KeyConcept Parametric Equations

If f and g are continuous functions of t on the interval I, then the set of ordered pairs $(f(t), g(t))$ represent a **parametric curve**. The equations

$$x = f(t) \text{ and } y = g(t)$$

are parametric equations for this curve, t is the parameter, and I is the parameter interval.

7-5 Study Guide and Intervention

Parametric Equations

Graph Parametric Equations **Parametric equations** are used to describe the horizontal and vertical components of an equation. **Parameters** are arbitrary values, usually time or angle measurement.

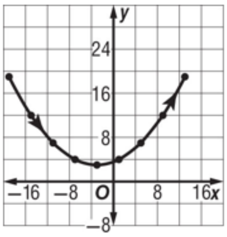
Example 1: Sketch the curve given by the parametric equations $x = -3 + 4t$ and $y = t^2 + 3$ over the interval $-4 \le t \le 4$.

Make a table of values for $-4 \le t \le 4$.

t	x	y
–4	–19	19
–3	–15	12
–2	–11	7
–1	–7	4
0	–3	3

t	x	y
0	–3	3
1	1	4
2	5	7
3	9	12
4	13	19

Plot the (x, y) coordinates for each t–value and connect the points to form a smooth curve.



Example 2: Write $x = 4t - 2$ and $y = t^2 + 1$ in rectangular form.

$x = 4t - 2$	Parametric equation for x
$\frac{x+2}{4} = t$	Solve for t.
$y = \left(\frac{x+2}{4}\right)^2 + 1$	Substitute $\frac{x+2}{4}$ for t in the equation for y.
$= \frac{x^2+4x+4}{16} + 1$	Square $x + 2 - 4$.
$= \frac{x^2}{16} + \frac{x}{4} + \frac{5}{4}$	Simplify.

The rectangular equation is $y = \frac{x^2}{16} + \frac{x}{4} + \frac{5}{4}$.

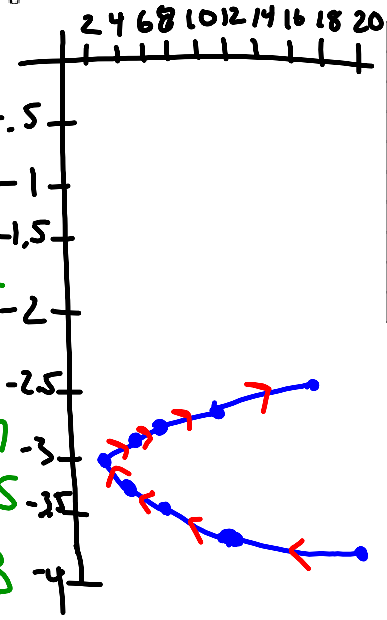
Exercises

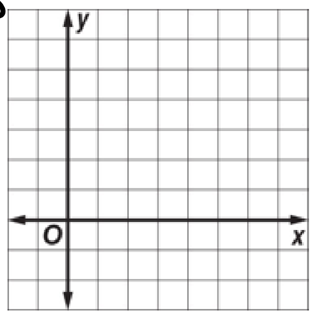
Sketch the curve given by each pair of parametric equations over the given interval.

1. $x = t^2 + 4$ and $y = \frac{t}{6} - 3$; $-4 \le t \le 4$

2. $x = \frac{t}{3}$ and $y = \sqrt{t} + 2$; $0 \le t \le 8$

t	x	y
-4	20	-3.7
-3	13	-3.5
-2	8	-3.3
-1	5	-3.2
0	4	-3
1	5	-2.8
2	8	-2.7
3	13	-2.5
4	20	-2.3





What is the rectangular form of the parametric equations? * substitution method for t

$x = t^2 + 4$

$y = \frac{t}{6} - 3$

$+3 \qquad +3$

$6(y+3) = \left(\frac{t}{6}\right)6$

$t = \boxed{6y + 18}$

$x = (6y+18)^2 + 4$

$x = (6y+18)(6y+18) + 4$

$x = 36y^2 + 216y + 328$

$-328 \qquad -328$

$x - 328 = 36y^2 + 216y$

$(36 \cdot 9) + x - 328 = 36(y^2 + 6y + 9)$

$6 \div 2 = \boxed{3}^2 = 9$

$1(x-4) = 36(y+3)^2$

$\frac{1(x-4)}{36} = \frac{36(y+3)^2}{36}$

$\boxed{\frac{1}{36}(x-4) = (y+3)^2}$

Write $y = 5 \sin \theta$ and $x = 3 \cos \theta$ in rectangular form. Then graph the equation.

