6-2 Matrix Multiplication and Inverses

① must be in the form of $m \times r$ and $r \times n$

② the size of the product is $m \times n$

③ order does matter

KeyConcept Matrix Multiplication

Words If A is an $m \times r$ matrix and B is an $r \times n$ matrix, then the product AB is an $m \times n$ matrix obtained by adding the products of the entries of a row in A to the corresponding entries of a column in B.

Symbols If A is an $m \times r$ matrix and B is an $r \times n$ matrix, then the product AB is an $m \times n$ matrix in which

$$c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + \cdots + a_{ir}b_{rj}.$$

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1r} \\ a_{21} & a_{22} & \cdots & a_{2r} \\ \vdots & \vdots & & \vdots \\ a_{i1} & a_{i2} & \cdots & a_{ir} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mr} \end{bmatrix} \cdot \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1j} & \cdots & b_{1n} \\ b_{21} & b_{22} & \cdots & b_{2j} & \cdots & b_{2n} \\ \vdots & \vdots & & \vdots & & \vdots \\ b_{r1} & b_{r2} & \cdots & b_{rj} & \cdots & b_{rn} \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} & \cdots & c_{1j} & \cdots & c_{1n} \\ c_{21} & c_{22} & \cdots & c_{2j} & \cdots & c_{2n} \\ \vdots & \vdots & & \vdots & & \\ c_{i1} & c_{i2} & \cdots & c_{ij} & \cdots & c_{in} \\ \vdots & \vdots & & \vdots & & \\ c_{m1} & c_{m2} & \cdots & c_{mj} & \cdots & c_{mn} \end{bmatrix}$$

EXAMPLE 1 **Multiply Matrices**

$3\times\boxed{2\quad 2}\times 3$

$$\begin{bmatrix} 4 & 5 \\ 1 & 9 \\ -6 & -3 \end{bmatrix} \bullet \begin{bmatrix} 7 & 0 & 1 \\ 2 & 6 & 8 \end{bmatrix}$$

$$\begin{bmatrix} (4)(7)+(5)(2) & (4)(0)+(5)(6) & (4)(1)+(5)(8) \\ (1)(7)+(9)(2) & (1)(0)+(9)(6) & (1)(1)+(9)(8) \\ (-6)(7)+(-3)(2) & (-6)(0)+(-3)(6) & (-6)(1)+(-3)(8) \end{bmatrix}$$

$$\begin{bmatrix} 38 & 30 & 44 \\ 25 & 54 & 73 \\ -48 & -18 & -30 \end{bmatrix}$$

$$\begin{bmatrix} 7 & 0 & 1 \\ 2 & 6 & 8 \end{bmatrix} \cdot \begin{bmatrix} 4 & 5 \\ 1 & 9 \\ -6 & -3 \end{bmatrix}$$

KeyConcept Properties of Matrix Multiplication

For any matrices A, B, and C for which the matrix product is defined and any scalar k, the following properties are true.

Associative Property of Matrix Multiplication	$(AB)C = A(BC)$
Associative Property of Scalar Multiplication	$k(AB) = (kA)B = A(kB)$
Left Distributive Property	$C(A + B) = CA + CB$
Right Distributive Property	$(A + B)C = AC + BC$

FOOTBALL The number of touchdowns (TD), field goals (FG), points after touchdown (PAT), and two-point conversions (2EP) for the three top teams in the high school league for this season is shown in the table below. The other table shows the number of points each type of score is worth. Use the information to determine the team that scored the most points.

Score	Points
TD	6
FG	3
PAT	1
2EP	2

Team	TD	FG	PAT	2EP
Tigers	27	7	21	2
Rams	24	12	18	3
Eagles	21	14	12	9

Let matrix X represent the Team/Score matrix, and let matrix Y represent the Score/Points matrix. Then find the product XY.

$$X = \begin{bmatrix} 27 & 7 & 21 & 2 \\ 24 & 12 & 18 & 3 \\ 21 & 14 & 12 & 9 \end{bmatrix}; Y = \begin{bmatrix} 6 \\ 3 \\ 1 \\ 2 \end{bmatrix}$$

$3 \times \boxed{4 \quad 4} \times 1$

$$\begin{bmatrix} (27)(6) + 7(3) + (21)(1) + 2(2) \\ (24)(6) + (12)(3) + (18)(1) + (3)(2) \\ (21)(6) + (14)(3) + (12)(1) + (9)(2) \end{bmatrix} = \begin{bmatrix} 208 \\ 204 \\ 198 \end{bmatrix}$$

Identity Matrix: Matrix with 1's on the diagonal, and zeros for all other entries.

Ex:

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

KeyConcept Inverse of a Square Matrix

Let A be an $n \times n$ matrix. If there exists a matrix B such that $AB = BA = I_n$, then B is called the **inverse** of A and is written as A^{-1}. So, $AA^{-1} = A^{-1}A = I_n$.

Determine whether $A = \begin{bmatrix} 5 & 6 \\ 4 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 5 & -6 \\ -4 & 5 \end{bmatrix}$ are inverse matrices.

If A and B are inverse matrices, then $AB = BA = I$.

$$AB = \begin{bmatrix} 5 & 6 \\ 4 & 5 \end{bmatrix} \bullet \begin{bmatrix} 5 & -6 \\ -4 & 5 \end{bmatrix} = \begin{bmatrix} (5)(5)+(6)(-4) & (5)(-6)+(6)(5) \\ (4)(5)+(5)(-4) & (4)(-6)+(5)(5) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$BA = \begin{bmatrix} 5 & -6 \\ -4 & 5 \end{bmatrix} \bullet \begin{bmatrix} 5 & 6 \\ 4 & 5 \end{bmatrix} = \begin{bmatrix} (5)(5)+(-6)(4) & (5)(6)+(-6)(5) \\ (-4)(5)+(5)(4) & (-4)(6)+(5)(5) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Yes A + B are inverse matrices

HW: p. 383,
1, 3, 5, 9, 19, 21, 57, 59