7-3 Study Guide and Intervention

Hyperbolas

Analyze and Graph Hyperbolas A **hyperbola** is the locus of all points in a plane such that the difference of their distances from two foci is constant. The standard form of the equation of a **hyperbola** is

$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$ when the **transverse axis** is horizontal, and

$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$ when the transverse axis is vertical. In both cases, $a^2 + b^2 = c^2$.

Example: Graph the hyperbola given by the equation $\frac{y^2}{16} - \frac{x^2}{4} = 1$.

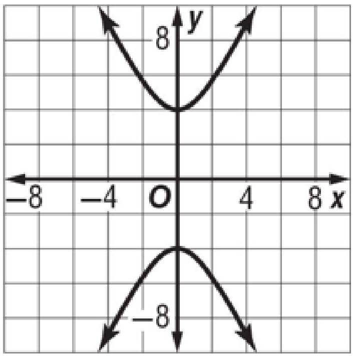
The equation is in standard form. Both h and k are 0, so the center is at the origin. Because the x-term is subtracted, the transverse axis is vertical. Use the values of a, b, and c to determine the vertices and foci of the hyperbola.

Because $a^2 = 16$ and $b^2 = 4$, $a = 4$ and $b = 2$. Use the values of a and b to find the value of c.

$c^2 = a^2 + b^2$	Equation relating a, b, and c
$c^2 = 4^2 + 2^2$	$a = 4$ and $b = 2$
$c = \sqrt{20}$ or about 4.47	Simplify.

Determine the characteristics of the hyperbola.

center: $(0, 0)$	(h, k)	foci:	$(0, \sqrt{20})$, $(0, -\sqrt{20})$	$(h, k \pm c)$
vertices: $(0, 4)$, $(0, -4)$	$(h, k \pm a)$	asymptotes:	$y = 2x$, $y = -2x$	$y - k = \pm \frac{a}{b}(x - h)$



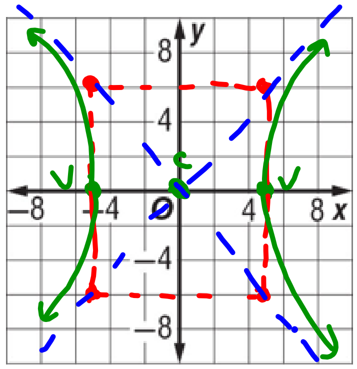
Graph the hyperbola given by each equation.

Center: (0,0)

1. $\frac{x^2}{25} - \frac{y^2}{36} = 1$

$a^2 = 25$ $36 = b^2$

$a = 5$ $6 = b$

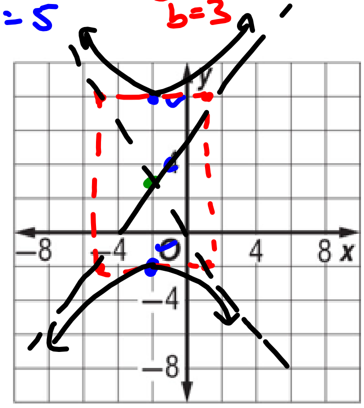


Center: (-2, 3)

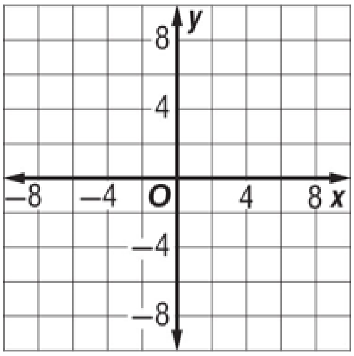
2. $\frac{(\overset{k}{y - 3})^2}{25} - \frac{(\overset{h}{x + 2})^2}{9} = 1$

$a^2 = 25$ $b^2 = 9$

$a = 5$ $b = 3$



3. $\frac{(x-1)^2}{16} - \frac{(y+2)^2}{36} = 1$



Graph the hyperbola given by $4x^2 - y^2 + 24x + 4y = 28$.

