LESSON 5-5

Multiple-Angle and Product-to-Sum Identities

Then

You used sum and difference identities. (Lesson 5-4)

Now

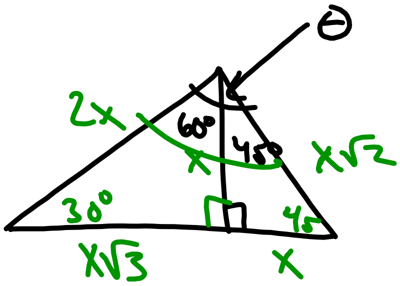
- Use double-angle, power-reducing, and half-angle identities to evaluate trigonometric expressions and solve trigonometric equations.

77. $d(x) = 2x^4 - x^3 - 6x^2 + 5x - 1$

$$\frac{\text{Factors of } -1}{\text{Factors of } 2} = \frac{\pm 1}{\pm 1, \pm 2} = \pm 1 \text{ or } \pm \frac{1}{2}$$

$\frac{1}{2}$

81.



$$\sin\theta = \sin(60+45)$$
$$= \sin 60 \cos 45 + \sin 45 \cos 60$$
$$= \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{1}{2}$$
$$= \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4}$$
$$= \frac{\sqrt{6}+\sqrt{2}}{4}$$

0-4 +
5-8 ✓
9↑ -

5-5 Study Guide and Intervention

Multiple-Angle and Product-to-Sum Identities

Use Multiple-Angle Identities By letting α and β both equal θ in each of the angle sum identities you have learned before, you can derive the following double-angle identities. The double-angle identities can then be used to derive the power-reducing identities.

KeyConcept Double-Angle Identities

$\sin 2\theta = 2 \sin \theta \cos \theta$

$\cos 2\theta = \cos^2 \theta - \sin^2 \theta$

$\cos 2\theta = 2 \cos^2 \theta - 1$

$\tan 2\theta = \dfrac{2 \tan \theta}{1 - \tan^2 \theta}$

$\cos 2\theta = 1 - 2 \sin^2 \theta$

Proof Double-Angle Identity for Sine

$\sin 2\theta = \sin(\theta + \theta)$	$2\theta = \theta + \theta$
$= \sin \theta \cos \theta + \cos \theta \sin \theta$	Sine Sum Identity where $\alpha = \beta = \theta$
$= 2 \sin \theta \cos \theta$	Simplify.

KeyConcept Power-Reducing Identities

$$\sin^2\theta = \frac{1-\cos 2\theta}{2} \qquad \cos^2\theta = \frac{1+\cos 2\theta}{2} \qquad \tan^2\theta = \frac{1-\cos 2\theta}{1+\cos 2\theta}$$

Proof Power-Reducing Identity for Sine

$\frac{1-\cos 2\theta}{2} = \frac{1-(1-2\sin^2\theta)}{2}$	Cosine Double-Angle Identity
$= \frac{2\sin^2\theta}{2}$	Subtract.
$= \sin^2\theta$	Simplify.

KeyConcept Half-Angle Identities

$$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}}$$

$$\cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}}$$

$$\tan\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$$

$$\tan\frac{\theta}{2} = \frac{1-\cos\theta}{\sin\theta}$$

$$\tan\frac{\theta}{2} = \frac{\sin\theta}{1+\cos\theta}$$

Proof Half-Angle Identity for Cosine

$\pm\sqrt{\frac{1+\cos\theta}{2}} = \pm\sqrt{\frac{1+\cos\left(2\cdot\frac{\theta}{2}\right)}{2}}$	Rewrite θ as $2\cdot\frac{\theta}{2}$.
$= \pm\sqrt{\frac{1+\cos 2x}{2}}$	Substitute $x = \frac{\theta}{2}$.
$= \pm\sqrt{\cos^2 x}$	Cosine Power-Reducing Identity
$= \cos x$	Simplify.
$= \cos\frac{\theta}{2}$	Substitute.

