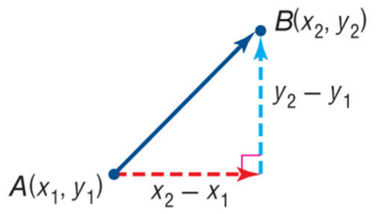
KeyConcept Component Form of a Vector

The component form of a vector $\overrightarrow{AB}$ with initial point $A(x_1, y_1)$ and terminal point $B(x_2, y_2)$ is given by

$$\langle x_2 - x_1, y_2 - y_1 \rangle.$$



EXAMPLE 1 Express a Vector in Component Form

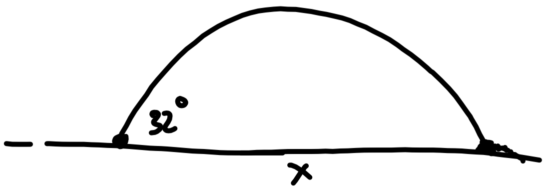
Find the component form of $\overrightarrow{AB}$ with initial point $A(1, –3)$ and terminal point $B(1, 3)$.

$\overrightarrow{AB}$	$= \langle x_2 - x_1, y_2 - y_1 \rangle$	Component form
	$= \langle 1 - 1, 3 - (-3) \rangle$	$(x_1, y_1) = (1, -3)$ and $(x_2, y_2) = (1, 3)$
	$= \langle 0, 6 \rangle$	Subtract.

EXAMPLE 1 Guided Practice

Find the component form of $\overrightarrow{AB}$ given initial point $A(–4, –3)$ and terminal point $B(5, 3)$.

71.



$y = t v_0 \sin\theta - \frac{1}{2} g t^2 + h_0$

$y = t(20)\sin 32° - \frac{1}{2}(9.8)t^2$

$y = 10.6 - 4.9t^2$

$y = 0$

$t = 2.16$ sec.

$x = t v_0 \cos\theta$

$x = (2.16)(20)(\cos 32°)$

$x = 36.6$ m

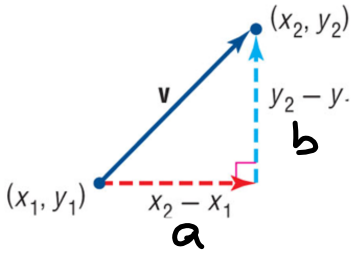
0-3 +
4-5 ✓
6 ↑ -

KeyConcept Magnitude of a Vector in the Coordinate Plane

If **v** is a vector with initial point (x_1, y_1) and terminal point (x_2, y_2), then the magnitude of **v** is given by

$$|\mathbf{v}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

If **v** has a component form of $\langle a, b\rangle$, then $|\mathbf{v}| = \sqrt{a^2 + b^2}$.



KeyConcept Vector Operations

If $\mathbf{a} = \langle a_1, a_2\rangle$ and $\mathbf{b} = \langle b_1, b_2\rangle$ are vectors and k is a scalar, then the following are true.

Vector Addition	$\mathbf{a} + \mathbf{b} = \langle a_1 + b_1, a_2 + b_2\rangle$
Vector Subtraction	$\mathbf{a} - \mathbf{b} = \langle a_1 - b_1, a_2 - b_2\rangle$
Scalar Multiplication	$k\mathbf{a} = \langle ka_1, ka_2\rangle$

Example 1: Find the magnitude of $\overrightarrow{XY}$ with initial point $X(2, -3)$ and terminal point $Y(-4, 2)$.

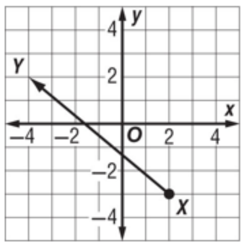
Determine the magnitude of $\overrightarrow{XY}$ using the Distance Formula.

$|\overrightarrow{XY}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(-4 - 2)^2 + [2 - (-3)]^2}$

$= \sqrt{(-6)^2 + 5^2}$

$= 61$ or about 7.8 units



Represent $\overrightarrow{XY}$ as an ordered pair.

