac{1}{\sin \theta} - \sin \theta$ Rewrite in terms of sine using a Reciprocal Identity.

$= \frac{1 - \sin^2 \theta}{\sin \theta}$ Rewrite using a common denominator.

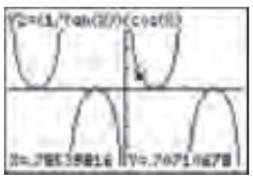
$= \frac{\cos^2 \theta}{\sin \theta}$ Pythagorean Identity

$= \frac{\cos \theta}{\sin \theta} \cdot \cos \theta$ Factor $\cos^2 \theta$.

$= \cot \theta \cos \theta$ ✓ Rewrite in terms of cot θ using a Quotient Identity.



$[-2\pi, 2\pi]$ scl: $\frac{\pi}{2}$ by $[-4, 4]$ scl: 1



$[-2\pi, 2\pi]$ scl: $\frac{\pi}{2}$ by $[-4, 4]$ scl: 1

Exercises

Test whether each equation is an identity by graphing. If it appears to be an identity, verify it. If not, find an x-value for which both sides are defined but not equal.

1. $\sin x + \cos x \cot x = \csc x$

2. $2 - \cos^2 x = \sin^2 x$