LESSON 5-4

Sum and Difference Identities

Then

You found values of trigonometric functions using the unit circle. (Lesson 4-3)

Now

- Use sum and difference identities to evaluate trigonometric functions.
- Use sum and difference identities to solve trigonometric equations.

7. $\frac{\sin x}{1+\sec x}\cdot\frac{(1-\sec x)}{(1-\sec x)}$

$= \frac{\sin x}{1+\frac{1}{\cos x}}$

$\cos x \cdot 1 + 1$

$= \frac{\sin x}{\frac{\cos x+1}{\cos x}}$

$\frac{(\sin \cos x)(-\cos x+1)}{(\cos x+1)(-\cos x+1)}$

$\frac{\sin x\cos x(-\cos x+1)}{1-\cos^2 x}$

$= \frac{\sin x\cos x(-\cos x+1)}{\sin^2 x}$

$= \cot x(-\cos x+1)$

$= -\cos x\cot x+\cot x$

13. $\frac{(\cos\theta)(1-\sin\theta)}{(1+\sin\theta)(1-\sin\theta)} = \sec\theta-\tan\theta$

$\frac{\cos\theta(1-\sin\theta)}{1-\sin^2\theta}$

$= \frac{\cos\theta(1-\sin\theta)}{\cos^2\theta}$

$= \frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}$

$= \sec\theta-\tan\theta$

5-4 Study Guide and Intervention

Sum and Difference Identities

Evaluate Trigonometric Functions You can use the **sum and difference identities** and the values of trigonometric functions of common angles to find the exact values of less common angles.

Sum Identities	Difference Identities
$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$	$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$
$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$	$\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$
$\tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$	$\tan(\alpha - \beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$

Example **Find the exact value of cos 375°.**

$\cos 375° = \cos(330° + 45°)$	330° and 45° are common angles with a sum of 375°.
$= \cos 330° \cos 45° - \sin 330° \sin 45°$	Cosine Sum Identity
$= \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} - \left(-\frac{1}{2}\right) \cdot \frac{\sqrt{2}}{2}$	$\cos 330° = \frac{\sqrt{3}}{2}$, $\cos 45° = \frac{\sqrt{2}}{2}$, $\sin 330° = -\frac{1}{2}$, $\sin 45° = \frac{\sqrt{2}}{2}$
$= \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4}$	Multiply.
$= \frac{\sqrt{6} + \sqrt{2}}{4}$	Combine the fractions.

Find the exact value of each trigonometric expression.

1. $\cos(-15^\circ)$

$\cos(30-45) = \cos 30 \cos 45 + \sin 30 \sin 45$

$\frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} + \frac{1}{2} \cdot \frac{\sqrt{2}}{2}$

$\frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4}$

$\frac{\sqrt{6}+\sqrt{2}}{4}$

4. $\cos \frac{11\pi}{12}$

$\frac{3(3\pi)}{3(4)} + \frac{\pi(2)}{(6)2}$

$\frac{9\pi}{12} + \frac{2\pi}{12}$

$\cos\left(\frac{3\pi}{4} + \frac{\pi}{6}\right)$

$\cos\frac{3\pi}{4}\cos\frac{\pi}{6} - \sin\frac{3\pi}{4}\sin\frac{\pi}{6}$

$-\frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \cdot \frac{1}{2}$

$-\frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}$

$\frac{-\sqrt{6}-\sqrt{2}}{4}$

6. $\dfrac{\tan \frac{\pi}{9} + \tan \frac{5\pi}{36}}{1 - \tan \frac{\pi}{9} \tan \frac{5\pi}{36}}$

$\tan\left(\frac{\pi(4)}{9(4)} + \frac{5\pi}{36}\right)$

$\tan \frac{9\pi}{36}$

$\tan \frac{\pi}{4} = \frac{\sin\frac{\pi}{4}}{\cos\frac{\pi}{4}} = \frac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = 1$

Simplify each expression.

7. $\cos 70^\circ \cos 20^\circ - \sin 70^\circ \sin 20^\circ$

Write each trigonometric expression as an algebraic expression.

9. $\cos(\arcsin x + \arccos x)$

12. Verify $\sin(270^\circ + \theta) = -\cos\theta$.

Sum and Difference Identities

Solve Trigonometric Equations You can solve trigonometric equations using the sum and difference identities along with algebraic methods and the same techniques you used before.

Example **Find the solutions of $\sin\left(\frac{\pi}{2} + x\right) + \cos\left(\frac{\pi}{2} + x\right) = 0$ on the interval $[0, 2\pi)$.**

$\sin\left(\frac{\pi}{2} + x\right) + \cos\left(\frac{\pi}{2} + x\right) = 0$	Original equation
$\sin\frac{\pi}{2}\cos x + \cos\frac{\pi}{2}\sin x + \cos\frac{\pi}{2}\cos x - \sin\frac{\pi}{2}\sin x = 0$	Cosine Sum Identity
$1(\cos x) + 0(\sin x) + 0(\cos x) - 1(\sin x) = 0$	Substitute.
$\cos x - \sin x = 0$	Simplify.
$\cos x = \sin x$	Add.

On the interval $[0, 2\pi)$, $\cos x = \sin x$ when $x = \frac{\pi}{4}$ and $x = \frac{5\pi}{4}$.

Find the solution of each equation on the interval $[0, 2\pi)$.

1. $\cos\left(\frac{\pi}{4} - x\right) - \sin\left(\frac{\pi}{4} - x\right) = -1$

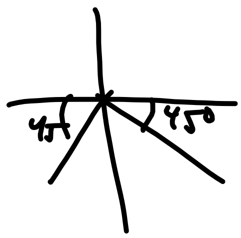
$\cos\frac{\pi}{4}\cos x + \sin\frac{\pi}{4}\sin x - \left(\sin\frac{\pi}{4}\cos x - \cos\frac{\pi}{4}\sin x\right) = -1$

$\frac{\sqrt{2}}{2}\cos x + \frac{\sqrt{2}}{2}\sin x - \frac{\sqrt{2}}{2}\cos x + \frac{\sqrt{2}}{2}\sin x = -1$

$\frac{\sqrt{2}\sin x}{\sqrt{2}} = \frac{-1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$

$\sin x = -\frac{\sqrt{2}}{2}$

$x = 315^\circ$ and 225°



4. $\tan(\pi - x) + \tan(\pi - x) = -2$

5. $\sin\left(x + \frac{\pi}{3}\right) + \sin\left(x - \frac{\pi}{3}\right) = 1$

HW: p. 341
1, 3, 5, 11-21 odd, 33, 35, 39