$(4x^2 + 24x \quad) + (-y^2 + 4y \quad) = 28$

$4(x^2 + 6x + 9 \;) - (y^2 - 4y \; + 4 \;) = 28 + 9(4) + 4(-1)$

$6 \div 2 = \boxed{3}^2 = 9 \qquad -4 \div 2 = \boxed{-2}^2 = 4$

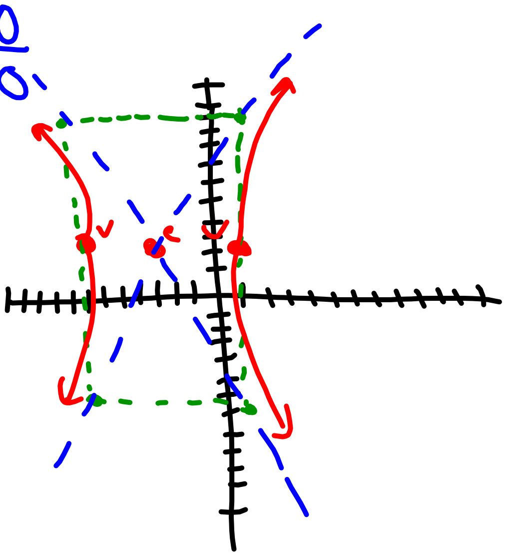
$$\frac{4(x+3)^2}{60} - \frac{(y-2)^2}{60} = \frac{60}{60}$$

$$\frac{(x+\overset{h}{3})^2}{15} - \frac{(y-\overset{k}{2})^2}{60} = 1$$

$a^2 = 15$ $\quad b^2 = 60$

$a = 3.87$ $\quad b = 7.75$

Center: (-3, 2)



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1, 3, 5, 9, 13, 15, 17, 21, 23, 27, 30

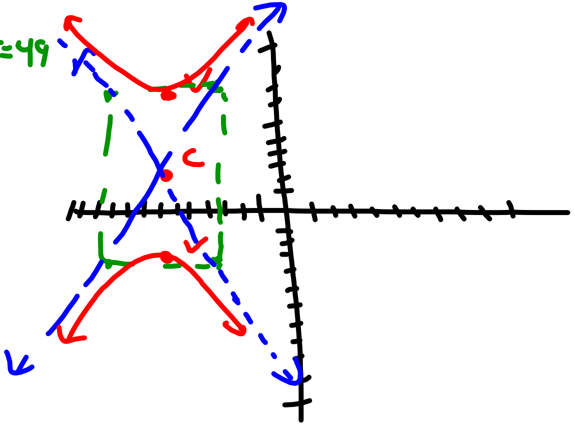
21.

$-5x^2 + 2y^2 - 70x - 8y = 287$

$(2y^2 - 8y) + (-5x^2 - 70x) = 287$

$2(y^2 - 4y + 4) - 5(x^2 + 14x) = 287 + 4(2) + 49(-5)$

$-4 \div 2 = (\boxed{-2})^2 = 4$ $\quad 14 \div 2 = (\boxed{7})^2 = 49$



$\frac{2(y-2)^2}{50} - \frac{5(x+7)^2}{50} = \frac{50}{50}$

$\frac{(y-\overset{k}{2})^2}{} - \frac{(x+\overset{h}{7})^2}{} = 1$

$a^2 = 25$ $\quad b^2 = 10$

$a = 5$ $\quad b = 3.16$

23. foci $(-1, 9)$ $(-1, -7)$ conjugate axis length = 14

$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$

$b = 7$

$c = 8$

$h = -1$ $\quad k = \frac{9 + -7}{2}$

$k = 1$

$a^2 + b^2 = c^2$

$a^2 + 7^2 = 8^2$

$a = \sqrt{15}$

center: $(-1, 1)$

$\frac{(y-1)^2}{15} - \frac{(x+1)^2}{49} = 1$

0 - 3+

4 - 5 ✓

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A. Write an equation for the hyperbola with foci (1, –5) and (1, 1) and transverse axis length of 4 units

$h = 1 \quad k = \frac{-5+1}{2} = -2$

center: $(1, -2)$

$\frac{(y+2)^2}{4} - \frac{(x-1)^2}{5} = 1$

$a = 2$

$c = 3$

$a^2 + b^2 = c^2$

$2^2 + b^2 = 3^2$

$b^2 = 5$

$b = \sqrt{5}$

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$\frac{(y+2)^2}{4} - \frac{(x-1)^2}{5} = 1$

B. Write an equation for the hyperbola with vertices (–3, 10) and (–3, –2) and conjugate axis length of 6 units.

$$\frac{(y-4)^2}{36}-\frac{(x+3)^2}{9}=1$$