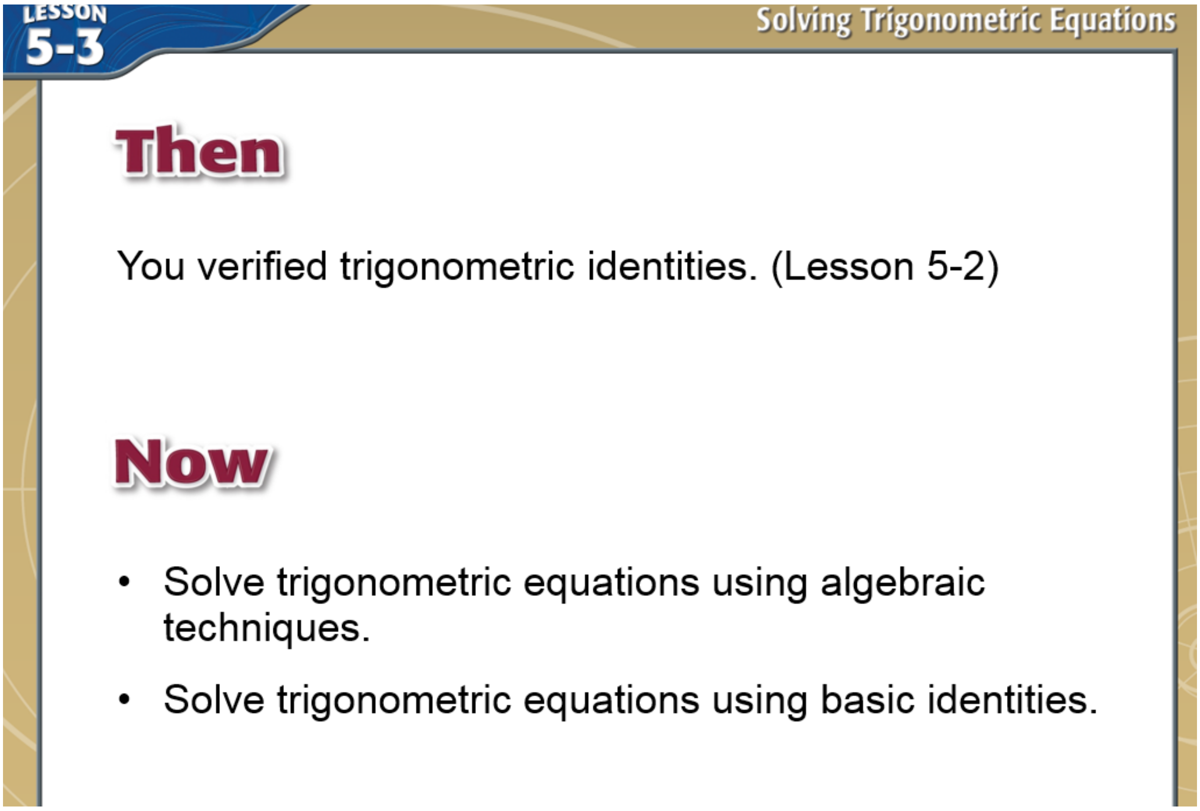


LESSON 5-3

Solving Trigonometric Equations

Then

You verified trigonometric identities. (Lesson 5-2)

Now

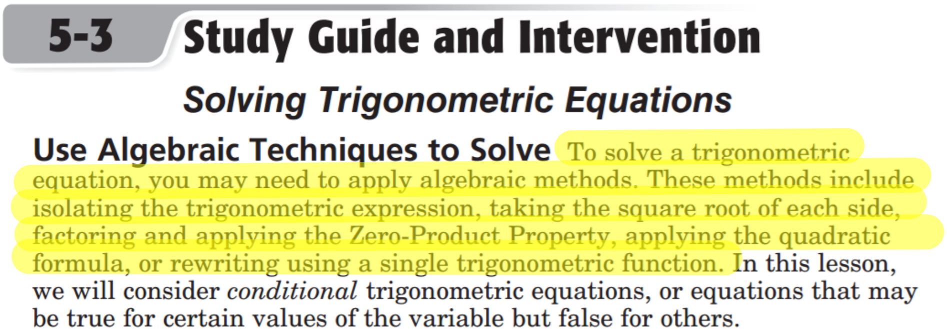
- Solve trigonometric equations using algebraic techniques.
- Solve trigonometric equations using basic identities.

