39. $F=(-4,0)$ $(\underset{x}{4},\underset{y}{-15})$

$(x-h)^2=4p(y-k)$

Focus: $(h, k+p) = (-4, 0)$

$h=-4$

$k+p=0$

$-p \quad -p$

$k=\boxed{-p}$

$k=-(-1)$

$k=1$

$(4--4)^2=4p(-15-k)$

$\frac{64}{4}=\frac{4p(-15-k)}{4}$

$16=p(-15-k)$

$16=p(-15+(+p))$

$16=-15p+p^2$

$-16 \qquad -16$

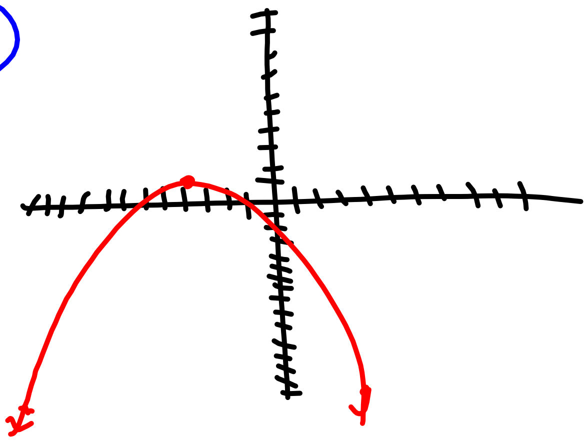
$0=p^2-15p-16$

$0=(p-16)(p+1)$

$\cancel{p=16}$ $p=-1$

$(x+4)^2=4(-1)(y-1)$

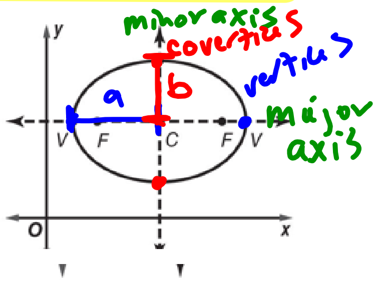
$(x+4)^2=-4(y-1)$



7-2 Study Guide and Intervention

Ellipses and Circles

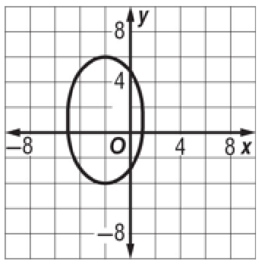
Analyze and Graph Ellipses and Circles An **ellipse** is the locus of points in a plane such that the sum of the distances from two fixed points, called **foci,** is constant. The standard form of the equation of an **ellipse** is $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$ when the **major axis** is horizontal. In this case, a^2 is in the denominator of the x-term. The standard form is $\frac{(y-k)^2}{a^2} + \frac{(x-h)^2}{b^2} = 1$ when the major axis is vertical. In this case, a^2 is in the denominator of the y-term. In both cases, $c^2 = a^2 - b^2$.



Example: Graph the ellipse given by the equation $\frac{(y-1)^2}{25} + \frac{(x+2)^2}{9} = 1$.

The equation is in standard form. Use the values of h, k, a, and b to determine the vertices and axes of the ellipse. Since $a^2 > b^2$, $a^2 = 25$ and $b^2 = 9$, or $a = 5$ and $b = 3$. Since a^2 is the denominator of the y-term, the major axis is parallel to the y-axis.

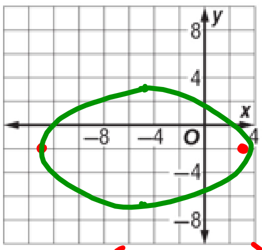
orientation:	vertical	
center:	$(-2, 1)$	(h, k)
vertices:	$(-2, 6)$, $(-2, -4)$	$(h, k \pm a)$
co-vertices:	$(-5, 1)$, $(1, 1)$	$(h \pm b, k)$
major axis:	$x = -2$	$x = h$
minor axis:	$y = 1$	$y = k$



Exercises

Graph the ellipse given by each equation.

