8-1 Study Guide and Intervention

Introduction to Vectors

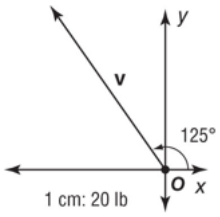
Geometric Vectors A vector is a quantity that has both magnitude and direction. The magnitude of a vector is the length of a directed line segment, and the direction of a vector is the directed angle between the positive x-axis and the vector. When adding or subtracting vectors, you can use the parallelogram or triangle method to find the resultant.

*Scalar quantities only have magnitude.

Example: Use a ruler and a protractor to draw an arrow diagram for each quantity described. Include a scale on each diagram.

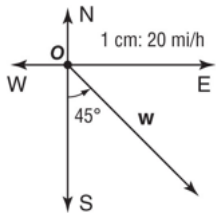
a. v = 60 pounds of force at 125° to the horizontal

Using a scale of 1 cm: 20 lb, draw and label a 60 ÷ 20 or 3-centimeter arrow in standard position at a 125° angle to the x-axis.



b. w = 55 miles per hour at a bearing of S45°E

Using a scale of 1 cm.: 20 mi/h, draw and label a 55 ÷ 20 or 2.75-centimeter arrow 45° east of south.



A. State whether a hockey puck shot northwest at 60 miles per hour is a *vector* quantity or a *scalar* quantity.

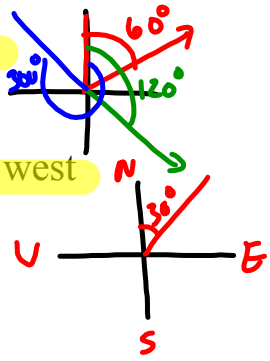
vector quantity

EXAMPLE 1 Identify Vector Quantities

B. State whether a tennis ball served at 110 miles per hour is a *vector* quantity or a *scalar* quantity.

Scalar quantity

Heading: Directional measurement where the angle is always measure clockwise from due north.



Bearing: Directional measurement between 0° and 90° east or west of the north-south line.

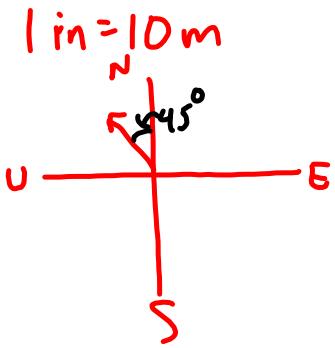
N 30° E

Measurement to the horizontal: standard "protractor" method of drawing an angle.

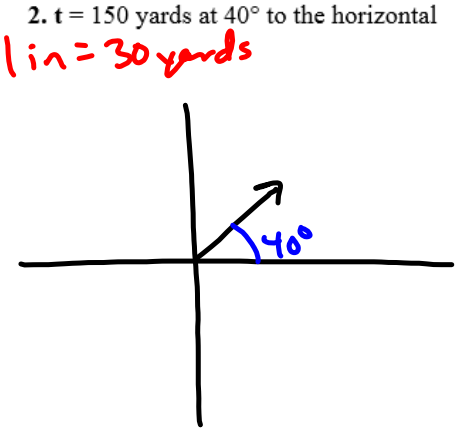
Exercises

Use a ruler and a protractor to draw an arrow diagram for each quantity described. Include a scale on each diagram.

1. $\mathbf{r}$ = 30 meters at a bearing of N45°W



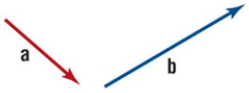
2. $\mathbf{t}$ = 150 yards at 40° to the horizontal



KeyConcept Finding Resultants

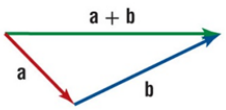
Triangle Method (Tip-to-Tail)

To find the resultant of **a** and **b**, follow these steps.



Step 1 Translate **b** so that the tail of **b** touches the tip of **a**.

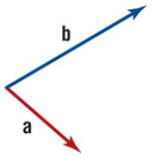
Step 2 The resultant is the vector from the tail of **a** to the tip of **b**.



Parallelogram Method (Tail-to-Tail)

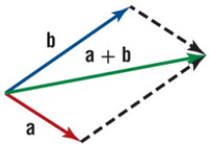
To find the resultant of **a** and **b**, follow these steps.

Step 1 Translate **b** so that the tail of **b** touches the tail of **a**.



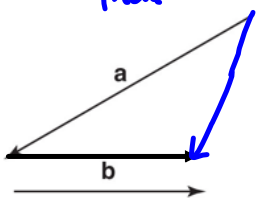
Step 2 Complete the parallelogram that has **a** and **b** as two of its sides.

Step 3 The resultant is the vector that forms the indicated diagonal of the parallelogram.