81.

$$9 \cdot \frac{180}{\pi}$$

$$\frac{1620}{\pi}$$

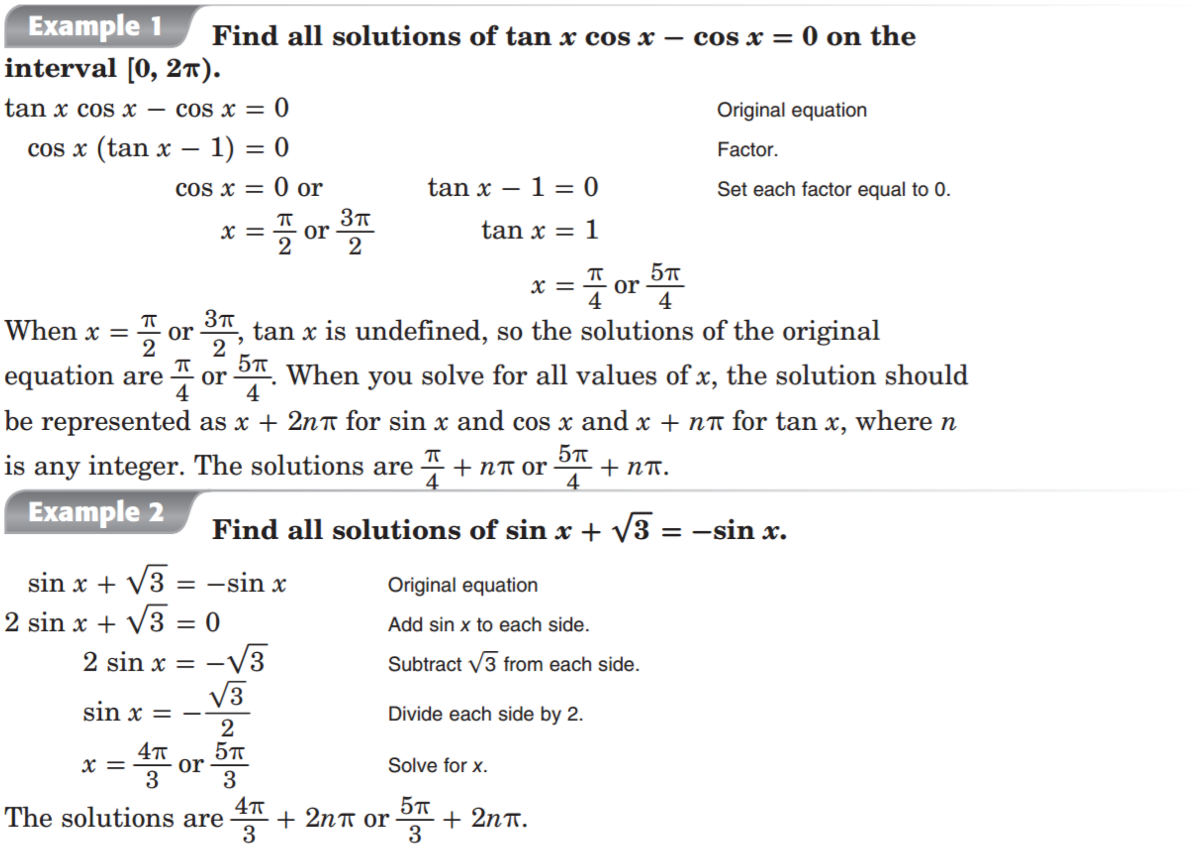
0 - 3 +
4 - 6 ✓
7 ↑ -



5-3 Study Guide and Intervention

Solving Trigonometric Equations

Use Algebraic Techniques to Solve To solve a trigonometric equation, you may need to apply algebraic methods. These methods include isolating the trigonometric expression, taking the square root of each side, factoring and applying the Zero-Product Property, applying the quadratic formula, or rewriting using a single trigonometric function. In this lesson, we will consider *conditional* trigonometric equations, or equations that may be true for certain values of the variable but false for others.