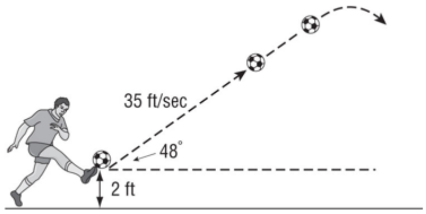
* solve for each trig function, then use the pythagorean identity

Hw: p. 469
3, 7, 11, 13, 19, 23, 33, 45-48

Projectile Motion Parametric equations are often used to simulate projectile motion. For an object launched at an angle θ with the horizontal at an initial velocity v_0, where g is the gravitational constant, t is time, and h_0 is the initial height, the horizontal distance x can be found by $x = tv_0 \cos\theta$ and the vertical position y by $y = tv_0 \sin\theta - \frac{1}{2}gt^2 + h_0$.

Example: Luigi is kicking a soccer ball. He kicks the ball with an initial velocity of 35 feet per second at an angle of 48° with the horizontal. The ball is 2 feet above the ground when he kicks it. How far will the ball travel horizontally before it hits the ground?

Step 1 Make a diagram of the situation.



Step 2 Write a parametric equation for the vertical position of the ball.

$y = tv_0 \sin\theta - \frac{1}{2}gt^2 + h_0$ — Parametric equation for vertical position

$= t(35)\sin(48) - \frac{1}{2}(32)t^2 + 2$ — $v_0 = 35$, $\theta = 48°$, $g = 32$, and $h_0 = 2$

Step 3 Graph the equation for the vertical position and the line $y = 0$. Use **5: INTERSECT** on the **CALC** menu of a calculator to find the point of intersection of the curve with $y = 0$. The value is about 1.7 seconds. You could also use **2: ZERO** and not graph $y = 0$.

Step 4 Determine the horizontal position of the ball at 1.7 seconds.

$x = tv_0 \cos\theta$ — Parametric equation for horizontal position

$= 1.7(35)\cos 48$ — $v_0 = 35$, $\theta = 48°$, and $t = 1.7$

≈ 39.8 — Use a calculator.

The ball will travel about 39.8 feet before it hits the ground.

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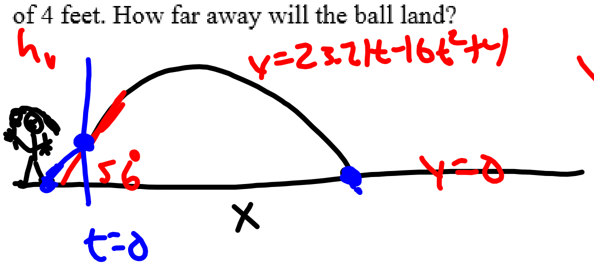
$x = (1.61)(28)\cos 56°$

$x = 25.2$ ft

Exercises

v_0

1. Julie is throwing a ball at an initial velocity of 28 feet per second and an angle of 56° with the horizontal from a height of 4 feet. How far away will the ball land?



$y = tv_0 \sin\theta - \frac{1}{2}gt^2 + h_0$

$y = t(28)\sin(56°) - \frac{1}{2}(32)t^2 + 4$

$y = 23.21t - 16t^2 + 4$

$t = -\cancel{.15}$ $\quad$ $t = 1.61$

2. Jerome hits a tennis ball at an initial velocity of 38 feet per second and an angle of 42° with the horizontal from a height of 1.5 feet. How far away will the ball land if it is not hit by his opponent?

0-3 +
4-5 ✓
6↑ -