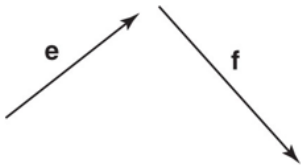
Find the resultant of each pair of vectors using either the triangle or parallelogram method. State the magnitude of the resultant in ~~centimeters~~ inches and its direction relative to the horizontal.

3.

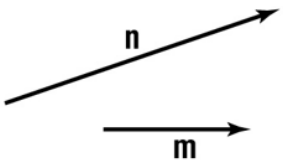


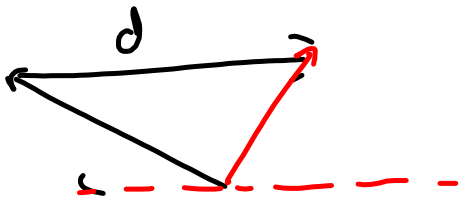
8 inches at 240°

4.

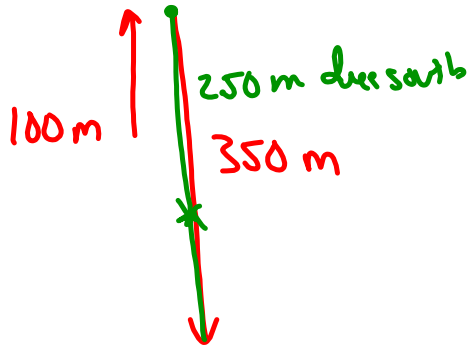


Draw a vector diagram of n + 2m.



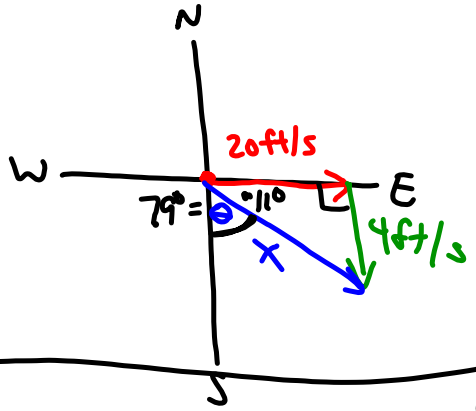


23.



0-3 +
4-5 V
6↑ −

ROWING Jamie rows her boat due east at a speed of 20 feet per second across a river directly toward the opposite bank. At the same time, the current of the river is carrying her due south at a rate of 4 feet per second. Find Jamie's speed and direction relative to the shore.



$20^2+4^2=x^2$

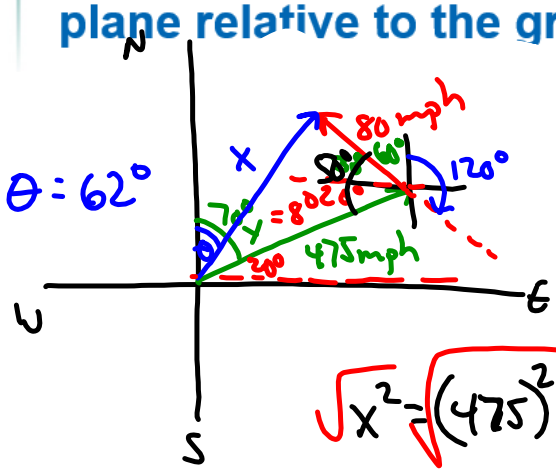
$x=20.4$ ft/s

$\tan^{-1}\left(\frac{4}{20}\right)=11°$

Jamies speed is 20.4 ft/s bearing S 79° E

Real-World EXAMPLE 5 Use Vectors to Solve Navigation Problems

AVIATION An airplane is flying with an airspeed of 475 miles per hour on a heading of 70°. If an 80-mile-per-hour wind is blowing from a true heading of 120°, determine the velocity and direction of the plane relative to the ground.



$\sqrt{x^2} = \sqrt{(475)^2 + (80)^2 - 2(475)(80)\cos 50°}$

$x = 428$ mph

$\frac{\sin 50}{428} = \frac{\sin y}{80}$

$80 \sin 50 = 428 \sin y$

$\frac{80 \sin 50}{428} = \frac{428 \sin y}{428}$

$\sin^{-1}(.14) = \sin^{-1}(\sin y)$

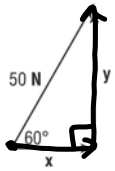
$y = 8°$

The plane travel 428 mph heading 62°

Vector Applications Vectors can be resolved into horizontal and vertical components.

Example: Suppose Jamal pulls on the ends of a rope tied to a dinghy with a force of 50 Newtons at an angle of 60° with the horizontal.

a. Draw a diagram that shows the resolution of the force Jamal exerts into its rectangular components.



Jamal's pull can be resolved into a horizontal pull **x** forward and a vertical pull **y** upward as shown.

b. Find the magnitudes of the horizontal and vertical components of the force.