$\overrightarrow{XY} = \langle x_2 - x_1, y_2 - y_1 \rangle$	Component form
$= \langle -4 - 2, 2 - (-3) \rangle$	$(x_1, y_1) = (2, -3)$ and $(x_2, y_2) = (-4, 2)$
$= \langle -6, 5 \rangle$	Subtract.

Example 2: Find each of the following for $\mathbf{s} = \langle 4, 2\rangle$ and $\mathbf{t} = \langle -1, 3\rangle$.

a. $\mathbf{s} + \mathbf{t}$

$\mathbf{s} + \mathbf{t} = \langle 4, 2\rangle + \langle -1, 3\rangle$ Substitute.

$= \langle 4 + (-1), 2 + 3\rangle$ or $\langle 3, 5\rangle$ Vector addition

b. $3\mathbf{s} + \mathbf{t}$

$3\mathbf{s} + \mathbf{t} = 3\langle 4, 2\rangle + \langle -1, 3\rangle$ Substitute.

$= \langle 12, 6\rangle + \langle -1, 3\rangle$ Scalar multiplication

$= \langle 11, 9\rangle$ Vector addition

Exercises

Find the component form and magnitude of the vector $\overrightarrow{AB}$ with the given initial and terminal points.

1. $A(12, 41), B(52, 33)$

$x_1 \quad y_1 \quad x_2 \quad y_2$

$\overrightarrow{AB} = \langle 52-12, 33-41 \rangle$

$= \langle 40, -8 \rangle$

$|AB| = \sqrt{(52-12)^2 + (33-41)^2}$

$= \sqrt{1600 + 64}$

$= 40.79$

Find each of the following for f = ⟨4, –2⟩, g = ⟨24, 21⟩, and h = ⟨–1, –3⟩.

5. 2g + h

$2\langle 24, 21 \rangle + \langle -1, -3 \rangle$

$\langle 48, 42 \rangle + \langle -1, -3 \rangle$

$\langle 47, 39 \rangle$

Unit Vectors A vector that has a magnitude of 1 unit is called a unit vector. A unit vector in the direction of the positive x-axis is denoted as $\mathbf{i} = \langle 1, 0\rangle$, and a unit vector in the direction of the positive y-axis is denoted as $\mathbf{j} = \langle 0, 1\rangle$. Vectors can be written as linear combinations of unit vectors by first writing the vector as an ordered pair and then writing it as a sum of the vectors **i** and **j**.

Example 1: Find a unit vector u with the same direction as v = ⟨–4, –1⟩.

$\mathbf{u} = \frac{1}{\lvert\mathbf{v}\rvert}\mathbf{v}$	Unit vector with the same direction as **v**
$= \frac{1}{\lvert\langle -4, -1\rangle\rvert}\langle -4, -1\rangle$	Substitute.
$= \frac{1}{\sqrt{(-4)^2 + (-1)^2}}\langle -4, -1\rangle$	$\lvert\langle a, b\rangle\rvert = \sqrt{a^2 + b^2}$
$= \frac{1}{\sqrt{17}}\langle -4, -1\rangle$	Simplify.
$= \langle \frac{-4}{\sqrt{17}}, \frac{-1}{\sqrt{17}}\rangle$ or $\langle \frac{-4\sqrt{17}}{17}, \frac{-\sqrt{17}}{17}\rangle$	Scalar multiplication

Example 2: Let $\overrightarrow{MP}$ be the vector with initial point $M(2, 2)$ and terminal point $P(5, 4)$. Write $\overrightarrow{MP}$ as a linear combination of the vectors i and j.

First, find the component form of $\overrightarrow{MP}$.

$\overrightarrow{MP} = \langle x_2 - x_1, y_2 - y_1\rangle$	Component form
$= \langle 5 - 2, 4 - 2\rangle$ or $\langle 3, 2\rangle$	$(x_1, y_1) = (2, 2)$ and $(x_2, y_2) = (5, 4)$

Then rewrite the vector as a linear combination of the standard unit vectors.