Example 1 **Find all solutions of $\tan x \cos x - \cos x = 0$ on the interval $[0, 2\pi)$.**

$\tan x \cos x - \cos x = 0$ Original equation

$\cos x (\tan x - 1) = 0$ Factor.

$\cos x = 0$ or $\tan x - 1 = 0$ Set each factor equal to 0.

$x = \frac{\pi}{2}$ or $\frac{3\pi}{2}$ $\tan x = 1$

$x = \frac{\pi}{4}$ or $\frac{5\pi}{4}$

When $x = \frac{\pi}{2}$ or $\frac{3\pi}{2}$, $\tan x$ is undefined, so the solutions of the original equation are $\frac{\pi}{4}$ or $\frac{5\pi}{4}$. When you solve for all values of x, the solution should be represented as $x + 2n\pi$ for $\sin x$ and $\cos x$ and $x + n\pi$ for $\tan x$, where n is any integer. The solutions are $\frac{\pi}{4} + n\pi$ or $\frac{5\pi}{4} + n\pi$.

Example 2 **Find all solutions of $\sin x + \sqrt{3} = -\sin x$.**

$\sin x + \sqrt{3} = -\sin x$ Original equation

$2 \sin x + \sqrt{3} = 0$ Add $\sin x$ to each side.

$2 \sin x = -\sqrt{3}$ Subtract $\sqrt{3}$ from each side.

$\sin x = -\frac{\sqrt{3}}{2}$ Divide each side by 2.

$x = \frac{4\pi}{3}$ or $\frac{5\pi}{3}$ Solve for x.

The solutions are $\frac{4\pi}{3} + 2n\pi$ or $\frac{5\pi}{3} + 2n\pi$.

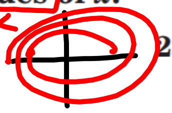
Exercises

Solve each equation for all values of x.

1. $\cos x = -1$

$x = 180^\circ + 360k$

$x = \pi + 2\pi k$



2. $\sin^3 x - 4\sin x = 0$

$x^3 - 4x$

$\sin x(\sin^2 x - 4) = 0$

$\sin x = 0$

$x = 0^\circ + 360k$

$x = 180^\circ + 360k$

$\sin^2 x - 4 = 0$

$+4 \quad +4$

$\sqrt{\sin^2 x} = \sqrt{4}$

$\sin x = \pm 2$

$\sin x = 2 \quad \sin x = -2$

3. $\sin x \cos x - 3\cos x = 0$

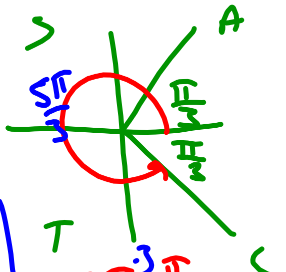
Find all solutions of each equation on the interval $[0, 2\pi)$.

