7-1 Study Guide and Intervention
Parabolas

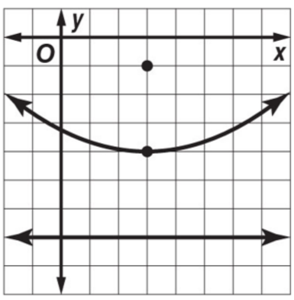
Analyze and Graph Parabolas A **parabola** is the locus of all points in a plane equidistant from a point called the **focus** and a line called the **directrix**. The standard form of the equation of a parabola that opens vertically is $(x - h)^2 = 4p(y - k)$. When p is negative, the parabola opens downward. When p is positive, it opens upward. The standard form of the equation of a parabola that opens horizontally is $(y - k)^2 = 4p(x - h)$. When p is negative, the parabola opens to the left. When p is positive, it opens to the right.

Example: For $(x - 3)^2 = 12(y + 4)$, identify the vertex, focus, axis of symmetry, and directrix. Then graph the parabola.

The equation is in standard form and the squared term is x, which means that the parabola opens vertically. Because $4p = 12$, $p = 3$ and the graph opens upward.

The equation is in the form $(x - h)^2 = 4p\,(y - k)$, so $h = 3$ and $k = -4$. Use the values of h, k, and p to determine the characteristics of the parabola.

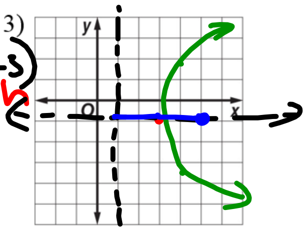
vertex: (3, –4)	(h, k)	directrix:	$y = -7$	$y = k - p$
focus: (3, –1)	$(h, k + p)$	axis of symmetry:	$x = 3$	$x = h$



Exercises

For each equation, identify the vertex, focus, axis of symmetry, and directrix. Then graph the parabola.

1. $(y + 1)^2 = 8(x - 3)$



$(y+1)^2 = 4(2)(x-3)$

k, p, h

vertex: (h, k)

$(3, -1)$

focus: $(h+p, k)$

$(3+2, -1)$

$(5, -1)$

axis sym: $y = k$

$y = -1$

directrix: $x = h - p$

$x = 3 - 2$

$x = 1$

x	y
4	1.8, -3.8

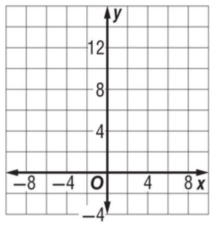
$(y+1)^2 = 8(4-3)$

$\sqrt{(y+1)^2} = \sqrt{8}$

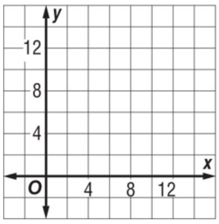
$y + 1 = \pm 2.8$

$-1 \quad -1$

2. $(x + 2)^2 = 4(y - 1)$



3. $(y-3)^2 = 2(x-6)$

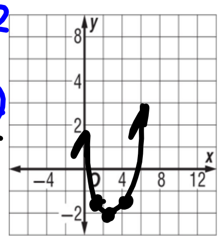


x	y
2	-1.9
4	-1.9

4. $\cancel{12}\left(\frac{1}{12}(x-3)^2\right) = (y+2)\,12$

$(\underset{h}{x-3})^2 = 12(\underset{k}{y+2})$

$(x-3)^2 = 4(3)(y+2)$



vertex: (h,k)

$(3,-2)$

focus: $(h,k+p)$

$(3,-2+3)$

$(3,1)$

axis of sym: $x=h$

$x=3$

directrix: $y=k-p$

$y=-2-3$

$y=-5$

Completing the Square Problems

Write x^2 – 8x – y = –18 in standard form. ~~Identify the vertex, focus, axis of symmetry, and directrix.~~

$$x^2 - 8x - y = -18$$
$$\underline{\qquad\quad +y \quad +y}$$
$$x^2 - 8x + 16 = y - 18 + 16$$
$$(x-4)^2 = 1(y-2)$$
$$4(\tfrac{1}{4})$$

$$-8 \div 2 = \left(\boxed{-4}\right)^2 = 16$$

Hw: p. 428
1, 3, 5, 7, 11, 13, 15, 21

B. Write an equation for and graph a parabola with vertex (3, –2) and directrix $y = -1$.

The directrix is a horizontal line, so the parabola opens vertically. Because the directrix lies above the vertex, the parabola opens down.

Use the equation of the directrix to find p.

$y = k - p$	Equation of directrix
$-1 = -2 - p$	$y = -1$, $k = -2$
$1 = -p$	Add 2 to each side.
$-1 = p$	Multiply each side by –1.

Substitute the values for h, k, and p into the standard form equation for a parabola opening vertically.

$4p(y - k) = (x - h)^2$	Standard form
$4(-1)[y - (-2)] = (x - 3)^2$	$p = -1$, $h = 3$, and $k = -2$
$-4(y + 2) = (x - 3)^2$	Simplify.

The equation for the parabola is $(x - 3)^2 = -4(y + 2)$.

Use a table of values to graph the parabola.

Example: Write an equation for and graph a parabola with focus (–4, –3) and vertex (1, –3).