$\overrightarrow{MP} = \langle 3, 2\rangle$	Component form
$= 3\mathbf{i} + 2\mathbf{j}$	$\langle a, b\rangle = a\mathbf{i} + b\mathbf{j}$

EXAMPLE 4 Find a Unit Vector with the Same Direction as a Given Vector

Find a unit vector u with the same direction as v = ⟨4, –2⟩.

$\mathbf{u} = \frac{1}{|\mathbf{v}|}\mathbf{v}$ Unit vector with the same direction as **v**.

$|v| = \sqrt{(4)^2 + (-2)^2}$

$|v| \approx 4.5 = 2\sqrt{5}$

$u = \frac{1}{2\sqrt{5}}\langle \frac{4}{1}, -2 \rangle$

$u = \langle \frac{4}{2\sqrt{5}}, -\frac{2}{2\sqrt{5}} \rangle$

$u = \langle \frac{2}{\sqrt{5}}, -\frac{\sqrt{5}}{5} \rangle$

$u = \langle \frac{2\sqrt{5}}{5}, -\frac{\sqrt{5}}{5} \rangle = \frac{2\sqrt{5}}{5}\vec{i} - \frac{\sqrt{5}}{5}\vec{j}$

Exercises

Find a unit vector u with the same direction as the given vector.

1. $\mathbf{p} = \langle 4, -3 \rangle$

$\vec{u} = \frac{1}{|\vec{p}|}\vec{p}$

$|\vec{p}| = \sqrt{(4)^2 + (-3)^2} = 5$

$\frac{1}{5}\langle 4, -3 \rangle$

$\langle \frac{4}{5}, \frac{-3}{5} \rangle = \frac{4}{5}\vec{i} - \frac{3}{5}\vec{j}$

2. $\mathbf{w} = \langle 10, 25 \rangle$

Let $\overrightarrow{MN}$ be the vector with the given initial and terminal points. Write $\overrightarrow{MN}$ as a linear combination of the vectors i and j.

3. $M(2, 8), N(-5, -3)$

4. $M(0, 6), N(18, 4)$

EXAMPLE 6 Find Component Form

Find the component form of the vector v with magnitude 7 and direction angle 60°.

$\mathbf{v} = \langle \lvert \mathbf{v} \rvert \cos\theta,\ \lvert \mathbf{v} \rvert \sin\theta \rangle$	Component form of **v** in terms of |**v**| and θ
$= \langle 7\cos 60°,\ 7\sin 60° \rangle$	|**v**| = 7 and θ = 60°
$= \left\langle 7\left(\frac{1}{2}\right),\ 7\left(\frac{\sqrt{3}}{2}\right) \right\rangle$	$\cos 60° = \frac{1}{2}$ and $\sin 60° = \frac{\sqrt{3}}{2}$
$= \left\langle \frac{7}{2}, \frac{7\sqrt{3}}{2} \right\rangle$	Simplify.

EXAMPLE 7 Direction Angles of Vectors

A. Find the direction angle of p = ⟨2, 9⟩ to the nearest tenth of a degree.

$\tan\theta = \frac{b}{a}$	Direction angle equation
$\tan\theta = \frac{9}{2}$	$a = 2$ and $b = 9$
$\theta = \tan^{-1}\frac{9}{2}$	Solve for θ.
$\theta \approx 77.5°$	Use a calculator.

Find the component form of v with the given magnitude and direction angle.

5. $|\mathbf{v}| = 18$, $\theta = 240°$

$\langle 18 \cdot \cos 240°, 18 \sin 240° \rangle$

$\langle -9, -15.58 \rangle$

6. $|\mathbf{v}| = 5$, $\theta = 95°$

HW: p. 497,
1, 5, 11, 13, 21, 25, 29, 35,
39, 43, 45, 51

Find the direction angle of each vector to the nearest tenth.

7. $-4\mathbf{i} + 2\mathbf{j}$

8. $\langle 2, 17 \rangle$