5. $2\cos x = 1$

$\frac{\cancel{2}\cos x}{\cancel{2}} = \frac{1}{2}$

$\cos x = \frac{1}{2}$

$\boxed{x = \frac{\pi}{3} \text{ and } \frac{5\pi}{3}}$



$\frac{2\pi}{1}\cdot\frac{3}{3} - \frac{\pi}{3} =$

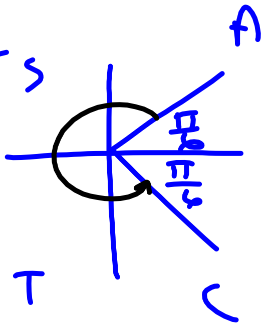
8. $2\cos x - \sqrt{3} = 0$

$+\sqrt{3} \quad +\sqrt{3}$

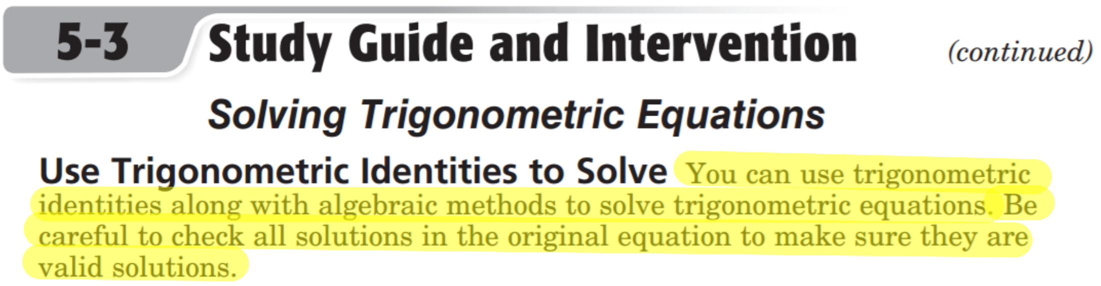
$\frac{\cancel{2}\cos x}{\cancel{2}} = \frac{\sqrt{3}}{2}$

$\cos x = \frac{\sqrt{3}}{2}$

$\boxed{x = \frac{\pi}{6} \text{ or } \frac{11\pi}{6}}$



$\frac{2\pi}{1}\cdot\frac{6}{6} - \frac{\pi}{6} = \frac{11\pi}{6}$



5-3 Study Guide and Intervention *(continued)*

Solving Trigonometric Equations

Use Trigonometric Identities to Solve You can use trigonometric identities along with algebraic methods to solve trigonometric equations. Be careful to check all solutions in the original equation to make sure they are valid solutions.

Example 1 **Find all solutions of $2\tan^2 x - \sec^2 x + 3 = 1 - 2\tan x$ on the interval $[0, 2\pi)$.**

$2\tan^2 x - \sec^2 x + 3 = 1 - 2\tan x$	Original equation
$2\tan^2 x - (\tan^2 x + 1) + 3 = 1 - 2\tan x$	$\sec^2 x = \tan^2 x + 1$
$\tan^2 x + 2 = 1 - 2\tan x$	Simplify.
$\tan^2 x + 2\tan x + 1 = 0$	Simplify.
$(\tan x + 1)^2 = 0$	Factor.
$\tan x = -1$	Take the square root of each side.
$x = \frac{3\pi}{4}$ or $\frac{7\pi}{4}$	Solve for x on $[0, 2\pi)$.

Example 2 **Find all solutions of $1 + \cos x = \sin x$ on the interval $[0, 2\pi)$.**

$1 + \cos x = \sin x$	Original equation
$(1 + \cos x)^2 = (\sin x)^2$	Square each side.
$1 + 2\cos x + \cos^2 x = \sin^2 x$	Multiply.
$1 + 2\cos x + \cos^2 x = 1 - \cos^2 x$	Pythagorean Identity
$2\cos^2 x + 2\cos x = 0$	Simplify.
$2\cos x(\cos x + 1) = 0$	Factor.
$\cos x = 0$ or $\cos x = -1$	Zero Product Property
$x = \frac{\pi}{2}, \pi, \frac{3\pi}{2}$	Solve for x on $[0, 2\pi)$.

Solve each equation for all values of x.

1. $\tan^2 x = 1$

2. $2\boxed{\sin^2 x} - \cos x = 1$

$2(1-\cos^2 x) - \cos x = 1$

$2 - 2\cos^2 x - \cos x = 1$

$-2 + 2\cos^2 x + \cos x \quad -2 + 2\cos^2 x + \cos x$

$0 = 2\cos^2 x + \cos x - 1$

$0 = (2\cos x - 1)(\cos x + 1)$

$2\cos x - 1 = 0$

$+1 \quad +1$

$\frac{2\cos x}{2} = \frac{1}{2}$

$\cos x = \frac{1}{2}$

$\cos x + 1 = 0$

$-1 \quad -1$

$\cos x = -1$

$\frac{\pi}{3} + 2\pi k$

$\pi + 2\pi k$

$\frac{5\pi}{3} + 2\pi k$

$3 \sin x$

Find all solutions of each equation on the interval $[0, 2\pi)$.

5. $\cos x = \sin x$

6. $\sqrt{3} \cos x \tan x - \cos x = 0$

HW: p. 331

1-17 odd