1. $\frac{(x+5)^2}{64 = a^2} + \frac{(y+2)^2}{25 = b^2} = 1$



vertices: $(h \pm a, k)$

$= (-5 \pm 8, -2)$

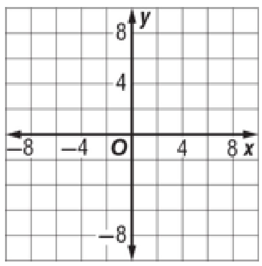
$= (3, -2)\ (-13, -2)$

covertices: $(h, k \pm b)$

$(-5, -2 \pm 5)$

$(-5, 3)\ (-5, -7)$

2. $\frac{(x+2)^2}{25} + \frac{(y+1)^2}{9} = 1$



<u>Eccentricity</u>: value, e, that tells us how circular an ellipse is.

$$e = \frac{c}{a}$$

Ex: Find the eccentricity of the previous ellipse.

$e = \frac{\sqrt{39}}{8}$

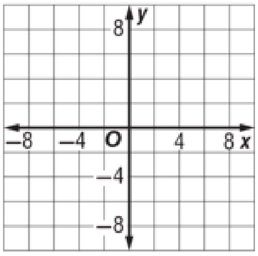
$e = .78$

$c = \sqrt{a^2 - b^2}$

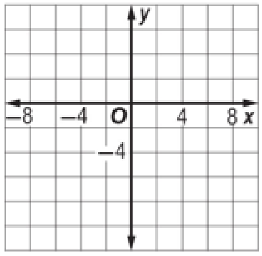
$= \sqrt{64 - 25}$

$= \sqrt{39}$

3. $\frac{(y-1)^2}{16} + \frac{(x+3)^2}{9} = 1$



4. $\frac{(y+3)^2}{64} + \frac{(x-2)^2}{25} = 1$



A. Write an equation for an ellipse with a major axis from (5, –2) to (–1, –2) and a minor axis from (2, 0) to (2, –4).

Use the major and minor axes to determine *a* and *b*.

Half the length of major axis

$$a = \frac{5-(-1)}{2} \text{ or } 3$$

Half the length of minor axis

$$b = \frac{0-(-4)}{2} \text{ or } 2$$

The center of the ellipse is at the midpoint of the major axis.

$$(h,k) = \left(\frac{5+(-1)}{2}, \frac{-2+(-2)}{2}\right)$$ Midpoint formula

$= (2, -2)$ Simplify.

The *y*-coordinates are the same for both endpoints of the major axis, so the major axis is horizontal and the value of *a* belongs with the x^2-term. An equation for the ellipse is $\frac{(x-2)^2}{9} + \frac{(y+2)^2}{4} = 1$.