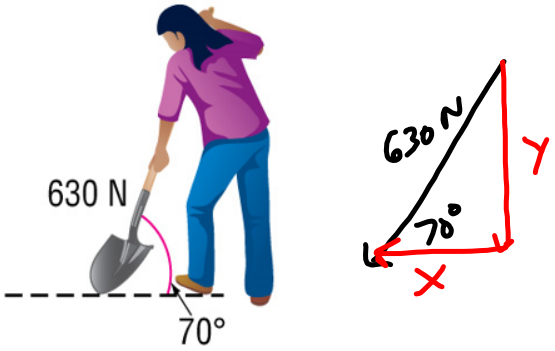
The horizontal and vertical components of the force form a right triangle. Use the sine or cosine ratios to find the magnitude of each force.

$\cos 60° = \frac{\lvert x \rvert}{50}$	Right triangle definitions of cosine and sine	$\sin 60° = \frac{\lvert y \rvert}{50}$
$\lvert x \rvert = 50 \cos 60°$	Solve for x and y.	$\lvert y \rvert = 50 \sin 60°$
$\lvert x \rvert = 25$	Use a calculator.	$\lvert y \rvert \approx 43.3$

The magnitude of the horizontal component is about 25 Newtons, and the magnitude of the vertical component is about 43 Newtons.

Real-World EXAMPLE 6 Resolve a Force into Rectangular Components

A. GARDENING While digging in his garden, Will pushes a shovel into the ground with a force of 630 newtons at an angle of 70° with the ground. Draw a diagram that shows the resolution of the force that Will exerts into its rectangular components.

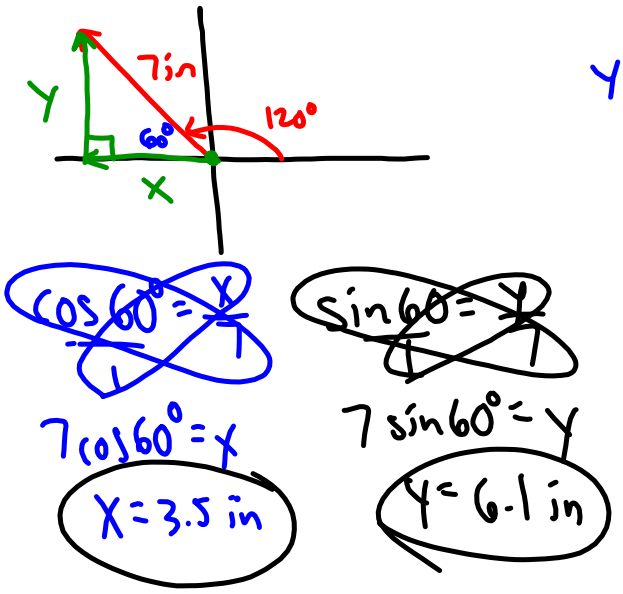


Answer: horizontal component ≈ 215.47 N; vertical component ≈ 592.01 N

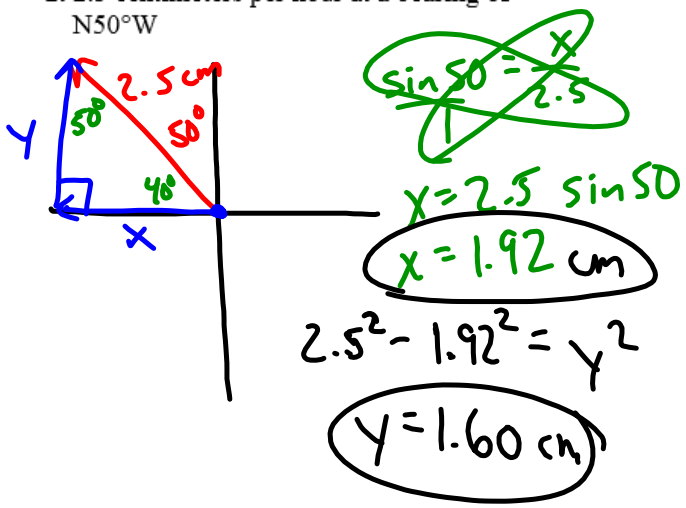
Exercises

Draw a diagram that shows the resolution of each vector into its rectangular components. Then find the magnitudes of the vector's horizontal and vertical components.

1. 7 inches at a ~~bearing~~ of 120° from the horizontal



2. 2.5 centimeters per hour at a bearing of N50°W



3. **YARDWORK** Nadia is pulling a tarp along level ground with a force of 25 pounds directed along the tarp. If the tarp makes an angle of 50° with the ground, find the horizontal and vertical components of the force. What is the magnitude and direction of the resultant?

4. **TRANSPORTATION** A helicopter is moving 15° north of east with a velocity of 52 km/h. If a 30-kilometer per hour wind is blowing from a bearing of 250°, find the helicopter's resulting velocity and direction.