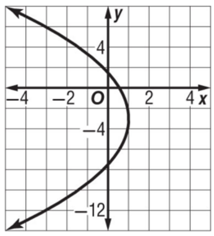
Because the focus and vertex share the same y-coordinate, the graph is horizontal. The focus is $(h + p, k)$, so the value of p is $-4 - 1$ or -5. Because p is negative, the graph opens to the left.

Write the equation for the parabola in standard form using the values of h, p, and k.

$(y - k)^2 = 4p(x - h)$	Standard form
$[y - (-3)]^2 = 4(-5)(x - 1)$	$p = -5$, $h = 1$, and $k = -3$
$(y + 3)^2 = -20(x - 1)$	Simplify.

The standard form of the equation is $(y + 3)^2 = -20(x - 1)$.

Graph the vertex, focus, and parabola.



13. $y^2 - 180x + 10y + 565 = 0$

$+180x \qquad -565 \quad -565$

a)

$y^2 + 10y + 25 = 180x - 565 + 25$ $\qquad 10 \div 2 = (\boxed{5})^2 = 25$

$(y+5)^2 = 180x - 540$

$(y+5)^2 = \underset{=4(45)}{180}(x-3)$

b) vertex: (h, k)

$(3, -5)$

focus: $(h+p, k)$

$(3+\underline{45}, -5$

45 ft

0 - 3+

4 - 5 ✓

6↑ -

C. Write an equation for and graph a parabola that has focus (–1, 7), opens up, and contains (3, 7).

$(x-h)^2 = 4p(y-k)$

focus $= (h, k+p) = (-1, 7)$

vertex $= (h, k)$

$k + p = 7$

$-k \qquad -k$

$p = \boxed{7-k}$

$p = 7-5$

$p = 2 \qquad p = -2$

x y

$(3+1)^2 = 4p(7-k)$

$\frac{16}{4} = \frac{4p(7-k)}{4}$

$4 = p(7-k)$

$\sqrt{4} = \sqrt{(7-k)^2}$

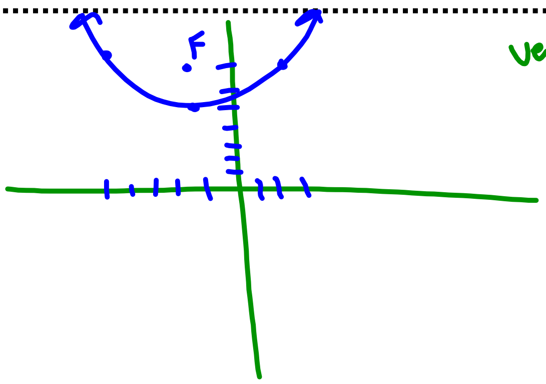
$\pm 2 = 7-k$

$-7 \qquad -7$

$\frac{-5}{-1} = \frac{-k}{-1} \qquad k = 5 \qquad -k = -9, \; k = 9$

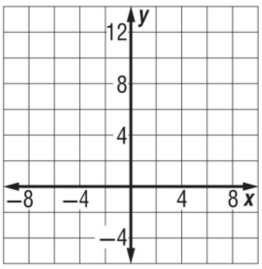
vertex: $(-1, 5)$ or $\cancel{(-1, 9)}$

$p = 2 \qquad \cancel{p = -2}$

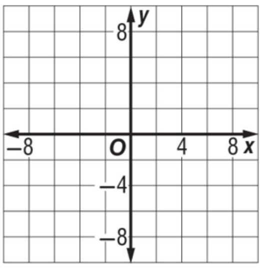


Write an equation for and graph a parabola with the given characteristics.

1. focus (–1, 5) and vertex (2, 5)

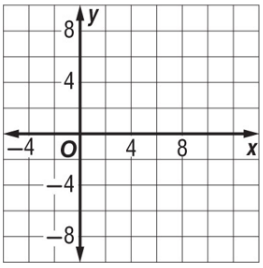


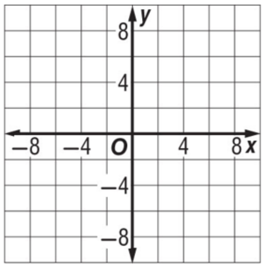
2. focus (1, 4) ; opens down; contains (–3, 1)



3. directrix $y = 6$; opens down; vertex (5, 3)

4. focus (1.5, 1); opens right; directrix $x = 0.5$





$(x-h)^2 = 4p(y-k)$

vertex: (h, k)

$h = 5 \quad k = 3$

directrix: $k - p = y$

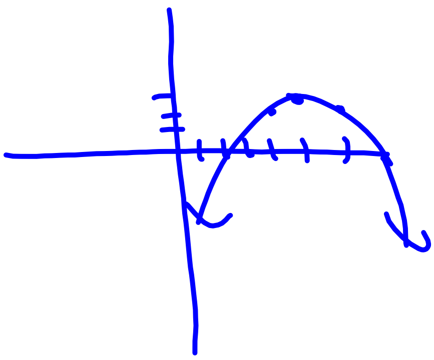
$3 - p = 6$

$-3 \qquad -3$

$\frac{-p}{-1} = \frac{3}{-1} \quad p = -3$

$(x-5)^2 = 4(-3)(y-3)$

$(x-5)^2 = -12(y-3)$



x	y
4	2.92
6	2.92

$(6-5)^2 = -12(y-3)$

$y = 2.92$

HW: p. 428

27, 33, 37, 39, 41