Example **If $\sin\theta = \frac{1}{4}$ on the interval [0, 90°], find the exact value of $\sin 2\theta$.**

To use the double-angle identity for $\sin 2\theta$, we must first find $\cos\theta$.

$\sin^2\theta + \cos^2\theta = 1$	Pythagorean Identity
$\left(\frac{1}{4}\right)^2 + \cos^2\theta = 1$	Substitute $\frac{1}{4}$ for $\sin\theta$.
$\cos^2\theta = \frac{15}{16}$	Simplify.
$\cos\theta = \frac{\sqrt{15}}{4}$	Solve.

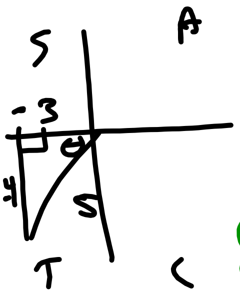
Now find $\sin 2\theta$.

$\sin 2\theta = 2\sin\theta\cos\theta$	Sine Double-Angle Identity
$= 2\left(\frac{1}{4}\right)\left(\frac{\sqrt{15}}{4}\right)$	$\sin\theta = \frac{1}{4}$, $\cos\theta = \frac{\sqrt{15}}{4}$
$= \frac{\sqrt{15}}{8}$	Simplify.

Q3

1. If $\tan\theta = \frac{4}{3}$ (opp/adj) on the interval $\left(\pi, \frac{3\pi}{2}\right)$, find $\sin 2\theta$, $\cos 2\theta$, and $\tan 2\theta$.

$\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta} = \dfrac{2\left(\frac{4}{3}\right)}{1-\left(\frac{4}{3}\right)^2} = \dfrac{\frac{8}{3}}{\frac{(9)}{(9)}1-\frac{16}{9}} = \dfrac{\frac{8}{3}}{-\frac{7}{9}} = \frac{8}{3}\cdot -\frac{9}{7} = \boxed{-\frac{24}{7}}$



$\sin 2\theta = 2\sin\theta\cos\theta$
$= 2\left(-\frac{4}{5}\right)\left(-\frac{3}{5}\right)$
$= \frac{24}{25}$

$\cos 2\theta = 2\cos^2\theta - 1$
$= 2\left(-\frac{3}{5}\right)^2 - 1$
$= 2\left(\frac{9}{25}\right) - 1$
$= \frac{18}{25} - 1$
$= -\frac{7}{25}$

Find the exact value of each expression.

2. $\sin 22.5^\circ$

$\sin \frac{45^\circ}{2} = \pm\sqrt{\dfrac{1-\cos\theta}{2}}$
$= \pm\sqrt{\dfrac{1-\cos 45^\circ}{2}}$
$= \pm\sqrt{\dfrac{\frac{(2)}{(2)}1-\frac{\sqrt{2}}{2}}{2}}$
$= \pm\sqrt{\dfrac{\frac{2-\sqrt{2}}{2}}{2}}$
$\pm\sqrt{\left(\frac{2-\sqrt{2}}{2}\right)\cdot\frac{1}{2}} = \pm\sqrt{\frac{2-\sqrt{2}}{4}} = \pm\frac{\sqrt{2-\sqrt{2}}}{2}$

3. $\cos \frac{11\pi}{12}$

4. Solve $\cos\theta \sin 2\theta = 0$ on the interval $[0, 2\pi)$.

$\cos\theta = 0$

$\theta = \frac{\pi}{2}, \frac{3\pi}{2}$

$\sin 2\theta = 0$

$2\sin\theta\cos\theta = 0$

$\frac{2\sin\theta}{2} = \frac{0}{2}$ $\qquad \cos\theta = 0$

$\sin\theta = 0$

$\boxed{\theta = 0, \pi}$

HW: p. 352
1, 5, 7, 9, 11, 13, 25, 27, 29, 33, 35