B. Write an equation for an ellipse with vertices at (3, –4) and (3, 6) and foci at (3, 4) and (3, –2).

$$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$$

$h = 3$

$k = \frac{-4+6}{2} = 1$

vertices: $(h, k \pm a) = (3, 6)$

$k + a = 6$

$1 + a = 6$

$a = 5$

$a^2 - b^2 = c^2$

$25 - b^2 = 9$

$-25 \quad -25$

$-b^2 = -16$

$\frac{-b^2}{-1} = \frac{-16}{-1}$

$b^2 = 16 \quad b = 4$

Foci: $(h, k \pm c) = (3, 4)$

$k + c = 4$

$1 + c = 4$

$c = 3$

Answer: $\frac{(y-1)^2}{25} + \frac{(x-3)^2}{16} = 1$

$$\frac{(x-3)^2}{16} - \frac{(y-1)^2}{25} = 1$$

HW: p. 438

1, 3, 5, 7, 11, 15, 19

3. $x^2+9y^2-14x+36y+49=0$

$\qquad\qquad\qquad\qquad\quad -49 \quad -49$

$-14\div 2 = (-7)^2 = 49$

$(x^2-14x+49)+(9y^2+36y) = -49+49$

$(x^2-14x+49)+9(y^2+4y+4) = -49+49+4(9)$ $\quad 4\div 2 = (2)^2 = 4$

$$\frac{(x-7)^2}{36}+\frac{\cancel{9}(y+2)^2}{\cancel{36}}=\frac{36}{36}$$

$$\frac{(x-7)^2}{36=a^2}+\frac{(y+2)^2}{4=b^2}=1$$

vertices: $(h\pm a, k)$

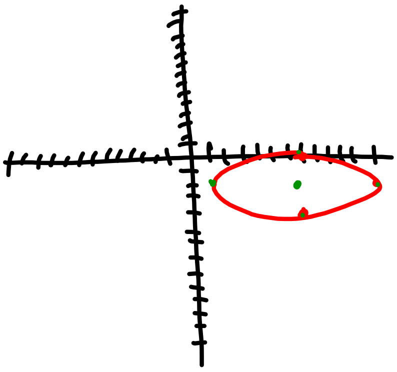
: $(7\pm 6, -2)$

= $(13,-2)$ $(1,-2)$

covertices: $(h, k\pm b)$

: $(7, -2\pm 2)$

: $(7,0)$ $(7,-4)$



11. Foci $(-6,9)$ and $(-6,-3)$

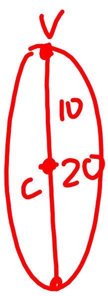
length of major axis $= 20$

$h=-6$

$k=\frac{9+-3}{2}=3$

$a=10$

$$\frac{(x-h)^2}{b^2}+\frac{(y-k)^2}{a^2}=1$$



Foci: $(h, k\pm c) = (-6, 9)$

$k+c=9$

$3+c=9$

$c=6$

$c^2=a^2-b^2$

$36=100-b^2$

$8=b$

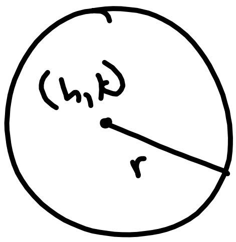
$$\boxed{\frac{(x+6)^2}{64}+\frac{(y-3)^2}{100}=1}$$

0-2+

3-4✓

5↑-

Determine Types of Conic Sections If you are given the equation for a conic section, you can determine what type of conic is represented using the characteristics of the equation. The standard form of an equation for a circle with center (h, k) and radius r is $(x-h)^2+(y-k)^2=r^2$.



Example: Write each equation in standard form. Identify the related conic.

a. $\mathbf{4x^2 + 9y^2 + 24x - 36y + 36 = 0}$

$4x^2 + 9y^2 + 24x - 36y + 36 = 0$	Original equation
$4(x^2 + 6x + ?) + 9(y^2 - 4y + ?) = -36 + ? + ?$	Complete the square.
$4(x^2 + 6x + 9) + 9(y^2 - 4y + 4) = -36 + 36 + 36$	$\left(\frac{6}{2}\right)^2 = 9$, $\left(-\frac{4}{2}\right)^2 = 4$
$4(x+3)^2 + 9(y-2)^2 = 36$	Factor.
$\frac{(x+3)^2}{9} + \frac{(y-2)^2}{4} = 1$	Divide each side by 36.

Write each equation in standard form. Identify the related conic.

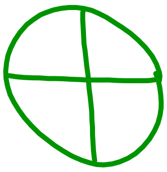
1. $y^2 + 2y + 6x^2 - 24x = 5$

2. $y^2 + 2y + x^2 - 24x = 14$

Write each equation in standard form. Identify the related conic.

4. $(x^2 + 4x) + (y^2 - 2y) - 49 = 0$

$+49 \quad +49$



$4 \div 2 = (2)^2 = 4$

$-2 \div 2 = (-1)^2 = 1$

$(x^2 + 4x + 4) + (y^2 - 2y + 1) = 49 + 4 + 1$

$$\frac{(x+2)^2}{54} + \frac{(y-1)^2}{54} = \frac{54}{54}$$

circle

5. $4x^2 + 8x + 5y^2 - 30y - 11 = 0$ $+11$ $+11$

6. $6x^2 + 24x + 2y - 10 = 0$

$(4x^2+8x\quad)+(5y^2-30y\quad) = 11 + 1(4) + 9(5)$

$4(x^2+2x+1)+5(y^2-6y+9)$

$2 \div 2 = (1)^2 = 1$

$-6 \div 2 = (-3)^2 = 9$

$$\frac{4(x+1)^2}{60} + \frac{5(y-3)^2}{60} = \frac{60}{60}$$

$$\frac{(x+1)^2}{15} + \frac{(y-3)^2}{12} = 1$$ ellipse

Hw: p. 438
25, 27, 29, 31, 33, 39, 41, 49